# Common Mode Rejection Ratio in Differential Amplifiers

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# Common Mode Rejection Ratio in Differential Amplifiers

Ramón Pallás-Areny, *Senior Member, IEEE,* and John G. Webster, *Fellow, IEEE*

*Abstract*—**We have analyzed the common mode rejection ratio (CMRR) for a differential amplifier (DA) using a single operational amplifier and for an instrumentation amplifier (IA) using three operational amplifiers, and we have derived the complete equations for the case when op amps have finite differential and common mode gains. Amplitude and phase measurements support our theoretical predictions. We conclude that, at low frequencies, for the single-op-amp DA the use of a trimming potentiometer is better than relying on low-tolerance resistors, because of the higher CMRR achieved. The DA yields a fixed 90° phase shift for the CMRR at frequencies above 1 kHz, giving a clear advantage if synchronous demodulation is used for further processing of amplitude-modulated signals. For the three-op-amp IA, it is extremely important for input buffers to be "coupled" and to be built from a matched op amp pair. The best CMRR is obtained when the differential gain is concentrated in the input stage, but in any case it decreases at frequencies above 1 kHz because of the reduced CMRR for the differential stage at these frequencies.**

## I. Introduction

OP AMP-based differential amplifiers are a common building block in instrumentation circuits working at frequencies lower than 100 kHz. At higher frequencies, discrete transistors usually replace op amps, for example, in oscilloscopes. Differential amplifiers are valuable because of their ability to reject power-line and other common-mode interference which follows from their high common mode rejection ratio (CMRR). So far, however, no complete analysis of this important parameter has been published.

Szynowski [1] derived the CMRR for the classical three-op-amp IA using amplifiers with finite common-mode gain, but otherwise he assumed infinite differential gain for the input op-amp pair. White [2] considered the finite differential gain of input op amps, but was not concerned with the common-mode gain. Kirshner [3] studied the CMRR degradation due to imbalance of input op amps, but otherwise he did not consider the limited CMRR for op amps. Most published analyses are satisfactory at low frequencies, but are not valid above 1 kHz. Also, some oversimplifications when analyzing these circuits may lead to the wrong conclusions.

In this paper, we analyze the CMRR performance of a DA based on a single op amp and the classical three-op-amp IA, in the frequency band from 0.1 Hz to 25 kHz (determined by the range of our measurement instruments). First, a theoretical analysis shows what parameters determine the CMRR. Then we present experimental results that verify the theoretical models developed. We have paid particular attention to phase measurements, since they have emerged as being very important in determining whether or not we were obtaining the best CMRR a given circuit could yield.

## II. Theoretical CMRR Analysis

It has been shown elsewhere [1], [3], [4], [5], that the CMRR for the three-op-amp IA depends heavily on the input stage. Therefore, we first analyze the CMRR for the one-op-amp DA and then we consider the influence of the op-amp pair in the first stage.

### *A. Differential Amplifier Using a Single Operational Amplifier*

Fig. 1 shows the first circuit analyzed. It consists of a nonideal op amp and four resistors. The output voltage is

$$v_o = A_d(v_b' - v_a') + A_c(v_b' + v_a')/2 \qquad (1)$$

where $A_d$ is the differential mode gain and $A_c$ is the common mode gain of the op amp. The common mode rejection ratio for the op amp is $CMRR_{OA} = A_d/A_c$. For the circuit we take the usual definitions for the differential input signal, $v_D = v_b - v_a$, and for the common mode input signal, $v_C = (v_b + v_a)/2$. Thus the circuit output voltage is

$$v_o = G_d v_D + G_c v_C. \qquad (2)$$

It can be easily shown that the respective expressions for the differential gain, $G_d$, and common mode gain, $G_c$, for the circuit are

$$G_d = \frac{1}{2}\,\frac{\dfrac{R_4}{R_3 + R_4}(A_d - A_c/2) + \dfrac{R_6}{R_5 + R_6}(A_d + A_c/2)}{1 + (A_d - A_c/2)\dfrac{R_3}{R_3 + R_4}} \qquad (3)$$

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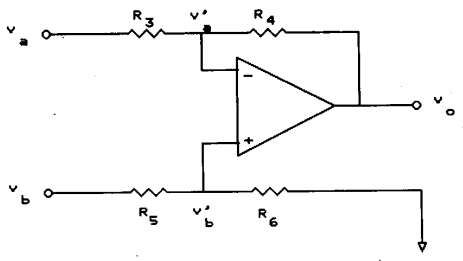


Fig. 1. Differential amplifier using a single op amp. The op amp is considered to have a limited common mode rejection ratio. The resistors can be high-precision components or ordinary components when including a potentiometer, in $R_6$, for example, to optimize the common mode rejection ratio.

$$G_c = \frac{\frac{R_4}{R_3 + R_4}(-A_d + A_c/2) + \frac{R_6}{R_5 + R_6}(A_d + A_c/2)}{1 + (A_d - A_c/2)\frac{R_3}{R_3 + R_4}}. \tag{4}$$

Therefore, the common mode rejection ratio for this differential stage, $CMRR_D$, is

$$CMRR_D = \frac{G_d}{G_c} = \frac{1}{2}\frac{A_d\left(\frac{R_6}{R_5 + R_6} + \frac{R_4}{R_3 + R_4}\right) + \frac{A_c}{2}\left(\frac{R_6}{R_5 + R_6} - \frac{R_4}{R_3 + R_4}\right)}{A_d\left(\frac{R_6}{R_5 + R_6} - \frac{R_4}{R_3 + R_4}\right) + \frac{A_c}{2}\left(\frac{R_6}{R_5 + R_6} + \frac{R_4}{R_3 + R_4}\right)}. \tag{5}$$

If the op amp considered had $A_c = 0$, then the only factor contributing to $CMRR_D$ would be mismatching of the resistors. We can thus define a common mode rejection ratio for the resistors, $CMRR_R$. By taking $A_c = 0$ in (5) we obtain

$$CMRR_R = \frac{1}{2}\frac{2R_4R_6 + R_4R_5 + R_3R_6}{R_3R_6 - R_4R_5}. \tag{6}$$

Equation (5) can then be written

$$CMRR_D = \frac{CMRR_R CMRR_{OA} + 1/4}{CMRR_R + CMRR_{OA}} \tag{7}$$

Whenever $CMRR_R CMRR_{OA} \gg 1/4$, (7) simplifies and leads to the following well-known rule [1]:

$$\frac{1}{CMRR_D} \approx \frac{1}{CMRR_R} + \frac{1}{CMRR_{OA}}. \tag{8}$$

An immediate conclusion is that in order for $CMRR_D$ to be high, both the resistors and the amplifier must have a high CMRR unless we are sure that they have different signs. Further, if we assume that the circuit is implemented using four resistors with the same tolerance $t$, from (6) we deduce that the worst-case condition will be when $R_3 = R_{30}(1 + t)$, $R_6 = R_{60}(1 + t)$, $R_4 = R_{40}(1 - t)$, $R_{50}(1 - t)$. $R_{30}$, $R_{40}$, $R_{50}$, and $R_{60}$ are the respective nominal values, whose ratio we call $k$,

$$k = \frac{R_{60}}{R_{50}} = \frac{R_{40}}{R_{30}} \tag{9}$$

Therefore, the minimal value for $CMRR_R$ is

$$CMRR_R = \frac{k(1 - t^2) + 1 + t^2}{4t} \approx \frac{k + 1}{4t}. \tag{10}$$

This means that when relying on high-precision resistors to obtain a high CMRR, we must consider not only $t$ but also the actual value for $k$. $CMRR_R$ is more sensitive to $t$ than to $k$, but because very small-tolerance resistors are expensive, a high value for $k$ is most desirable, provided that $A_d$ is large enough at the frequencies considered. For example, 0.1% resistors can result on a $CMRR_R$ of only 54 dB when $k = 1$ and 69 dB when $k = 10$. We may be tempted to design for example with $k = 100$ in order to have $CMRR_D > 89$ dB. But in addition to the restricted bandwidth for the differential signal, we must remember that $CMRR_D$ is also limited by the op amp (8), and at frequencies beyond 1 kHz many op amps have a $CMRR_{OA}$ well below 89 dB.

Because $CMRR_{OA}$ is very high at low frequencies and starts to decrease at frequencies between 100 Hz and 1 kHz, depending on the op-amp model, we can interpret (8) and (10) in the following way. In a worst-case condition ($CMRR_R$ and $CMRR_{OA}$ with the same sign), the frequency where $CMRR_D$ starts to decrease because of $CMRR_{OA}$ depends on $k$; a larger $k$ lowers that frequency. But also a larger $k$, as long as it does not exceed moderate values, yields a higher $CMRR_D$ at low frequencies. Therefore, for reasonable values for $t$ and $k$, the value for $CMRR_D$ will be determined at low frequencies by the matching of resistors and at high frequencies by the op amp.

A method to overcome the need for low-tolerance resistors is by using a potentiometer to trim the value for one of them, usually $R_6$. this also allows us to take advantage of a fact implicit in (8): if we can achieve $CMRR_R = -CMRR_{OA}$, then we have an infinite $CMRR_D$. In order to study this possibility, we must consider the frequency dependence of $CMRR_{OA}$, which according to op amp data sheets we can assume has the form

$$CMRR_{OA} = CMRR_{OA}(0)\frac{w_r}{s + w_r} = CMRR_{OA}(0)\frac{1}{1 + s/w_r} \tag{11}$$

where $w_r$ is the frequency where $CMRR_{OA}$ has decreased by 3 dB. Then (8) can be expressed as

$$\frac{1}{CMRR_D} \approx \frac{1}{CMRR_R} + \frac{1}{CMRR_{OA}(0)} + \frac{1}{CMRR_{OA}(0)} \frac{s}{w_r}. \tag{12}$$

Therefore, if instead of adjusting the potentiometer until $R_6/R_5 = R_4/R_3$, we adjust it until $CMRR_R = -CMRR_{OA}(0)$, we then will have

$$\frac{1}{CMRR_D} = \frac{1}{CMRR_{OA}(0)} \frac{s}{\omega_r}. \tag{13}$$

That is, we obtain an increased $CMRR_D$ at frequencies below $\omega_r$. (We assume that at frequencies higher than this, in (8) it is the op amp that determines $CMRR_D$, and we assume that the resistors have a low enough tolerance). Further, from (13) we deduce that the output voltage due to the common mode input will now have a 90° phase shift with respect to the input. Therefore, for the case of amplitude-modulated signals that are further processed by synchronous demodulators, the final contribution of input common mode signals to the in-phase input will be very low even if the resulting $CMRR_D$ is not very high. This is advantageous, for example, in ac-resistance measurements in the presence of stray capacitances and with common mode voltages. We notice that in order to have $CMRR_R = CMRR_{OA}$ we do not need low-tolerance resistors. But if there is any thermal drift, it must be the same percentage for all of them. Therefore, because temperature coefficients for potentiometers and resistors are, in general, different, the use of a single potentiometer instead of $R_6$ seems less appropriate than the use of a resistor in series with a potentiometer with a low nominal resistance value.

Another conclusion from (13) is that the use of a single potentiometer does not allow us to improve $CMRR_D$ at frequencies beyond $\omega_r$, where it remains limited by $CMRR_{OA}$.

### B. Three-Op-Amp Instrumentation Amplifier

The need for a high input impedance, for both differential and common mode, calls for improvements to the circuit shown in Fig. 1. The usual solution consists of adding a pair of "coupled" input buffers with gain, Fig. 2(a). We wish to know when and why the three-op-amp IA is better than just placing a pair of "noncoupled" buffers in front of a DA based on a single op amp as shown in Fig. 2(b). The reason for the possible difference between these two circuits stems in the particular way the cascaded differential stages combine their respective CMRRs that, as far as we know, has not yet been described in a complete way. This means that the classical three-op-amp IA should not be considered as a combination of a single-op-amp DA and two simple input buffers with gain. Rather, we must emphasize that the input buffers are in some way "coupled."

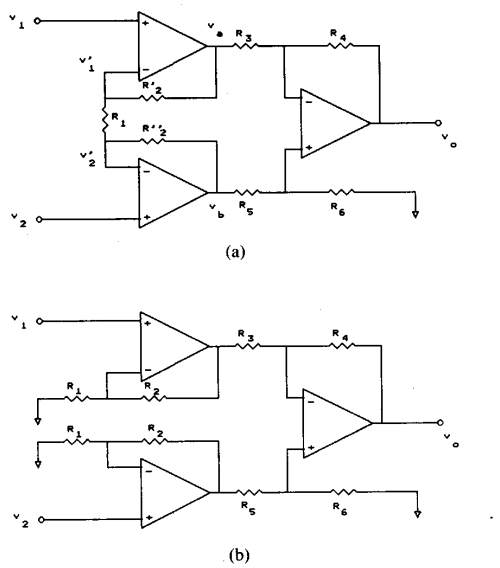


Fig. 2. (a) Three-op-amp instrumentation amplifier with two coupled input buffers. (b) Three-op-amp instrumentation amplifier with two independent (noncoupled) input buffers with gain.

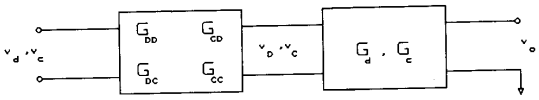

Fig. 3. Two-stage system with differential input and single-ended output.

Fig. 3 shows a two-stage differential system with single-ended output. The first stage has a differential output, and is best described by four transfer functions: $G_{DD}$, $G_{CC}$, $G_{CD}$, and $G_{DC}$ [6]. $G_{DD}$ is the differential mode gain: output differential mode signal/input differential mode signal, when there is no input common mode signal. $G_{CC}$ is the common mode gain: output common mode signal/input common mode signal, when there is no input differential mode signal. $G_{CD}$ is the differential to common mode gain, that is, the output common mode signal due to the input differential signal. $G_{DC}$ is the common mode to differential gain, that is, the output differential signal due to the input common mode signal. Therefore, we have

$$v_o = G_d v_D + G_c v_C \tag{14}$$

where $G_d$ and $G_c$ are, respectively, the differential and common mode gain for the second stage. By applying the preceding definitions we have

$$v_D = G_{DD} v_d + G_{DC} v_c \tag{15a}$$

$$v_C = G_{CD} v_d + G_{CC} v_c. \tag{15b}$$

The complete expression for the output voltage is then

$$v_o = v_d(G_d G_{DD} + G_c G_{CD}) + v_c(G_d G_{DC} + G_c G_{CC}). \tag{16}$$

The CMRR for the complete system, $CMRR_T$, is defined as the ratio between the output voltage due to an input differential mode signal and the output voltage due to an input common mode signal of equal amplitude [7]. That is,

$$CMRR_T = \frac{v_o/v_d \,|\, v_c = 0}{v_o/v_c \,|\, v_d = 0}. \tag{17}$$

This leads to

$$CMRR_T = \frac{G_d G_{DD} + G_c G_{CD}}{G_d G_{DC} + G_c G_{CC}} \tag{18}$$

which holds independently of the particular implementation for each stage. If the second stage were perfect from the point of view of differential measurements, that is if $G_c = 0$, then the CMRR for the system would be limited only by the first stage. We can thus define a CMRR for the first stage, $CMRR_F$,

$$CMRR_F = CMRR_T|_{G_c=0} = G_{DD}/G_{DC}. \tag{19}$$

In a similar way, if the first stage were perfect, then the CMRR would be limited only by the second stage. From the point of view of differential measurements, a stage with differential output can be considered as perfect when there is no conversion from common mode to differential mode, and no conversion from differential mode to common mode. This is the case when $G_{CD} = 0$. Then from (18) we obtain the common mode rejection ratio for the second stage,

$$CMRR_S = \frac{G_d}{G_c}\frac{G_{DD}}{G_{CC}}\bigg|_{\substack{G_{CD}=0\\G_{DC}=0}} = CMRR_D \frac{G_{DD}}{G_{CC}}\bigg|_{\substack{G_{CD}=0\\G_{DC}=0}} \tag{20}$$

where $CMRR_D = G_d/G_c$ is the CMRR for the second stage when considered alone and $G_{DD}/G_{CC}$ is the first-stage differential mode gain divided by its common mode gain. Therefore, a first conclusion from this analysis is that, in general, the CMRR for the second stage is not the same when considered alone or when it is included in a circuit. Only when $G_{DD} = G_{CC}$ will it be the same in both cases.

By substituting (20) and (19) in (18) we obtain

$$CMRR_T = \frac{CMRR_D G_{DD}/G_{CC} + G_{CD}/G_{CC}}{1 + CMRR_D G_{DD}/G_{CC} CMRR_F}. \tag{21}$$

From (21), when $G_{CD}/G_{DD} \ll CMRR_D$ we have the following rule for combining the common mode rejection ratio for cascaded stages:

$$\frac{1}{CMRR_T} \approx \frac{1}{CMRR_F} + \frac{1}{CMRR_S}. \tag{22}$$

The validity of this approximation can be judged only after analyzing the particular circuit being considered. In any case, (22) shows that $CMRR_T$ will not be better than the common mode rejection for a single stage unless we are sure that $CMRR_F$ and $CMRR_S$ have different signs. We must also remember that (20) says that the contribution of the second stage to the $CMRR_T$ is increased by a factor contributed by the first stage. Equation (22) suggests two different approaches to obtain a high value for $CMRR_T$. One is to have a high CMRR for each stage; the other is to make $CMRR_F = -CMRR_S$. Whether each of these approaches is possible or not depends on the particular circuit being considered.

Starting with the circuit in Fig. 2(b) if the respective transfer functions for the buffers are $H_1(s)$ and $H_2(s)$, then we have

$$v_o = \frac{v_d}{2}\{[H_2(s) + H_1(s)]G_d + [H_2(s) - H_1(s)]G_c/2\} + v_c\{[H_2(s) - H_1(s)]G_d + [H_2(s) + H_1(s)]G_c/2\}. \tag{23}$$

Therefore, we have for this circuit

$$G_{DD} = [H_1(s) + H_2(s)]/2 \tag{24a}$$

$$G_{CD} = [H_2(s) - H_1(s)]/4 \tag{24b}$$

$$G_{DC} = [H_2(s) - H_1(s)] \tag{24c}$$

$$G_{CC} = [H_1(s) + H_2(s)]/2. \tag{24d}$$

We observe that $G_{CC} = G_{DD}$ and by applying (20) we deduce that $CMRR_S = CMRR_D$. That is, even if the first stage were perfect, i.e., $H_1(s) = H_2(s)$, there would not be any improvement of the CMRR for this circuit as compared to the one-op-amp DA. For this circuit, (19) gives

$$CMRR_F = \frac{1}{2}\frac{H_1(s) + H_2(s)}{H_2(s) - H_1(s)}. \tag{25}$$

This confirms that a "perfect" input stage would imply $H_2(s) = H_1(s)$. Note that this conclusion holds independent of the gain for the input amplifiers. For unity gain buffers the condition $H_2(s) = H_1(s)$ depends only on the op amps, and, therefore, is feasible. For buffers with gain it depends on passive components, whose tolerances make it very difficult to achieve this condition, unless trimpots are added.

The analysis of the circuit in Fig. 2(a) when both input op amps have finite differential and common mode gains follows. The respective outputs for each input amplifier are

$$v_a = A_{d1}(v_1 - v_1') + A_{c1}(v_1 + v_1')/2 \tag{26a}$$

$$v_b = A_{d2}(v_2 - v_2') + A_{c2}(v_2 + v_2')/2. \tag{26b}$$

From the analysis of currents at inverting nodes of input op amps we have

$$v_a = v_1'(1 + R_2'/R_1) - v_2'R_2'/R_1 \tag{27a}$$

$$v_b = -v_1'R_2''/R_1 + v_2'(1 + R_2''/R_1). \tag{27b}$$

From these two equations we can obtain $v_1'$ and $v_2'$ and then substitute them in (26). From the resulting equations we can obtain $v_a$ and $v_b$, and from them $v_D$ $(= v_b - v_a)$ and $v_C(= (v_a + v_b)/2)$. The final results for the transfer

functions corresponding to the first stage are

$$2DG_{DD} = A_{d1} + A_{c1}/2 + A_{d2} + A_{c2}/2 + 2A_{d1}A_{d2} - A_{c1}A_{c2}/2 \tag{28a}$$

$$4DG_{CD} = A_{d2} - A_{d1} + A_{c2}/2 - A_{c1}/2 + \frac{A_{d1}A_{c2} - A_{d2}A_{c1} + (R_2''/R_1 - R_2'/R_1)(2A_{d1}A_{d2} - A_{c1}A_{c2}/2)}{1 + R_2'/R_1 + R_2''/R_1} \tag{28b}$$

$$DG_{DC} = A_{d2} + A_{c2}/2 - A_{d1} - A_{c1}/2 + A_{d1}A_{c2} - A_{d2}A_{c1} \tag{28c}$$

$$2DG_{CC} = A_{d1} + A_{c1}/2 + A_{d2} + A_{c2}/2 + \frac{2A_{d1}A_{d2} - A_{c1}A_{c2}/2 + (R_2''/R_1 - R_2'/R_1)(A_{d1}A_{c2} - A_{d2}A_{c1})}{1 + R_2'/R_1 + R_2''/R_1} \tag{28d}$$

where $D$ is the determinant of the resulting system of equations. Because we are interested in ratios between the transfer functions given by (28), it is not necessary to evaluate $D$. For the particular case where both input op amps are assumed to be identical and with zero common mode gain, (28) leads to the same results obtained by White [2].

By applying (19), we have for this circuit

$$CMRR_F = \frac{1}{2}\frac{A_{d1} + A_{c1}/2 + A_{d2} + A_{c2}/2 + 2A_{d1}A_{d2} - A_{c1}A_{c2}/2}{A_{d2} + A_{c2}/2 - A_{d1} - A_{c1}/2 + A_{d1}A_{c2} - A_{d2}A_{c1}}. \tag{29}$$

This shows that in order to achieve a high value for $CMRR_F$, input op amps must be matched not only for their CMRR [1], but also for both differential and common mode gains. In other words, it is not the ratio $A_d/A_c$ what should be matched, but the absolute values for $A_d$ and $A_c$. Furthermore, it is neither necessary for $A_d$ to be very high nor for $A_c$ to be very small. General purpose op amps are acceptable provided they are matched.

From (28a) and (28d) we deduce that for this circuit we will have, in general, $G_{DD}$ different from $G_{CC}$, as opposed to findings for the circuit in Fig. 2(b). Because we wish to have a large $CMRR_S$, we desire $G_{DD} >> G_{CC}$ when $G_{CD} = G_{DC} = 0$ (20). From (28b) and (28c) we deduce that these last conditions require that both input op amps be identical. In this particular case we would have $A_{d1} = A_{d2} = A_d$, $A_{c1} = A_{c2} = A_c$, and

$$\frac{G_{DD}}{G_{CC}} = \frac{2A_d + A_c + 2A_d^2 - A_c^2/2}{2A_d + A_c + (2A_d^2 - A_c^2/2)/(1 + R_2'/R_1 + R_2''/R_1)}. \tag{30}$$

Therefore, when both input op amps are matched we have the guarantee that $G_{DD} >> G_{CC}$, and consequently $CMRR_S > CMRR_D$. Observe that for the moment $R_2''$ and $R_2'$ do not have to be matched. For the particular case when $A_d >> A_c$; that is, provided that this CMRR for the input op amps is very large, we have

$$\frac{G_{DD}}{G_{CC}} \approx 1 + R_2'/R_1 + R_2''/R_1 = G. \tag{31}$$

This is, therefore, a very important factor in the design of the three-op-amp IA. The larger $G$ is, the larger the improvement of $CMRR_D$ when included in this circuit, as compared to the case when the second stage is used alone.

Note that the matching of $R_2''$ and $R_2'$ affects both $G_{CC}$ and $G_{CD}$ (28d and 28b). But when op amps are matched, then it only affects $G_{CD}$. From (18) we deduce that provided $G_d/G_c = CMRR_D$ is large enough, $G_{CD}$ will not usually be very important, and consequently the matching between $R_2''$ and $R_2'$ is not important. Therefore, from now on we will assume $R_2'' = R_2' = R_2$.

When the circuit in Fig. 2(a) is based on matched resistors, then $CMRR_D$ is determined by the tolerance of the resistors, $t$, the ratio $k$ of the resistors, and $CMRR_{OA}$ ((7) and (9)). For a given $CMRR_D$, according to (18), $CMRR_T$ depends on the four transfer functions for the first stage, and, therefore, on input op-amp matching.

Equation (22) shows what will happen when we assume that $CMRR_F$ and $CMRR_S$ have the same sign. Whenever $CMRR_S >> CMRR_F$ or $CMRR_F >> CMRR_S$, then $CMRR_T$ will be determined respectively by $CMRR_F$ or $CMRR_S$. Because $CMRR_D$ is fixed, we can obtain a large value for $CMRR_S$ by designing $G$ very large (while taking into account the limited bandwidth for input op amps). The value for $CMRR_F$ will be determined by the imbalance for the input op amps.

At frequencies lower than the corner frequency of the CMRR of the op amps, $A_d >> A_c$, i.e., CMRR is very high for op amps at low frequencies. At these frequencies, from (29) we obtain

$$CMRR_F \approx \frac{1}{2}\frac{A_{d1} + A_{d2} + 2A_{d1}A_{d2}}{A_{d2} - A_{d1} + A_{d1}A_{c2} - A_{d2}A_{c1}} \approx \frac{A_{d1}A_{d2}}{A_{d2} - A_{d1} + A_{d1}A_{c2} - A_{d2}A_{c1}} \tag{32}$$

$$\frac{1}{CMRR_F} \approx \frac{1}{A_{d1}} - \frac{1}{A_{d2}} + \frac{1}{CMRR_2} - \frac{1}{CMRR_1} \tag{33}$$

where $CMRR_1 = A_{d1}/A_{c1}$ and $CMRR_2 = A_{d2}/A_{c2}$. Equation (33) shows that both the open-loop differential gain and the CMRR must be matched for the input op amps. The first of these conditions is difficult to meet because, according to op amp data sheets, $A_d$ usually starts to decrease at frequencies well below the CMRR corner frequency. For the LM741C, for example, $A_d$ starts to decrease below 10 Hz while the CMRR does not decrease until about 500 Hz. Therefore, even though the contributions of open-loop gain and CMRR mismatching are the same, the first condition may be worse. If we model the open-loop differential frequency response as being determined by a dominant pole, $A_d = A_0\omega_d/(s + \omega_d)$, then we have

$$\frac{1}{A_{d1}} - \frac{1}{A_{d2}} \approx \frac{A_{20} - A_{10}}{A_{10}A_{20}} \frac{s + \omega_e}{\omega_e} \tag{34}$$

where

$$\omega_e = \frac{\omega_{d1}\omega_{d2}(A_{20} - A_{10})}{A_{20}\omega_{d2} - A_{10}\omega_{d1}}. \tag{35}$$

If, for example, $A_{20}$ and $w_{d2}$ are both 10% higher than $A_{10}$ and $\omega_{d1}$, then even if $CMRR_2 = CMRR_1$, from (33) to (35) we deduce that starting at frequencies lower than $\omega_{d1}/2$, $CMRR_F$ will decrease 20 dB/decade from a dc value 11 times higher than $A_{10}$. Nonmatched op amps will clearly result in a low $CMRR_T$ even at low frequencies, even if precision resistors are used in the second stage.

Some manufacturers of matched op-amp pairs specify the CMRR matching as the difference between the CMRR for each op amp when defined as $A_c/A_d$. In these cases, the specified CMRR matching gives the difference between the two terms on the right in (33). But no matching in $A_d$ is specified.

If instead of relying on low-tolerance resistors we use a potentiometer to adjust the value for $R_6$, then at a low frequency it is possible to trim $CMRR_D$ to any desired value. Therefore, we can seek an infinite value for $CMRR_T$ by nulling the denominator of (21), which, when assuming $R_2''/R_1 = R_2'/R_1$, leads to

$$-CMRR_D = \frac{1}{2}\frac{A_{d1} + A_{c1}/2 + A_{d2} + A_{c2}/2 + (2A_{d1}A_{d2} - A_{c1}A_{c2}/2)/G}{A_{d2} + A_{c2}/2 - A_{d1} - A_{c1}/2 + A_{d1}A_{c2} - A_{d2}A_{c1}}. \tag{36}$$

At very low frequencies (36) simplifies to

$$CMRR_D \approx \frac{1}{2}\frac{A_{d1} + A_{d2} + 2A_{d1}A_{d2}/G}{A_{d1} - A_{d2} - A_{d1}A_{c2} + A_{d2}A_{c1}}. \tag{37}$$

At these frequencies, $CMRR_D$, $A_{d1}$, $A_{d2}$, $A_{c1}$, and $A_{c2}$ are real numbers. Thus we can fulfill (37). Therefore, instead of seeking a high value for $CMRR_D$ it is better to adjust it until it compensates for the imbalance between op amps 1 and 2. In fact, the more different op amps 1 and 2 are, the smaller $CMRR_D$ must be in order to achieve a large $CMRR_T$. But at frequencies where the differential gains are no longer a real number, it will not be possible to fulfill (37). As mentioned before, $A_d$ usually has a dominant pole at frequencies that are rather low, less than 10 Hz in low-cost general purpose models. Therefore, potentiometer trimming will not allow us to improve $CMRR_T$ at frequencies higher than $\omega_e$, as given by (35).

## III. Experimental Results

We have measured the CMRR for the circuits in Figs. 1 and 2 for several op amps and different values for the differential mode gain, using a Hewlett-Packard 3582A spectrum analyzer. At low frequencies, the input common mode signal was so highly attenuated that we could not measure the exact value for the CMRR but only provide a lower limit. Thus we have presented the results in tabular instead of graphical form.

We first built the circuit in Fig. 1 using 1250 Ω, 0.1% resistors for $R_3$, $R_4$, $R_5$, and $R_6$. Therefore, $G_d = 1$. Table I shows the results for the common mode gain $G_c$ when using an LM741C and an OP-27GP. We repeated the measurements for several different positions for the resistors. Some of them gave higher gains, others lower gains, and some a 180° phase shift. The results scarcely changed when we substituted another op amp of the same model. These results support (8) and (10). The minimal $CMRR_R$ for 0.1% tolerance resistors is 54 dB. We obtained 62 dB. This value is so low that it predominates over $CMRR_{OA}$ at low frequencies, so that in spite of the better $CMRR_{OA}$ for the OP-27GP as compared to the LM741C, we obtained a similar $CMRR_D$. But the phase for $G_c$ clearly indicates that at frequencies above 1 kHz there was already some influence of $CMRR_{OA}$.

The lower $CMRR_D$ for the OP-27GP at the higher frequencies is somewhat surprising, because it is far better than the LM741C. The phase for $G_c$ gives the explanation: it happened that $CMRR_{OA}$ had an opposite sign to $CMRR_R$ in the case of the LM 741C, while the sign was the same in the case of the OP-27GP. When we placed the resistors so that at low frequencies the phase for $G_c$ was 180°, then for the OP-27 we obtained $G_c = -62.7$ dB at low frequencies and $G_c = -60.3$ dB at 25 kHz, while for the same resistors the LM741C gave, respectively, −62.6 dB and −48.5 dB.

Next, we changed the value of resistors $R_3$ and $R_4$ to 130 Ω, 0.1%. The measured value of $G_d$ at low frequency was 19.6 dB. The results for $G_c$ are also given in Table I. From (10), the minimal $CMRR_R$ is now 69 dB; we obtained about 82 dB (19.6 dB + 62.6 dB) at low frequencies for the LM741C. This is higher than in the case when $k = 1$, in spite of the poorer resistor matching we had, as inferred from the higher common mode gains at high frequencies. Because $CMRR_R$ is now higher, the effect of $CMRR_{OA}$ is observed at lower frequencies. Because the

TABLE I
$G_c$ FOR THE CIRCUIT IN FIG. 1 FOR TWO DIFFERENT OP AMPS (when using 0.1% precision resistors, when $G_d = 1$ and $G_d = 10$, and when using a trimming potentiometer)

| $f$ (Hz) | 1 | 10 | 100 | 1 000 | 10 000 | 25 000 | Condition |
|---|---|---|---|---|---|---|---|
| LM741C (dB) | −61.6 | −61.2 | −61.7 | −61.8 | −61.8 | −60.1 | 0.1% res. |
| $G_d = 1$ | 0° | 0° | 0° | 1° | 20° | 57° | |
| OP-27GP (dB) | −61.4 | −60.9 | −61.2 | −61.2 | −58.9 | −54.1 | 0.1% res. |
| $G_d = 1$ | 0° | 0° | 0° | −5° | −48° | −74° | |
| LM741C (dB) | −62.6 | −62.6 | −63.5 | −57.6 | −38.6 | −31.2 | 0.1% res. |
| $G_d = 10$ | 0° | 0° | 0° | 60° | 77° | 64° | |
| OP-27GP (dB) | −63.3 | −63.3 | −63.3 | −63.3 | −58.6 | −53.2 | 0.1% res. |
| $G_d = 10$ | 0° | 0° | 0° | 6° | 40° | 42° | |
| LM741C (dB) | < −100 | < −100 | −91.2 | −77.4 | −57.0 | −49.4 | trimming |
| $G_d = 1$ | 180° | 180° | 180° | 104° | 92° | 87° | |
| OP-27GP (dB) | < −100 | < −100 | −95.0 | −92.0 | −83.0 | −76. | trimming |
| $G_d = 1$ | 180° | 180° | 180° | 167° | 119° | 89° | |

TABLE II
COMMON MODE GAIN FOR THE CIRCUITS IN FIG. 2 FOR DIFFERENT INPUT STAGE OP AMPS AND GAINS (The second stage was based on an OP-27GP with $G_d = 1$, and we trimmed its CMRR at low frequency before connecting the first stage.)

| $f$ (Hz) | 1 | 10 | 100 | 1 000 | 10 000 | 25 000 | Condition |
|---|---|---|---|---|---|---|---|
| LM741C (dB) | −53.4 | −53.4 | −53.4 | −53.2 | −36.8 | −32.7 | noncoupled |
| $G = 2$ | 0° | 0° | −2° | −16° | −53° | −63° | buffers |
| LM741C (dB) | −86.0 | −86.0 | −83.2 | −69.7 | −49.0 | −41.2 | coupled |
| $G = 2$ | 0° | 0° | −50° | −85° | −87° | −90° | buffers |
| LM1458 (dB) | −91.2 | −91.2 | −91.0 | −90.5 | −71.1 | −63.0 | coupled |
| $G = 2$ | 0° | 0° | −22° | −49° | −88° | −95° | buffers |
| LM1458 (dB) | −90.0 | −87.6 | −85.8 | −75.8 | −54.9 | −49.8 | coupled |
| $G = 21$ | 0° | 0° | −18° | −81° | −104° | −130° | buffers |

$CMRR_{OA}$ is higher for the OP-27GP, its influence is lower and allows us to obtain a higher $CMRR_D$ up to high frequencies. Notice, however, that at low frequencies resistor mismatching reduces $CMRR_D$ more than 20 dB below $CMRR_{OA}$, in accordance with (8).

The use of a trimming potentiometer in the circuit in Fig. 1 permits us to overcome that limitation. Table I shows the results obtained when $k = 1$, using 1 kΩ, 5% resistors for $R_3$, $R_4$, and $R_5$. $R_6$ was the series combination of a 820-Ω 5% resistor and a 200-Ω 15-turn potentiometer. We adjusted the potentiometer until a minimal output was obtained at frequencies below 100 Hz. As predicted by (12), we overcame the limitation posed by $CMRR_R$. According to (13), at low frequencies we obtained a $CMRR_D$ even higher than $CMRR_{OA}$, as deduced from the results for the LM 741C, in which the typical $CMRR_{OA}$ is 90 dB. The higher $\omega_r$ for the OP-27GP resulted in a better $CMRR_D$ at the higher frequencies. The 90° phase shift at frequencies above $\omega_r$ predicted by (13) was also observed.

We built the circuit in Fig. 2(b) using a trimming potentiometer in the differential stage, based on the OP-27GP, and $R_2 = 10$ kΩ, $R_1 = 10$ kΩ. This resulted in a differential gain of 2. The potentiometer was trimmed at low frequency before connecting the input stage. The results for $G_c$ are those in the last line in Table I. When connecting the input op-amp pair, we obtained the results in Table II for the overall common mode gain. We observe that noncoupled buffers resulted in a 46 dB increase in low-frequency common mode gain (from −100 dB to about −54 dB), as predicted by (22) and (25). By interchanging two resistors in the input stage, we obtained a common mode gain of −38.1 dB at 1 Hz and −30.3 dB at 25 kHz. This underlines the importance of input stage matching, according to (25).

Next we built the circuit in Fig. 2(a) using the very same components, i.e., by connecting together the terminals of resistors $R_1$ instead of connecting them to ground. The results in Table II show the large improvement in common mode gain we obtained, as expected. The results did not change when we changed the positions of resistors $R_2$. While for the one-op-amp circuit we had never observed a significant phase-shift for $G_c$ at 100 Hz, now we had −50°. Equation (34) gives a possible explanation. The same result, however, can be explained by a very different $CMRR_{OA}$ for the input op-amp pair.

The use of a coupled input buffer based on matched op amps should improve the results. This is what we obtained when using an LM1458 dual op amp. Table II shows that the common mode gain remained low even at 1000 Hz, and was lower than when using two LM741C's. Note, however, that there is still a significant phase shift in this gain at 100 Hz. This, together with the lower CMRR when compared with $CMRR_D$, is a clear indication that dual units are not always very well matched. Some quad units we tested performed far worse than the LM1458.

A final set of experiments concerned the problem of the effect of the differential gain $G$ for the first stage in the resulting $CMRR_T$. We changed resistor $R_1$ to 1 kΩ, thus giving $G = 21$ (=26.8 dB) for the circuit in Fig. 2(a). The input op-amp pair was an LM1458. At low frequencies we obtained a common mode gain not very different from its value when $G = 1$, and therefore a $CMRR_T$ 26 dB higher than in the previous case. At 25 kHz the improvement was only about 13 dB, but high enough to achieve a $CMRR_T$ similar in magnitude to $CMRR_D$. The phase shift, however, was far from 90°. This is the price to be paid for the still significant imbalance in input op amps.

## IV. Conclusions

A one-op-amp DA based on precision resistors has a common mode rejection ratio less than $(k + 1)/4t$, where $k$ is the differential gain and $t$ the tolerance for the resistors. Therefore, in order to obtain a high $CMRR_D$ without any trimming component it is advisable to work with high differential gains. But, in any case, the limited $CMRR_{OA}$ will result in a decreased $CMRR_D$ at frequencies higher than about 1 kHz. This high gain also requires, of course, that the op amp have a broad bandwidth.

The use of a trimming potentiometer allows us to increase $CMRR_D$ at frequencies lower than the corner frequency for $CMRR_{OA}$, usually about 1 kHz, and also to obtain an output that is at 90° with respect to the input common mode voltage. The improvement is higher at the lower frequencies and does not depend on the op-amp model. Further, $CMRR_D$ does not depend on the differential gain for the circuit. No precision resistors are required, but the values of the resistors used must have a constant ratio.

An optimal $CMRR_D$ resulting in an output that lags or leads the common mode input by 90° has two advantages. It allows us to easily verify whether or not we are obtaining the best CMRR for the circuit while adjusting it. Also, it permits a further reduction of the resulting output error for amplitude-modulated signals by using synchronous demodulation, provided we are interested only in the in-phase component. To improve only the out-of-phase (quadrature) component, a slight misadjustment of the potentiometer will yield an output error signal either in phase or at 180° with respect to the input common mode voltage.

For the three-op-amp IA, the $CMRR_T$ increases when two coupled buffers are used as the first stage. This is due to the different equations for the CMRR for the first stage depending on whether the buffers are coupled (33) or not (25), and also to the effective increase of the CMRR of the second stage by the ratio between the differential and common mode gains for the first stage. For a pair of non-coupled buffers, this ratio is always 1; worse yet, passive component matching is also important in this case. A solution based on two simple buffers with gain added to an integrated DA can therefore ruin the CMRR performance of the DA.

The best approach for obtaining a high-CMRR broadband instrumentation amplifier is to use two well-matched op amps as coupled input buffers, and design this stage with a high differential gain, as long as this gain is permitted by the op-amps' differential open-loop gain. No matter how well matched the input op-amp pair is, at high frequencies the CMRR will decrease because of the limited CMRR for the differential (second) stage. Values higher than 80 dB for a differential gain of 1 at 25 kHz are feasible although difficult to measure.

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