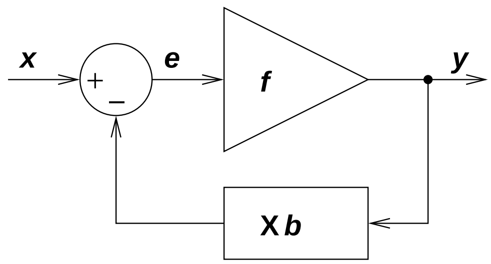

Figure 1: A feedback amplifier

the stated “laws” (square for FETs, three-halves for triodes, exponential for BJTs); that complementary pairs are perfectly symmetrical; that the tubes used in push-pull are identical; and that transformers are perfect. Though none of these is true, the results may yet be useful.

Our analysis is stateless; that is, we assume no frequency-dependent elements. Real amplifiers have such elements, but we can see the essential behavior without considering them.

The core of this paper is the numerically-derived spectra that we obtain for a variety of amplifiers, each being considered both with and without feedback. We begin, however, with a formal derivation of analytic estimates for the lowest order spectral lines, which we can use to check the validity of the numerical work; and a bit of circuit analysis. The reader may wish to skip this preliminary analysis and proceed to the discussion of the spectra that begins on page 10 under “Our spectra,” touching down at figure 2 on the way.

Feedback in a nonlinear system

In figure 1 the amplifier is modeled by a function f that is in general nonlinear. The feedback path is assumed to be a linear path that multiplies by a constant b. Thus, the equation relating the output y to the input x is

$$y = f(x - by). \tag{1}$$

If f were linear, say $f(e) = Ae$ we could solve for y to get the familiar Black's formula,

$$y = \frac{Ax}{1 + Ab}. \tag{2}$$

If A is very large then $y \approx x/b$, allowing us to reliably make amplifiers with gain $1/b$ using amplifiers with large, but uncontrolled gain. However, the distortion we are interested in is due to the *nonlinearity* of f.

Assume that f may be expressed as a power series

$$y = A_1 e + A_2 e^2 + A_3 e^3 + \cdots \tag{3}$$

with no offset term, so if $e = 0$ then $y = 0$. To account for the feedback we can substitute $(x - by)$ for e to obtain

$$y = A_1(x - by) + A_2(x - by)^2 + A_3(x - by)^3 + \cdots. \tag{4}$$

In general, we can solve for y, producing a power series that represents the entire transfer function of the feedback amplifier. This series

$$y = a_1 x + a_2 x^2 + a_3 x^3 + \cdots \tag{5}$$

can be obtained by taking derivatives of equation (4):

$$\begin{aligned}
a_1 &= \left.\frac{dy}{dx}\right|_{x=0} = \frac{A_1}{1 + A_1 b} && (6)\\
a_2 &= \left.\frac{1}{2}\frac{d^2y}{dx^2}\right|_{x=0} = \frac{A_2}{(1 + A_1 b)^2} && (7)\\
a_3 &= \left.\frac{1}{6}\frac{d^3y}{dx^3}\right|_{x=0} = \frac{(A_3 A_1 - 2A_2^2)b}{(1 + A_1 b)^5} && (8)\\
&\cdots
\end{aligned}$$

We see that a_1 is the gain we would expect if the amplifier were linear, and the higher-order terms are the distortion. If we make the input a sinusoid $x(t) = C \cos \omega t$, expand powers using the multiple angle formulas,[3] and collect like terms, we get a Fourier series showing the harmonic components. Considering only the first three terms of equation (5) we get:

$$y = \left(C a_1 + \frac{3}{4} C^3 a_3 \right) \cos \omega t + \frac{1}{2} C^2 a_2 \cos 2\omega t + \frac{1}{4} C^3 a_3 \cos 3\omega t. \tag{9}$$

Thus, for small signals (C small) the relative size of the second harmonic and third harmonic distortion terms are:

$$\begin{aligned}
\mathrm{HD}_2 &\approx \frac{1}{2} C \frac{a_2}{a_1} = \frac{1}{2} \frac{A_2}{A_1(1 + A_1 b)} && (10)\\
\mathrm{HD}_3 &\approx \frac{1}{4} C^2 \frac{a_3}{a_1} = \frac{1}{4} \frac{(A_3 A_1 - 2A_2^2)b}{A_1(1 + A_1 b)^4} && (11)
\end{aligned}$$

[3] For example, $(\cos \alpha)^2 = \frac{1}{2} + \frac{1}{2} \cos 2\alpha$ and $(\cos \alpha)^3 = \frac{3}{4} \cos \alpha + \frac{1}{4} \cos 3\alpha$.