Tape Flux Measurement Theory and Verification*

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Tape flux measurement is useful for standardizing program levels on magnetic tape records, and for measuring the performance of tapes and recording and reproducing heads. A ring-core head is the most satisfactory tape-flux measuring instrument, but calibration of the flux-to-voltage sensitivity is difficult in the general practical case. By using a "symmetrical head" construction, however, the average sensitivity of front and rear gaps is accurately calculated quite easily. For practical measurements, a "high-efficiency head" has the advantage of requiring only one (not two) measurements. The flux-efficiency of the core, and the frequency- and wavelength-response factors are all calculated and experimentally verified for both the symmetrical and the high-efficiency designs. A transfer to unidirectional flux and magnetometer flux measurement gives further verification. A flux measurement accuracy of better than 3% is achieved at medium wavelengths of 0.25 to 2.0 mm.

1. INTRODUCTION The knowledge of the recorded signal amplitude on a magnetic tape record has several practical uses: (*a*) for determining and standardizing references for the levels of audio programs on magnetic tape records; (*b*) for specifying the recording-performance properties of magnetic media (e.g., the distortion vs flux, etc.); (*c*) for measuring and specifying the sensitivity of reproducing heads (and also, by reciprocity, recording heads).

The quantity for specifying the recorded signal, discussed previously by the author [1] is the "*magnetic tape shortcircuit flux*" Φ_{sc} usually shortened to tape flux, or just flux. At an intuitive level, we may say that the shortcircuit flux is that flux from a magnetic tape record which flows through a magnetic shortcircuit placed in intimate contact with the record.

A more precise definition is as follows: The tape flux is the total flux of a recorded track which passes through a half-plane normal to both the plane of the tape, and to the direction of the tape flux (see Fig. 1). This half-plane is contained within a semi-infinite block of infinite permeability (a magnetic shortcircuit) which is in intimate contact with the tape surface. This half-plane is located halfway between magnetization nodes on the tape; in the case of a recorded sinewave, the rms value is the total flux divided by $\sqrt{2}$. The SI unit for flux is the weber.

By way of interpretation, we should note that the shortcircuit flux is the quantity which is measured by the "ideal" heads which are mentioned in many standards and which have been discussed by the author previously [1]. The definition given here precludes all of the known errors in making and using an ideal head: "flux which passes through a half-plane" means a measurement without gap-length loss, gap defects, or nonmagnetic spacing between the laminations; "normal to the direction of the flux" means adjustment for zero azimuth error; "semi-infinite block" means a head which is wider than the track (no fringing effect), and very long compared to

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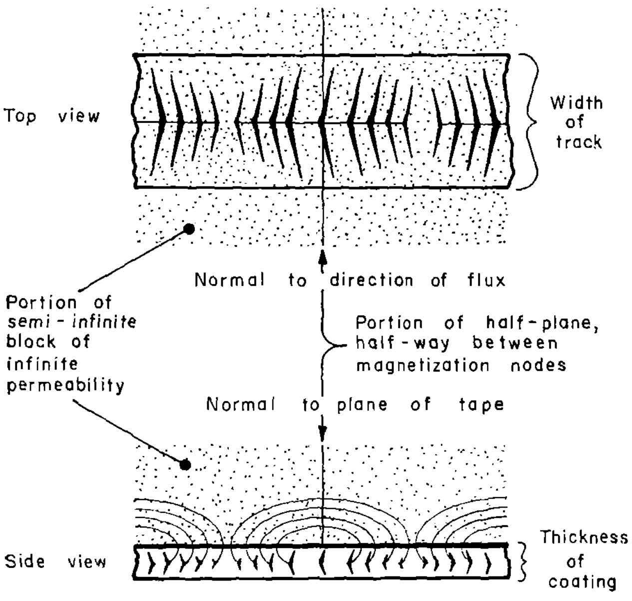


Fig. 1. Simplified illustration for the definition of shortcircuit flux, showing the sinusoidal magnetization of the coating, the resulting flux, the relationship of the coating to the semi-infinite block, and the relationship of the flux to the half-plane of measurement. ("Half-plane" and "semi-infinite block" refer to the fact that they are bounded by the plane of the tape.)

the longest wavelength (no head-length effect, which also means no effect from wavelength comparable to track width); "infinitely permeable" means that all of the flux is collected and also precludes secondary-gap effect; "intimate contact" precludes additional spacing over that inherent in the medium itself. Any system which meets these criteria directly or by calibration and correction can therefore be used to measure the tape flux.

Several possible methods for measuring tape flux exist [1], but the primary measurement method to be discussed here is the "calibrated ring-core head". The theory and experimental verification of the calibration itself are first given. Then the "Ampex Operating Level" flux is measured with a calibrated ring-core head, and by the vibrating sample magnetometer technique, and the results are found to agree quite well. The practical measurement of tape flux using such a calibrated ring-core head will be described in detail in another paper [2].

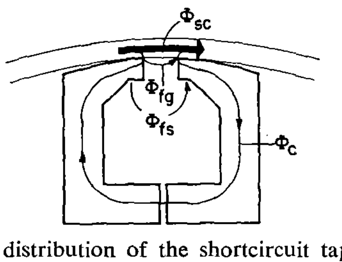


Fig. 2. The distribution of the shortcircuit tape flux (Φ_{sc}) in a ring-core head: front gap flux (Φ_{fg}) and front stray flux (Φ_{fs}) are lost, and the core flux (Φ_c) is transduced to an output voltage.

2. CALIBRATION OF A RING-CORE REPRODUCING HEAD

The measurement of shortcircuit tape flux Φ_{sc} by a calibrated ring-core head involves the following steps:

a) The tape flux is collected by the ring core, as shown in Fig. 2. We may represent this process by the equation

$$\Phi_c = \Phi_{sc} \cdot \eta_\Phi(f) \cdot F(\lambda), \tag{1}$$

where Φ_c is the core flux, Φ_{sc} is the shortcircuit tape flux to be measured, $\eta_\Phi(f)$ is the flux efficiency of the core structure (a function of the reproduced frequency f), and $F(\lambda)$ is the wavelength response factor of the core structure.

b) The core flux is then transduced to an emf E by a coil wound on the core. This process is described by Faraday's law of induction:

$$E = N \cdot d\Phi_c/dt = 2\pi f N \Phi_c, \tag{2}$$

for a sinusoidal wave of frequency f, where N is the number of turns on the coil.

c. We actually measure the terminal voltage V of the coil, not its emf E. Therefore, one must consider the transmission of the network shown in Fig. 3, consisting

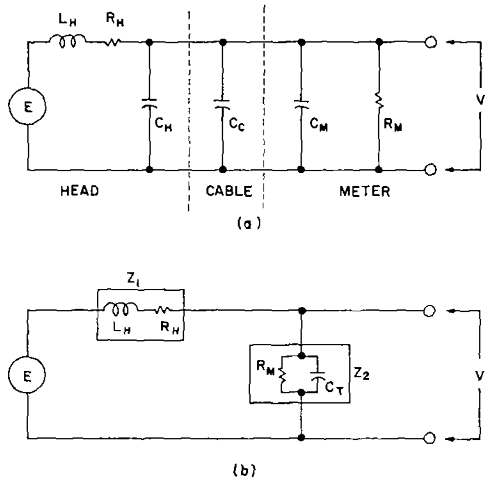


Fig. 3. The electrical network between the head emf E and the measured voltage V. L_H = inductance of the coil on the head; R_H = dc resistance of the coil on the head; C_H = capacitance of the coil on the head; C_C = capacitance of the cable from head to voltmeter C_M = input capacitance of the voltmeter; R_M = input resistance of the voltmeter. **a**. Complete circuit. **b**. Simplified circuit for the calculation of the network transmission factor; $C_T = C_H + C_C + C_M$.

of the resistance, inductance, and capacitance of the coil itself, the capacitance of the connecting cable, and the input resistance and capacitance of the voltmeter. We may represent this by:

$$V = E \cdot T(f), \tag{3}$$

where $T(f)$ is the network transmission factor as a function of frequency.

Combining these equations, we have

$$\Phi_{sc} = V/[2\pi f \cdot N \cdot \eta_\Phi(f) \cdot F(\lambda) \cdot T(f)]. \tag{4}$$

We must know each of these five factors—f, N, $\eta_\Phi(f)$, $F(\lambda)$, and $T(f)$—in order to calibrate the head to measure the total shortcircuit tape flux. Note that it is often convenient in practice to combine the product of N and $\eta_\Phi(f)$, and call it the "effective number of turns" at a given frequency: $N_{eff}(f) = \eta_\Phi(f) \cdot N$. The effective number of turns can be measured for a finished head. The actual number of turns and the flux efficiency are design and construction data usually unknown to a user.

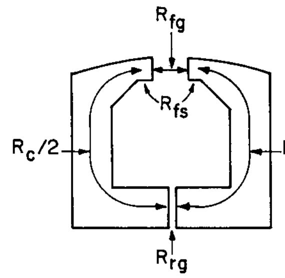


Fig. 4. Head reluctances, for calculation of the head flux efficiency. R_{fg} = front gap reluctance; R_{fs} = front stray reluctance; R_c = core reluctance (a function of reproduced frequency); and R_{rg} = rear gap reluctance.

To convert the measured total flux to the flux per unit recorded track width, the recorded track width must be measured accurately; then the measured flux is divided by the measured width.

A head for measuring purposes can easily be designed and made for which the wavelength response factor $F(\lambda)$ is unity at medium wavelengths such as 0.5 to 1 mm; this will be discussed in Sec. 2.2. The coil can also be designed and made such that the electrical transmission factor $T(f)$ is unity at the measuring frequencies to be used (250 to 1000 Hz) (Sec. 2.3). The major problem is to determine the core flux efficiency factor $\eta_\Phi(f)$.

The core flux efficiency factor is

$$\eta_\Phi(f) = \Phi_c(f)/\Phi_{sc} = R_f/[(R_f+R_r(f)], \tag{5}$$

where R_f is the front reluctance and $R_r(f)$ is the rear reluctance as a function of frequency. As may be seen in Fig. 4, the front reluctance consists of the front gap reluctance R_{fg} in parallel with the front stray reluctance R_{fs}; the rear reluctance consists of the core reluctance R_c (which is frequency-dependent), and the rear gap reluctance R_{rg}. The "circuit" in Fig. 5 shows the division of the flux between the front path (lost flux) and core path (measured flux).

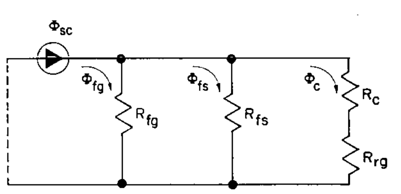


Fig. 5. "Circuit" showing the distribution of the shortcircuit tape flux Φ_{sc} into the flux lost into the front gap (Φ_{fg} through R_{fg} and Φ_{fs} through R_{fs}), and the useful flux through the core (Φ_c through R_c and R_{rg}).

It is often more convenient to consider the reluctance *ratio*. We define $X = R_f/R_r$, giving

$$\eta_\Phi = X/(X+1); \tag{6}$$

this relationship is plotted in Fig. 6.

The reluctance of magnetic circuits is calculated from

$$R = l/(\mu A), \tag{7}$$

where R is the circuit reluctance in reciprocal henries (H^{-1}); l is the path length of the magnetic circuit in meters; μ is the absolute permeability of the circuit, where $\mu = \mu_0 \cdot \mu_r$; μ_r is the relative permeability of the circuit (dimensionless), and μ_0 is the permeability of free space, $4\pi \times 10^{-7}$ henries per meter (H/m); and A is the area of the circuit in square meters.

The length,[1] area and permeability of both the front gap and the core are easily measured; therefore, the corresponding reluctances R_{fg} and R_c may be calculated accurately. The front gap depth (and therefore reluctance) of course varies as tape is run over the head face wearing away the materials. The other two important reluctances (front stray, R_{fs}, and rear gap, R_{rg}) are more difficult to determine: First, the effective length and area for the front stray reluctances are not simply determined—this is not really a "circuit" problem at all, but a field problem; an approximate formula, however, is available (see Appendix A). Second, the length of a closed gap in the rear of the core depends on the surface finishes, contact pressure, etc., which are not known exactly. In addition, the front gap spacer may cause the back gap to "wedge" open, causing an appreciable gap.

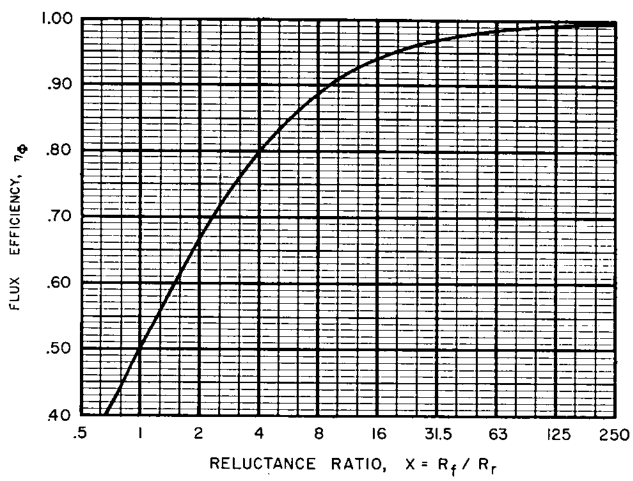


Fig. 6. Core flux efficiency η_Φ *vs* front-reluctance to rear-reluctance ratio X.

There are, however, two particular cases which yield simple solutions to the problem of knowing the reluctances accurately:

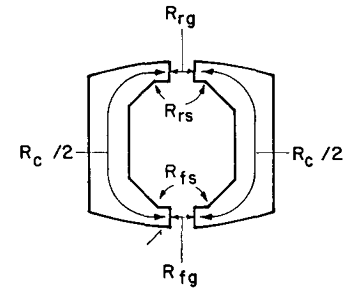
a) A "symmetrical" head may be made with long—25 μm (1 mil)—front and rear gaps of approximately equal depths, with both gaps finished for use, as shown in Fig. 7. The head is mounted so that a tape flux measurement can be made with each gap. Then the average flux efficiency from a tape flux measurement with each of the two gaps of the symmetrical head is

$$\eta_{\Phi avg} = 0.500/[1+R_c(R_{fg}+R_{rg})^{-1}]. \tag{8}$$

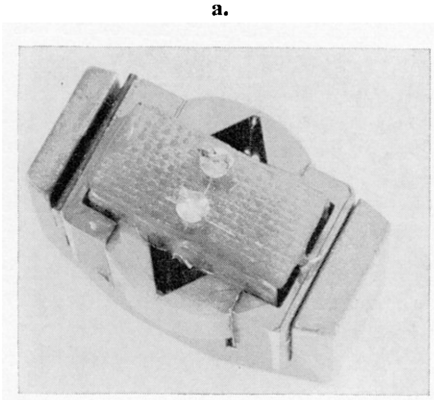
In an easily achieved design $R_{fg} + R_{rg} = 100\ R_c$, so that $\eta_{\Phi avg} = 0.495$.

By this technique, we avoid the usual calibration problems: First, there is no "unknown" reluctance of a closed rear gap. Second, the exact value of the stray gap reluctances is not critical, as it is a second-order effect. Third, as the head wears, the flux efficiencies from the front and rear gaps will change, but the average $\eta_{\Phi avg}$ is

[1] Gap length and core length are in the direction of the tape length; gap width and core width are in the direction of the tape width; and gap depth is perpendicular to the plane of the tape.

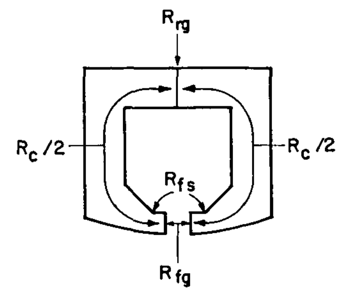


a.

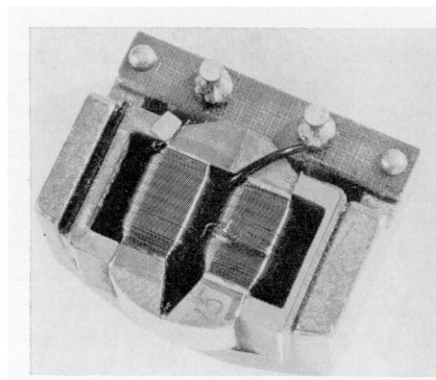


b.

Fig. 7. Symmetrical head configuration. Both gaps are used for reproducing signals, and the average efficiency is calculated and used. **a.** Schematic representation. **b.** Photograph of actual heads used.



a.



b.

Fig. 8. The high-efficiency head configuration. **a.** Schematic representation. **b.** Photograph of actual heads used.

still the correct value, and can change less than 1% (from 0.495 to 0.500).

Having to make two measurements and take the average is somewhat inconvenient but is perfectly satisfactory for a primary standard.

b) A high-efficiency head may be made with a deep rear gap and a long—25 μm (1 mil)—front gap, as shown in Fig. 8. The front-reluctance to rear-reluctance ratio can be made so large that the efficiency approaches 1.00. Under this condition small inaccuracies in the reluctance calculations again produce only second-order errors. Practical values of flux efficiency of 0.98 to 0.99 are typical. The calculated flux efficiency is verified by comparison with the flux efficiency of the symmetrical head. The high-efficiency head is an especially good construction for a *secondary* standard, since the efficiency is essentially unchanged by wear of the front gap depth.

Thus, with these two designs available, one may easily construct and verify both a primary and secondary measuring standard.

2.1 CALCULATION OF THE FLUX EFFICIENCY

Calculation of the flux efficiency first requires calculation of the several reluctances; flux efficiency follows directly.

2.1.1 Flux Efficiency of the Symmetrical Head

The lamination shape shown in Fig. 9 was used (Ampex Part No. 12018 or 80065). The total magnetic path length in the core l_c is approximately 34 mm; depth is 2.5 mm and gap width 6.9 mm, giving a core area[2] A_c of 17 μm^2. The cores are made of 150 μm (6 mil) thickness laminations of 4-79 molybdenum permalloy; the relative permeability μ_r = 20 000 at 1000 Hz (according to the manufacturer), so that the permeability at 1000 Hz is $\mu_c = \mu_r \cdot \mu_0$ = 20 000 · 1.26 μH/m = 25 mH/m. Thus from Eq. (7), R_c = 80 kH^{-1}. The mica gap shims make the gap lengths l_g = 25 μm (1 mil); the gap width was 6.9 mm, and depth 500 μm (20 mil), giving a gap area A_g = 3.5 μm^2, and a gap reluctance, from Eq. (7), of R_g = 5.7 MH^{-1}. The stray reluctance is calculated and experimentally verified in Appendix A; the value R_s = 14 MH^{-1} is typical. Therefore, the

[2] Throughout this paper, unit exponents apply *only* to the unit to which they are affixed, not to the associated decimal multiplier. Thus, 12 μm^2 = 12 × 10^{-6} m^2; 10 kH^{-1} = 10 × 10^3 H^{-1}. (The more unusual convention, e.g., in ANSI Standard Y10.19-1967, specifies that the exponent applies to symbol *and multiplier*, so that 12 μm^2 = 12 (μm)^2 = 12 × 10^{-12} m^2.)

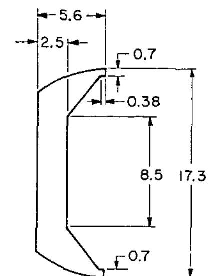


Fig. 9. Lamination used for the symmetrical heads. All dimensions in millimeters.

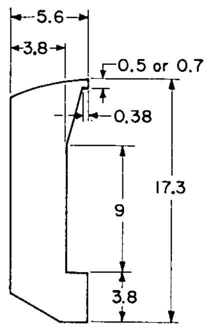


Fig. 10. Lamination used for the high-efficiency heads. All dimensions in millimeters.

total reluctance of the gaps is $(R_{fg})(R_{fs})/(R_{fg}+R_{fs}) = 4$ MH^{-1}. The average efficiency at 1000 Hz, from Eq. (8), is $\eta_{\Phi avg} = 0.5/(1+0.01) = 0.495$, or 1% (0.09 dB) less than 0.50.

Since this head is to be used only at or below 1000 Hz, its flux efficiency will not be calculated at other frequencies: at lower frequencies the change of core permeability and resulting change of flux efficiency is negligible; at frequencies above 1000 Hz, the flux efficiency would need to be recalculated and/or measured (see Sec. 2.4.2.1).

If the core reluctance had been 160 kH^{-1}, η_Φ would have been 0.49, an error of 1%; if it had been 40 kH^{-1}, η_Φ would have been 0.498, an error of 0.6%. Thus even large errors of core permeability affect the flux efficiency only very slightly.

2.1.2 Flux Efficiency of the High-Efficiency Head

The lamination shape shown in Fig. 10 was used (Ampex Part No. 120449, with 0.76 mm front gap depth before finishing; or No. 12875, with 0.5 mm depth). The total magnetic path length in the core l_c is approximately 32 mm; depth is 3.8 mm and gap width 6.9 mm, giving a core area $A_c = 26$ μm^2. Cores were made of both 160 μm (6 mil) and 75 μm (3 mil) thickness laminations of 4-79 molybdenum permalloy; the relative permeabilities[3] at 1000 Hz were $\mu_r = 20\,000$ and 45 000. (The 160 μm thickness laminations were used in earlier experiments; the 75 μm laminations were used for a large group of heads made later. Measured efficiencies were sensibly identical.) The core permeabilities were therefore $\mu_c = 25$ mH/m or 56 mH/m, respectively. Thus, from Eq. (7), the core reluctance is $R_c = 50$ kH^{-1} or 22 kH^{-1}, respectively.

A direct calculation of the rear gap reluctance is not possible when a "zero gap length" is used, because the *effective* rear gap length cannot be measured directly. In a practical head, however, the front gap spacer causes a "wedge angle" at the rear gap, as shown in Fig. 11. The average gap length (from the geometry shown in the figure) is $(2/17) \times 25$ $\mu m = 3$ μm. From the depth of 3.8 mm and the width of 6.9 mm, the area is $A_{rg} = 26$ μm^2; $\mu_0 = 1.26$ μH/m; therefore $R_{rg} = 90$ kH^{-1}. The rear-gap reluctance was determined experimentally, as described in Appendix B. There the value $R_{rg} = 21$ kH^{-1} was found; therefore, the worst-case "wedge" reluctance of 90 kH^{-1} may be used with the knowledge that the actual efficiency may be a little bit greater. The total rear reluctance then is $R_r = R_c + R_{rg} = 150$ or 110 kH^{-1}.

The mica shim makes the front gap length 25 μm (1 mil); the gap width was 6.9 mm, and depth was reduced in finishing to 0.35 mm (14 mil), giving a gap area $A_g = 2.4$ μm^2, and a front gap reluctance $R_{fg} = 8.3$ MH^{-1}. The front stray reluctance (from Appendix A) is approximately 14 MH^{-1}, giving a total front reluctance $R_f = (R_{fg})(R_{fs})/(R_{fg}+R_{fs}) = 5$ MH^{-1}.

With these values ($R_f = 5$ MH^{-1}, $R_r = 150$ to 110 kH^{-1}) we calculate $X = 5/0.15 = 33$ to $X = 5/0.11 = 45$, and η_Φ at 1000 Hz $= 0.971$ to 0.978. These values will be confirmed, and the exact sensitivity calibrated, by use of the symmetrical head as a primary reference (Sec. 2.4). Other experimental methods have also been used (Sec. 4).

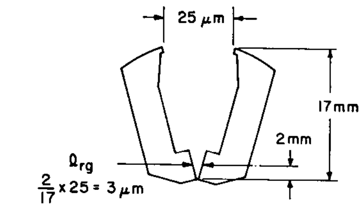


Fig. 11. Calculation of the length of the rear gap which is caused by "wedging" from the front gap spacer.

2.2 CALCULATION OF THE WAVELENGTH RESPONSE

When heads are to be used to make tape flux measurements over a wide range of wavelengths, a detailed wavelength response analysis is required. In the present case, only medium wavelengths of about 0.5 to 1 mm will be measured, and we will simply show that the wavelength response factor is sufficiently close to unity at these wavelengths. The wavelength response $F(\lambda)$ will be divided into the gap-length response factor $F_g(\lambda)$ and the head-length response factor $F_h(\lambda)$. The theory and experiments here were with full-track recordings with a head core wider than the tape (6.9 mm head width, 6.3 mm tape width). When the recorded track is wider than the head core, an additional wavelength-dependent "fringing factor" $F_f(\lambda)$ must be introduced [3].

2.2.1 Gap-Length Response Factor

The gap-length response factor $F_g(\lambda)$ has been accurately calculated by Westmijze [4], and more extensive results have been tabulated by Wang [5]. Figure 12 is drawn from Wang's data. For the present heads, $l_g = 25$ μm, $f_{max} = 1000$ Hz, $v = 38$ cm/s (15 in/s), $\lambda_{min} = 380$ μm, so that $l_g/\lambda_{min} = 0.066$, and the gap length response factor F_g is 0.991. Actually, a 700 Hz signal will be measured, for which $\lambda = 550$ μm, $l_g/\lambda = 0.045$, and $F_g = 0.996$. Although the theory corresponds

[3] These values were both as taken from the manufacturer's literature, and as verified by our experiments. "Rings" from the same stock used to make the laminations were heat-treated simultaneously with the laminations. The permeability of these rings was calculated from measurements of the emf in one winding due to a known current at 1000 Hz in another winding; it was also calculated from the inductance of a single winding on the ring. Measured permeability was 40 000 at 1000 Hz with the 75 μm laminations.

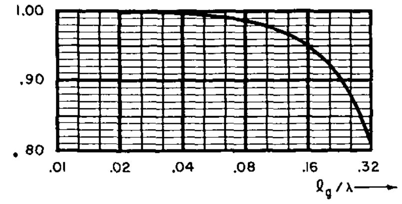


Fig. 12. Gap-length response factor *vs* gap-length to recorded-wavelength ratio, l_g/λ.

exactly to the practical case, an experimental verification is given in Sec. 2.4.

2.2.2 Head-Length Response Factor

The head-length response factor $F_h(\lambda)$ has been accurately calculated by Duinker and Geurst [6] for a round, unshielded head. The core actually used here is circular at the tape contact, with a radius corresponding to a diameter of $D = 17$ mm; but the sides are flat, not circular (the "length" of the head is 11 mm). Thus $D/\lambda > 10$. The total wrap angle used in the measurements ranged between 0 radians (tape tangent to head face) and 0.2 rad (12°), corresponding to values of half-wrapping angle α of 0 to 0.1 rad. Duinker and Geurst show in their Table III that, for $D/\lambda > 3$, and α between 0 and 0.2 rad, the head-length response factor lies between 1.000 and 1.011; the interpolated value for $\alpha = 0.1$ rad would be 1.005. Thus, for the wavelength actually used, an error of less than 0.5% would be expected.

Since the core shape actually used is not exactly the same as that for which the response is calculated, a practical verification of the long-wavelength response was deemed necessary (see Sec. 2.4).

2.3 CALCULATION OF THE NETWORK TRANSMISSION FACTOR

The emf developed by the head winding is transmitted through the coupling network shown in Fig. 3a to the measuring voltmeter. Figure 3b simplifies the circuit, and lumps the head, cable and meter capacitances into one total capacitance C_T.

From circuit theory,

$$T(f) = \frac{Z_2}{Z_1+Z_2} = \frac{1/L_HC_T}{s^2+[(R_H/L_H)+(1/R_MC_T)]s+[(R_M+R_H)/R_MC_HL_H]} = \frac{K_1}{(K_3K_1-\omega^2)+j\omega K_2} \quad (9)$$

and the magnitude of transmission is

$$|T(f)| = \frac{K_1}{\sqrt{[(K_3K_1-\omega^2)^2+(\omega K_2)^2]}}, \quad (10)$$

where $s = j\omega$, $\omega = 2\pi f$, f = frequency at which response is computed, in Hz, and $K_1 = 1/(L_HC_T)$, $K_2 = (R_H/L_H) + [1/(R_MC_H)]$, and $K_3 = (R_M+R_H)/R_M$.

The apparatus used here had the following characteristics: cable (1 m of RG58 A/U, 90 pF/m)—$C_C = 90$ pF; meter (Hewlett-Packard 400 L, on 10 mV range)—$C_M = 25$ pF, $R_M = 10$ MΩ. For the symmetrical head, $L_H = 530$ mH, $R_H = 195$ Ω, $C_H = 15$ pF (both terminals of coil to frame, 25 pF); therefore $C_T = 130$ pF. For this head, at $f = 0$ Hz (dc), the transmission T is 0.99998; at 250 Hz, $T = 1.00015$; at 700 Hz, $T = 1.0013$; at 1000 Hz, $T = 1.0027$. Thus, in the region of interest, T = unity, to the accuracy required. For the high-efficiency head, $L_H = 210$ mH, $R_H = 40$ Ω, $C_H = 30$ pF (both terminals of coil to frame, 60 pF); $C_T = 145$ pF; at 700 Hz, $T = 1.0008$. We will therefore take $T = 1.00$ for both heads, in the present calculations.

2.4 EXPERIMENTAL VERIFICATIONS

Now that the several transmission factors have been calculated, it remains to verify their correctness experimentally.

2.4.1 Verification of Flux Efficiency Calculation

2.4.1.1 Symmetrical Head

Three "identical" symmetrical heads were constructed according to the design described in Sec. 2.1.1. All of the gaps were lapped to a depth of 500 μm, and all windings were $N = 2000$ turns of No. 40 wire, giving $R_{dc} = 195$ Ω. Since (from Sec. 2.1.1) $\eta_\Phi = 0.495$, and $N = 2000$, $N_{eff} = 990$. From Sec. 2.1.1, the gap reluctance (calculated from its dimensions and geometry) is about 4 MH^{-1} for each gap, or 8 MH^{-1} total; therefore, $L_{calc} = N^2/R = (2000)^2/(8\times10^6) = 500$ mH. The measured head inductances were 525 to 530 mH (see Table I). A fourth symmetrical head had gaps lapped to 380 μm depth; for this head, $L_{calc} = 410$ mH, $L_{meas} = 420$ mH. A recording of Ampex Operating Level (700 Hz at 38 cm/s, $\lambda = 0.5$ mm) was reproduced first with the front gap, then with the rear gap, of each of these heads; the measured output voltages are given in Table I. For all of these heads, then, 5.08 mV from this 700 Hz recording corresponds to $N_{eff} = 990$ turns.

Table I. Head output voltages from symmetrical heads.

Head No.	Gap Depth (μm)	Measured Inductance (mH)	Measured Output Voltage (mV) Front Gap	Rear Gap	Average Voltage (mV)
395	560	530	5.08	5.08	5.08
446	510	525	5.08	5.08	5.08
448	510	527	5.08	5.08	5.08
447	380	420	5.25	4.90	5.08

All of this data for output voltage is ± 0.03 mV, because of flux variations of the recorded signal; thus differences of less than about ± 0.6% would not have been seen. A more sophisticated measurement technique would allow a more accurate measurement; however, even the present data shows that both symmetry and repeatability are achieved.

2.4.1.2 High-Efficiency Head

Four "identical" high-efficiency heads were constructed according to Sec. 2.1.2. The winding was $N = 1000$ turns of No. 37 wire, giving $R_{dc} = 40$ Ω. Gap depths were about 380 μm. From Sec. 2.1.1, the gap reluctance (calculated from its dimensions and geometry) is about 5 MH^{-1}, so that $L_{calc} = (1000)^2/(5 \times 10^6) = 200$ mH. The measured values were 211 to 214 mH (see Table II). The same recording of the Ampex Operating Level

Table II. Head output voltages from high-efficiency heads.

Head No.	Gap Depth (μm)	Measured Inductance (mH)	Measured Output Voltage (mV)
(27.8.67)	380	212	5.08
449	380	214	5.08
450	360	212	5.08
451	360	211	5.08

used above was again reproduced with this group of heads, and the measured output voltages are given in Table II.

From Eq. (4), we see that the effective number of turns on any unknown head, N_{eff}, may be found by comparing it with a reference head with known effective number of turns, $N_{eff\ ref}$, according to

$$N_{eff} = N_{eff\,ref} \cdot \frac{V}{V_{ref}} \cdot \frac{f_{ref}}{f} \cdot \frac{F_{ref}(\lambda)}{F(\lambda)} \cdot \frac{T_{ref}(f)}{T(f)} \cdot \frac{(\Phi/w)_{ref}}{(\Phi/w)} \cdot \frac{w_{ref}}{w}. \quad (11)$$

When the same recording is reproduced by both heads at the same speed, $f = f_{ref}$ and $(\Phi/w) = (\Phi/w)_{ref}$. For these symmetrical (reference) heads and high-efficiency heads the reproduced width is the same for both heads, so $w = w_{ref}$; since the core shapes and sizes are essentially identical, and the gap lengths are identical, F (0.5 mm) $= F_{ref}$(0.5 mm); the transmission factors are T (700 Hz) $= T_{ref}$(700 Hz) $= 1$. Therefore, for this particular case Eq. (11) reduces to

$$N_{eff} = N_{eff\,ref}(V/V_{ref}). \quad (12)$$

For these heads, $V = V_{ref} = 5.08$ mV, and $N_{eff\ ref} = 990$; therefore, $N_{eff} = 990$ for the high-efficiency head. From this we calculate for the high-efficiency head that $\eta = N_{eff}/N = 990/1000 = 0.99$, which compares quite well with the calculated value (Sec. 2.1.2) of 0.98. The consistency of the four high-efficiency heads is good—the deviation was less than the limit of accuracy for the measuring system, which is $\pm$ 0.6%, as mentioned in Sec. 2.4.1.1.

2.4.2 Verification of Frequency Response

We have shown previously that the overall frequency response of a head is the product of three factors, namely, the flux efficiency of the core *vs* frequency, $\eta_\Phi(f)$; the action of Faraday's law of induction, $E = 2\pi f N \Phi_c$; and the network transmission factor, $T(f)$.

Several means for the practical measurement of the overall frequency response of a reproducing head have been described in detail by Bick [7] and by the author [8]. Each of the methods has possible pitfalls under certain conditions. In the author's opinion, the most convenient method and that subject to the least number of practical errors in measurement is the following: Generate a constant magnetic field strength at the gap by means of a small wire fastened over the head gap and carrying a constant current *vs* frequency. Neither the wire size nor its exact location is critical since there is no wavelength effect here. A No. 30 to No. 40 enameled wire is convenient, held approximately over and parallel to the line of the gap by a simple jig, or even just by pressure-sensitive tape. It is most easily positioned by adjusting the location of the wire to give maximum head output voltage at any convenient frequency. Constant current (and therefore constant field around the wire) is insured by feeding the wire from a resistance which is much higher than the reactance of the wire at the highest frequencies used—for instance, a 600 Ω constant-voltage generator in series with a 600 Ω building-out resistor is convenient.

In most heads, the core permeability remains high compared to that of free space over the entire frequency range of interest. Under this condition, constant magnetic field strength produces constant flux at the gap. If the core permeability is very low—as it will be at very high frequencies, or with very thick laminations—the constant field will produce a falling flux with increasing frequency; a given recording on tape, on the other hand, produces constant flux at any frequency.

A further error may occur at high frequencies because the flux division ratio at the gap may be different for the "short" wavelength flux from the tape and the flux from the wire (which has no wavelength at all). This results in a rising core flux from the constant field, compared to a constant core flux from the tape. Thus at high frequencies either a rising or a falling response could be measured relative to the true value, and the two-speed method described in the referenced articles should be used. For most audio frequency head constructions, and for the heads described here in particular, the response over the audio frequency range is correctly measured by the constant field method described above.

The Faraday's law that causes the response to be proportional to frequency (20 dB/decade rise) is well known and proven, and needs no verification. This response may be accounted for by comparing the head output voltage to a 20 dB/decade rising response. It is often more convenient for data-taking purposes to insert between the head and the voltmeter a network having a 20 dB/decade falling response—that is, a passive R-C integrating network, or an integrating amplifier. When this is done, the measured response is just the product of the flux efficiency *vs* frequency and the transmission factor *vs* frequency.

With a given head it is not usually possible to separate the change of flux efficiency *vs* frequency (due to the eddy currents in the core) from the transmission factor response (due to the resonance of the head inductance with the self- and load-capacitance on the head). There is, however, a simple technique for separating the two effects if another head can be constructed which is *identical* to the first except for the number of turns on the coil. If the second head is made with a coil of very low inductance, causing the head resonance to be well above the highest frequency of interest, this causes the transmission factor to be exactly unity at the frequencies of interest. The response measurement on this second head therefore yields just the flux efficiency of the core *vs* frequency; since the two cores are identical, this is also the desired measurement of flux efficiency *vs* frequency of the original head.

This measured flux efficiency level *vs* frequency is then subtracted from the overall response level of the original head, giving the desired transmission level of the original head. Thus, we have separated the flux efficiency *vs* frequency from the transmission factor response of the original head.

2.4.2.1 Verification of Flux vs *Frequency for the High-Efficiency Head*

A high-efficiency core with 75 μm (3 mil) laminations was wound with 200 turns rather than the usual 1000 turns. The usual 25 μm gap was used, and the resulting inductance was 8 mH. For $C = 100$ pF, the resonant frequency would be about 180 kHz, and we may therefore assume that $T(f)$ equals unity for all frequencies below 18 kHz. Constant field was injected at the gap as described above. The output voltage was integrated with an operational amplifier in an integrating configuration. The integrated output voltage was constant up to 20 kHz (the limit of this particular measurement), within the accuracy of the measuring equipment (about $\pm$ 0.1 dB). Thus, $\eta_{\Phi}(f)$ is constant up to at least 20 kHz.

2.4.2.2 Verification of Transmission Factor vs *Frequency for the High-Frequency Head*

The transmission factor $T(f)$ for the circuit actually used with the 1000-turn high-efficiency head has been calculated in Sec. 2.3, and was measured by the constant input field method described above. The measured and calculated responses were within 0.1 dB of each other, and are shown as one curve in Fig. 13. For frequencies of 2 kHz and below, the transmission factor $T(f)$ is unity. When the load resistance (R_M of Fig. 3) was decreased to 30 kΩ, the frequency response was flat $\pm$ 0.1 dB from 20 Hz to 16 kHz; therefore, the frequency response factors could be neglected when making measurements in this frequency range.

2.4.3 Verification of Wavelength Response

According to generally accepted magnetic recording theory, a constant magnetizing field ("constant current") recording produces a constant tape flux at wavelengths which are very long compared to the coating thickness. In order to measure the wavelength response of the high-efficiency head, such a constant magnetizing field recording was made on a triple play tape with a 4.3 μm (170 μin) coating thickness, running at a speed of 76 cm/s (30 in/s). The thickness response factor for this tape is shown in the upper part of Fig. 14, as the short-dashed curve. The gap-length response for the 25 μm (1 mil) head is shown in the same figure as the long-dashed curve, and the sum of these two curves—the total expected short-wavelength response—is shown as the solid curve.

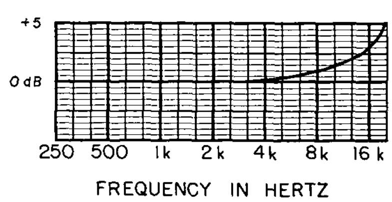


Fig. 13. Transmission level 20 log $T(f)$, in dB, of the high-efficiency head.

The measured response is shown in the lower part of Fig. 14. At short wavelengths (2 mm to 0.25 mm) the calculated and measured responses are essentially identical. (The measured response is 0.2 dB greater than the calculated value at 0.25 mm, for reasons not known.) At longer wavelengths of 2 mm to 8 mm, a $\pm$ 0.2 dB ripple occurs, with a rise to +2 dB at 16 mm wavelength (50 Hz). Since the head design used does not correspond exactly to that in any theoretically calculated cases, a quantitative comparison of calculated and measured values cannot be made at long wavelengths.

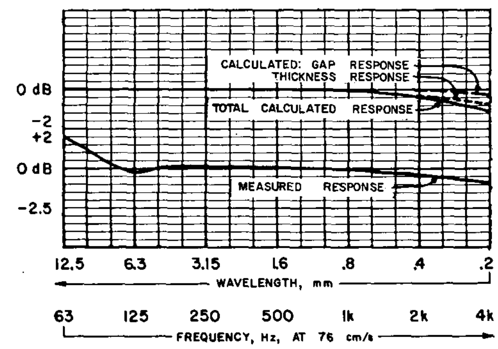


Fig. 14. Combined wavelength response of the constant magnetizing-field recording and the high-efficiency head. **Upper curves**: calculated; **lower curve**: measured.

The normal wrap angle (0.2 rad total angle) was used for these measurements, and the head was shielded with a ferromagnetic can having a "length" of 3.3 cm. (These correspond to an unmodified Ampex Model 300 or 350 head assembly.) The response was also measured without the shield, and with somewhat greater and lesser wrap angles. The response change was negligible (less than 0.1 dB) for wavelengths less than 8 mm.

Thus the wavelength response of the reproducer is seen to agree with the calculated values, and the wavelength response factor is 1.00, $\pm$ 1% for the wavelength range 2 mm to 0.25 mm, which is one-half to twice the design range.

2.5 SUMMARY OF HEAD CALIBRATION

The frequency response factors of the high-efficiency head, $\eta_{\Phi}(f)$ and $T(f)$, are 1.00 over the frequency range up to 2 kHz. The wavelength response factor $F(\lambda)$ is 1.00 over the wavelength range 0.4 mm to 2 mm. The core efficiency is 0.495 for the symmetrical head, and 0.99 for the high-efficiency head, and both have an effective number of turns $N_{eff} = 990$. The error for each of these calculated and experimentally verified factors is less than the resolution of the measuring instruments, which is about $\pm$ 1%; thus, the total error should be less than about $\pm$ 1.7%. To this error one must add the errors of the frequency measuring device (typical frequency counter error at 1000 Hz is 0.1% for a 1 s gate time); and the errors of the voltmeter (for example, Hewlett-Packard Model 400L, $\pm$ 3% of reading; Model 400EL, $\pm$ 1.5% of reading).

3. MEASUREMENT OF AMPEX OPERATING LEVEL WITH A CALIBRATED RING-CORE HEAD

The Ampex Operating Level is a recording of a particular flux which is used as a reference for magnetic recording flux levels. The wavelength is 0.5 mm, corresponding to 700 Hz at 38 cm/s. The flux is controlled by comparison with an arbitrary reference standard held by the Ampex Test Tape Laboratory. The magnitude is of interest because this flux is widely disseminated on Ampex Test Tapes.

The complete measurement data has been given bit-by-bit in the previous sections: Table II shows that the high-efficiency head has an output voltage of 5.08 mV (rms) at 700 Hz; Sec. 2 shows that N_{eff} (700 Hz) = 990; T (700 Hz) = 1.00; F (0.5 mm) = 1.00. Substituting these values in Eq. (4), we find that the rms tape flux for the Ampex Operating Level recording is 1.167 nWb; since the measured track width is 6.3 mm (248 mil), the rms tape flux per unit track width is 185 nWb/m. The measurement error is estimated to be less than 3%.

4. MEASUREMENT OF AMPEX OPERATING LEVEL BY TRANSFER TO UNIDIRECTIONAL MAGNETIZATION AND MEASUREMENT WITH A MAGNETOMETER

A measurement of a sinusoidal tape flux may also be made by transferring the sinusoidal flux to an equivalent unidirectional flux, and measuring the magnetic dipole moment with a magnetometer. Knowing the sample length, the moment is easily converted to the total flux, and this in turn to the rms value of the alternating flux.

4.1 Transfer to Unidirectional Flux

A 38 cm/s (15 in/s) full track recorder/reproducer was used. The reproducer first measured the head output voltage for the 700 Hz Ampex Operating Level Flux (as in Sec. 3) as 5.08 mV. Then the recorder was set up using a Hewlet-Packard Model 3300 Function Generator as the signal source for a 700 Hz sinewave. This signal voltage and a high-frequency bias voltage were resistively mixed (no capacitance in the signal leg), and fed to the recording head. The resistances were such that constant signal voltage produced constant recording-head current. A general-purpose tape was used, with a 10 μm (0.4 mil) coating thickness. The bias current was adjusted for maximum recording sensitivity with the 700 Hz signal; then the bias current level was increased 2 dB in order to provide the most stable recorded tape flux. The 700 Hz recorder input voltage was adjusted so that the reproducing head output voltage would be the same as it had been from the Ampex Operating Level Flux, *viz*, 5.08 mV. Therefore, the new recording has the same tape flux as the Ampex Operating Level.

The signal generator waveform was then changed to a squarewave having the same voltage as the peak value of the previously obtained sinusoid. The frequency was changed to 10 Hz, producing a recording of 3.8 cm wavelength. The recorded flux would therefore be short segments—1.9 cm each—of "unidirectional" flux, first of one polarity, then of the other polarity. (The boundary is easily found with *Edivue*.)

4.2 Measurement of Unidirectional Flux with a Magnetometer

A number of means are available for measuring unidirectional tape flux, including systems which measure the force on the sample when it is placed in a known magnetic field, or the induced voltage produced by relative motion between the sample and a coil. These methods are described in detail by Zijlstra [9]. A method commonly used now in magnetic tape laboratories is the vibrating sample magnetometer (VSM), and this method was used here.

From the squarewave tape, two samples of "unidirectional flux" of the opposite polarity were cut, each having a length of 6.35 mm (0.25 in). The VSM measures the magnetic dipole moment j and the value was 11.5 pWb·m for both samples.[4] The estimated error of measurement was less than 5%. Dividing the moment by the sample length gives the tape flux, Φ = 1.82 nWb.

It appears that the tape head senses the total tape flux, whereas the VSM senses only the longitudinal component of the flux. The perpendicular component is not too easily measured, but has been estimated to be about 15% of the total. For this ratio of perpendicular to total flux, the longitudinal component alone would be about 1% less than the total (vector) sum. Thus, for a longitudinal component of 1.82 nWb measured on the VSM, the corresponding total tape flux would be 1.84 nWb.

4.3 Conversion to the Corresponding Alternating Flux

Three correction factors are necessary to relate the total unidirectional flux measured by the magnetometer to the corresponding alternating flux per unit track width specified for Ampex Operating Level Flux.

First, the total flux must be divided by the measured tape width of 6.25 mm. Second, the peak flux must be divided by $\sqrt{2}$ to obtain the corresponding rms value. Third, the unidirectional flux value must be multiplied by the "thickness loss factor" for the wavelength λ of 0.5 mm (700 Hz at 38 cm/s). Since the tape thickness t was 10 μm, the thickness loss factor $(e^{-2\pi t/\lambda})$ would be $e^{-0.125} = 0.88$.

Thus the rms value of the corresponding 0.5 mm (700 Hz at 38 cm/s) alternating flux per width would be

$$\begin{aligned}\Phi/w &= [(1.84\,\text{nWb})\,(0.88)]/[(\sqrt{2})\,(0.00625\,\text{m})] \\ &= (1.84\,\text{nWb})\,(99.6/\text{m}) \\ &= 183\,\text{nWb/m}.\end{aligned}$$

4.4 Comparison of Results

The total tape flux per width according to the measurement with the calibrated heads was 185 nWb/m, with an error of less than 3%. The value calculated from the measurement with the magnetometer was 183 nWb/m, with an error of less than 5%. Thus the difference in the two measurements is less than 1%, which is ably less than the estimated error of either measurement.

[4] The measurement was actually 0.00918 emu. The conversion is that j (in emu) times $4\pi \cdot 10^{-10} \triangleq j$ (in Wb·m). The term, symbol, and unit "magnetic dipole moment, j, in Wb·m" are taken from IEC Publication 27, Item 86.

This agreement supports the procedures in Sec. 2 for calibrating the sensitivity of the ring-core reproducing head.

4.5 Sources of Error in Flux Transfer and VSM Measurements

There are an unusually large number of chances for error in the transfer to unidirectional flux and VSM measurement. Therefore, this is not a preferred method of flux measurement. We believe, however, that we have avoided these errors in the present measurements.

The known chances for errors are the following: First, the usual method of transfer to unidirectional flux is to measure the ac signal current for the recording, then to replace the ac generator with a dc generator, measure the current again, and adjust for the same current. Thus, both the ac and dc current measurement errors of the ammeters contribute to the total error. Also, if the impedance and/or grounding of the ac and the dc sources are not identical, a change in the bias current—and therefore in the recording sensitivity—may occur. In addition, it is difficult to keep the recording system free from unintentional direct currents, magnetizing fields, and bias asymmetries, all of which cause the tape flux of one polarity to differ from that of the other polarity. (The average value would be the desired value, under these conditions.) For all of these reasons, the measurement reported here used a generator which could be switched from a 700 Hz sinusoid to a 10 Hz squarewave of the same peak voltage (corresponding to head current). This produced a flux which was in fact measured to be identical for both polarities.

Second, the tape laboratories use their VSMs for relative measurements of bulk samples of magnetic materials, and the absolute measurement of magnetized samples of tape is somewhat different. The VSM is calibrated by means of a pure nickel sample of known saturation dipole moment per unit mass, and known mass. One must first of all be certain that the coils on the VSM are in fact able to fully saturate the calibrating nickel sample. Also, because the sensitivity of the pickup coils of the VSM varies with the sample size and position [9], it is necessary that calibrating (nickel) sample and unknown (tape) sample have identical sizes and positions.

Finally, the tape sample must be held rigidly so that it vibrates as a solid body rather than flapping.

5. CONCLUSIONS

The symmetrical head can be easily constructed of standard parts, and calibrated to measure medium-wavelength tape flux with an accuracy of ± 2%. This head, in turn, can be used to calibrate the efficiency of a "high-efficiency" head, whose practical advantage is that the efficiency does not change with wear of the head.

The Ampex Operating Level flux has been measured with the calibrated heads as 185 nWb/m ± 3%. The validity of the calibration of the heads has been verified by the process of measuring this same flux by the method of transfer to unidirectional flux and magnetometric measurement of the moment. The measurements by the two methods are identical within 1%, and this is taken as a proof of the validity of both flux measuring methods.

The calibrated-head method is preferred over the VSM method because it is much less subject to measurement mistakes and errors. The calibrated-head method is also recommended because it is usable in any tape recording laboratory; it requires no additional special equipment for its use. A calibrated high-efficiency flux measuring head for medium wavelengths (Ampex Part No. 4991005-1, similar to the head described here in Sec. 2.1.2) is commercially available from the Special Products Division of Ampex Corporation.

ACKNOWLEDGEMENT

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APPENDIX A

Determining the Stray Reluctance of a Gapped Circuit

A1 Calculation

An exact calculation of the front stray reluctance from the known physical dimensions is a difficult field problem. An approximate value might be determined from an electrolytic tank experiment, but this has not been attempted here. An approximate solution has been given by Unger and Fritzsch [10]. Figure A1 shows the geometry, and the approximate reluctance on each side of the gap is calculated from

$$R = \frac{2m\pi}{\mu_0 w_c \ln[(2m\pi L/l_g) + 1)]}.$$

The reluctance values have been calculated for "out-

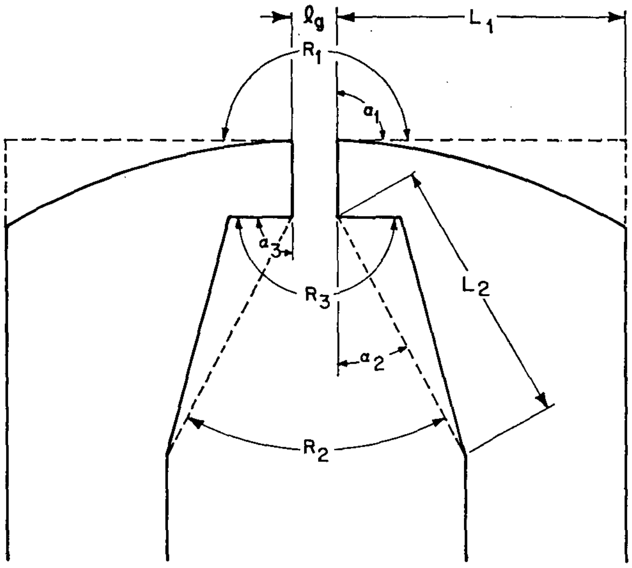

Fig. A1. Geometry of the head, for calculating the stray reluctances. **Solid outline**: actual core shape. **Dashed outline**: simplified core shape assumed in the calculation. For the stray reluctance outside, R_1, $a_1 = m_1\pi = 1.5$ rad, $L_1 = 5.5$ mm, $l_g = 25$ μm. For the maximum stray reluctance inside, R_2 $a_2 = m_2\pi = 0.4$ rad, $L_2 = 3$ mm. For the minimum stray reluctance inside, R_3, the values used for R_1 are used again.

side" and "inside" the gap, for gap lengths of 5, 25, and 100 μm; the results are given in Table A1. For the

Table AI. Calculated values for front stray reluctance R_{fs}.

			R_{fs} (MH^{-1})	
l_g (μm)	R_1 (MH^{-1})	R_2 (MH^{-1})	from R_1 and R_2	from $R_1/2$
5	43	14.5	11	21
25	50	20	14	25
100	67	28	20	33

outside reluctance R_1 the assumed geometry (dashed line in Fig. A1) and actual geometry (solid line) are nearly identical, and the calculated values should be applicable. For the inside reluctance, the formula assumes a taper from the gap area, as shown by the dashed line in Fig. A1, and the reluctance R_2. The actual head (solid line) has "tips" at right-angles to the gap plane, then a taper. The reluctance R_3 for the actual geometry would be something less than R_2, but not more than that outside (R_1). Thus the total front stray reluctance would lie between the values for the parallel combination of R_1 and R_2, and the value for half of R_1. This range is 11 to 33 MH^{-1} for gaps between 5 and 100 μm, having a "straight" or "tapered" inside. Therefore a value of 14 MH^{-1} will not be greatly in error for a range of geometries similar to those used in the main part of the paper (Figs. 9 and 10).

For narrow pole pieces (e.g., the 0.6 mm core widths used in cassettes) the reluctance of the top and bottom surfaces should also be included. This is easily done by increasing the value of w_c for R_1 by two gap depths.

A2 Experimental Measurement

The calculated front-gap stray reluctance may be experimentally verified by the following means: calculate the front-gap reluctance from the gap dimensions; build the corresponding head with a low-reluctance back gap (laminations of Fig. 10), measure the inductance, and calculate the total reluctance from the formula $R_t = N^2/L$, where N is number of turns (1000 total, in this case), and L the measured inductance; then, assuming all the reluctance to be at the front gap, find the stray reluctance as the difference between the total front reluctance and the calculated gap reluctance. This may be repeated for various gap depths, and for various gap lengths.

The results of the first experiment are shown in Table AII. The gap reluctance was calculated from the mea-

Table AII. Measurement of stray front reluctance from the physical dimensions and the measured inductance of one head with differing front gap depths.

Gap Depth d_g (μm)	R_{fg} (MH^{-1}) Calculated from Physical Dimensions	L (mH) Measured	R_f (MH^{-1}) Calculated from Inductance	R_{fs} (MH^{-1}) By Difference
173	12.8	139	7.2	16.5
76	29.2	108	9.3	13.5
0	"infinite"	65	16	16

Note: $l_g = 28$ μm except when $d_g = 0$; then $l_g = 0.5$ mm. $w_g = 6.9$ mm. $N = 1000$ turns.

sured gap length (25 μm), width (6.9 mm) and depth (173, 76, and 0 μm): $R_{fg} = l_g/(\mu_0 A_g)$. Then the measured inductance was used to calculate a total reluctance from the formula* $R_{fs} = R_{fg} \cdot R_f/(R_{fg} - R_f)$. The stray reluctance so calculated ranged from 13.5 MH^{-1} to 16.5 MH^{-1}.

In another similar experiment, ten heads of similar construction were used to calculate the front stray reluctance by the same procedure; the gap lengths ranged from 5 to 100 μm; the front gap depth was reduced on each of these heads from approximately 600 μm to approximately 350 μm, and the values of stray reluctance computed for both depths. The results are shown in Table AIII. For the 5 μm front gap length, the reluctance of the core and the rear gap is large enough to mask the front stray reluctance. (In fact, the "negative" reluctance of 190 and 220 kH^{-1} may be taken as a rough confirmation of the values found in Sec. 2.1.2 for the total rear reluctance, core plus rear gap.) Head 344 gave an unaccountably high value for R_{fs}, and this result will be omitted in the summary. The other seven heads gave values of R_{fs} of 11 to 16 MH^{-1}, with an average of approximately 14 MH^{-1}.

APPENDIX B

Measuring the Reluctance of the "Zero-Length" Rear Gap

In the main text, Sec. 2.1.2, we assumed a "wedge" at the rear gap (Fig. 11). Actually, the head construction used places the cores in a metal clamp as shown in Fig. B1, and the clamping may bend the cores sufficiently to eliminate the wedge and produce intimate contact. Thus the reluctance in "intimate contact" is of interest.

* This assumes all of the reluctance to be across the front gap. In fact, some of it is just around one half-core, as shown by the fact that the inductance of a single half-core in free space is 8 mH; two cores in series with no mutual coupling would therefore have an inductance of 16 mH, and the inductance due to the front stray reluctance for "zero" gap depth would be 65 − 16 = 50 mH, and the stray reluctance 20 MH^{-1}, rather than 16 MH^{-1}. Thus the error in neglecting this factor is small.

Table AIII. Measurement of stray front reluctance from physical dimensions and reluctance for ten heads.

Head No.	l_g (μm)	d_g (μm)	L (mH) Measured	R_{fg} (MH^{-1}) Calculated from Physical Dimensions	R_f (MH^{-1}) Calculated from Inductance	R_{fs} (MH^{-1}) By Difference
350	5	560	825	1.02	1.21	−0.19
351	5	610	862	0.94	1.16	−0.22
352	12.5	380	359	3.75	2.80	11
352	12.5	610	491	2.34	2.04	15.9
344	12.5	660	487	2.16	2.06	44.5
344	12.5	380	318	3.75	3.14	19.4
353	26	380	221	7.80	4.53	13.1
353	26	560	278	5.30	3.60	11.2
345	26	660	288	4.50	3.48	15.3
345	26	360	190	8.25	5.25	14.5
354	51	660	193	8.80	5.18	12.7
346	51	560	178	10.4	5.62	12.2
355	102	660	131	17.60	7.64	13.4
347	102	610	124	19.00	8.06	13.9

An experimental estimate of this "zero" gap length may be obtained from the measured inductance of a head having no intentional gaps in front *or* rear. Such a head, with $N = 1000$ turns, had a measured inductance of 4.8 H; thus the total reluctance was $R_t = N^2/L = 10^6/(4.8\ \text{H}) = 210\ \text{kH}^{-1}$. Subtracting the calculated core reluctance of 54 kH^{-1}, the reluctance of the gaps would have been 156 kH^{-1}. The front gap area was 0.5 mm by 6.9 mm. Since the rear gap area is eight times that of the front gap, we will assume for simplicity that all of the reluctance is due to the front gap. The effective front gap length in intimate contact would be $l_g = R\mu_0 A = (156\ \text{kH}^{-1})\ (1.26\ \mu\text{H/m})\ (0.5\ \text{mm})\ (6.9\ \text{mm}) = 0.7\ \mu\text{m}$. Using this estimated value as a rear-gap length and an area of 6.9 × 3.8 mm, the calculated rear-gap reluctance would be $R_{rg} = (0.7\ \mu\text{m})/[(1.26\ \mu\text{H/m})\ (26\ \mu\text{m}^2)] = 21\ \text{kH}^{-1}$, a value much smaller than the 90 kH^{-1} in the "wedge" condition assumed in Sec. 2.1.2 of the main paper.

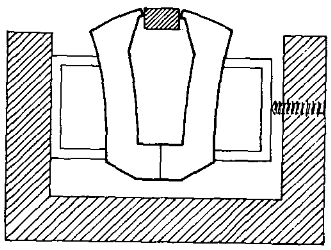

Fig. B1. Elimination of the "wedge angle" when the cores are clamped together tightly (effect greatly exaggerated for illustration).

APPENDIX C

Measurements on High-Efficiency Heads

In addition to calibrating a high-efficiency head by comparison with a symmetrical head of known efficiency, one can compare the consistency of sensitivities of "identical" heads, and the *ratios* of computed efficiencies for different gap lengths and depths, to *ratios* of measured efficiencies for the corresponding gap lengths and depths. For a truly high-efficiency head, the change of efficiency for changes of reluctances (gap depths and lengths) is very small, making these tests rather insensitive. They will show, however, that there is no gross error in the various other calculations and experiments.

C1. Consistency of Sensitivity of Identical Heads

A "negative" test for unaccounted-for reluctances is the consistency of sensitivity of identical heads. Table CI

Table CI. Consistency of flux efficiencies for seven pairs of "identical" heads.

Head No.	Gap Length l_g (μm)	Gap Depth d_g (μm)	Difference in Output Voltage Levels (dB)
350	5	560	0.05
351	5	610	
352	12.5	610	0.00
344	12.5	660	
352	12.5	380	0.00
344	12.5	380	
353	26	560	0.10
345	26	660	
353	26	380	0.10
345	26	360	
354	51	660	0.10
346	51	560	
355	102	660	0.00
347	102	610	

shows the differences in output voltage levels for seven pairs of heads with gap lengths from 5 to 100 μm, with approximately 600 μm gap depth. The heads with 12.5 and 25 μm gap lengths were also lapped down to approximately 380 μm gap depth and measured again. The Ampex Operating Level on 0.5 mm wavelength (700 Hz at 38 cm/s), was measured with these several heads. The greatest discrepancy is 0.1 dB (1%), which shows that no large uncontrolled errors are present. An additional group of 25 heads similar to No. 353 also showed similar consistency.

C2. Change of Sensitivity with Change of Gap Depth

Another test consists of calculating the flux efficiency of a head as the gap *depth* is changed; the corresponding change in sensitivity is measured when a given flux (the Ampex Operating Level) is reproduced and the gap depth is changed (by lapping). Table CII shows the

Table CII. Measured and calculated efficiencies versus gap depth.

Gap Depth d_g (μm)	Relative Output (%)	Relative Output × 0.99 (%)	Calculated $\eta(\Phi)$ (%)
394	97 ± 1	96 ± 1	97.1
335	98 ± 1	97 ± 1	97.5
281	100 ± 1	99 ± 1	97.6
234	100 ± 1	99 ± 1	97.8
173	100 ± 1	99 ± 1	98.0
127	100 ± 1	99 ± 1	98.5

calculated and measured efficiencies for different gap depths. Since only the *change* of sensitivity is measured (because the flux on the tape must be considered to be unknown), the sensitivity data must arbitrarily be fitted to the calculated efficiency data. The calculated and measured values agree fairly well if the efficiency at the "most sensitive" point is taken as 99%, R_{fg} is taken as 14 MH^{-1}, and R_r is taken as 150 kH^{-1}. (For this head, $l_g = 28$ μm, $w_g = 6.9$ mm, and $N = 1000$ turns.)

The accuracy of data required to make this type of test really valid is beyond that which is practical: the flux variations on the tape record are about ± 1% (0.1 dB); the meter scale divisions are also 1% (0.1 dB). These are considered good for ordinary measurements, yet the measuring errors are, in this case, as large as the data being measured! Thus these data "prove" only that no gross error has occurred.

C3. Change of Sensitivity with Change of Gap Length

Still another test consists of calculating the flux efficiency of a head as the gap *length* is changed; the corresponding change in sensitivity is measured when a given flux (Ampex Operating Level) is reproduced and the gap length changed.* This was achieved by constructing two each identical heads except for gap length. Table CIII shows that if a a rear reluctance of 90 kH^{-1} is assumed, the calculated and relative measured efficiencies agree quite well (errors of 1% or less). When efficiencies were calculated assuming the wedge rear gap, the results were considerably in error; thus the "closing-of-the wedge" effect of Appendix B apparently really does occur.

*The necessary corrections have been applied for the gap-length loss in the flux measurements: wavelength (700 Hz, 38 cm/s) = 550 μm;

l_g (μm)	5	12.5	26	51	102
l_g/λ	0.01	0.023	0.047	0.09	0.18
Gap loss factor	1.000	1.000	0.995	0.985	0.935
Gap loss level (dB)	0	0	0.05	0.15	0.60

Table CIII. Change in sensitivity with change of gap length.

Head No.	Gap Length l_g (μm)	Gap Depth d_g (μm)	$\eta(\Phi)$, Calculated $R_r \triangleq$ Wedge (%)	$\eta(\Phi)$, Calculated $R_r = 90$ kH^{-1} (%)	$\eta(\Phi)$, Measured (%) Re. to No. 354 and 355	Deviation I and II (%)
				I	II	
350	5	560	93.0	91.0	90.5	0.5
351	5	610	93.0	91.0	91.0	0.0
352	12.5	610	95.0	96.8	95.5	1.3
352	12.5	380	96.8	97.6	96.5	1.1
344	12.5	660	95.0	96.8	95.5	1.3
344	12.5	380	96.8	97.6	96.5	1.1
353	26	560	96.5	98.1	98.3	0.2
353	26	380	97.3	98.6	98.3	0.3
345	26	660	96.0	98.0	97.0	1.0
345	26	320	97.5	98.7	97.0	1.7
354	51	660	96.0	98.7	99.0	0.3
346	51	560	96.0	98.8	99.0	0.2
355	102	660	95.0	99.0	99.0	0.0
347	102	610	95.0	99.0	99.0	0.0

THE AUTHOR

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