

THE FREQUENCY DEPENDENCE OF RESISTANCE IN FOIL-WOUND INDUCTORS†

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(Received June 1, 1978; in final form August 1, 1978)

Foil wound air-cored inductors are widely used in power electronics applications, and often exhibit power dissipation much greater than can be accounted for by the d.c. resistance. This is shown to be due to non-uniform current flow. A scaling rule, relating the effects to turns and frequency, is demonstrated. The general principle of equal current sharing can be satisfied in a toroidal system.

1. INTRODUCTION

For some years aluminium foil or thin sheet has been used to wind air-cored inductors for power-electronics applications. The inductors are relatively easy to make and are structurally simple and strong. The foil is wound, together with a slightly wider insulating sheet of (say) melinex, usually on a cylindrical former. Compared with a copper winding with the same rating and inductance, an aluminium foil inductor tends to be slightly bulkier, considerably lighter, and probably somewhat cheaper in material costs. If the comparison is made between a conventional multi-layer wire coil and a foil coil of similar electrical properties, the latter tends to have better thermal dissipation because every turn provides its own all-metal heat path to the outside.

We became interested in foil coils at Loughborough about five years ago, initially not through their importance in power electronics, but as a vehicle to study self-resonance in capacitors. Using an equivalent circuit model¹ we were able to calculate the self-resonant frequencies of foil coils,² which are due to the large turn-to-turn capacitance, but through using the d.c. foil resistance we always seriously over-estimated the coil Q's. Our explanation was non-uniform current flow, and we began to study the theory of a one-turn foil coil.³ It was shown that the resistance of foil coils increases with frequency, and this "width effect" contributes to the overheating observed in some power electronics systems, particularly as chopper frequencies are increased,

even though the inductors are apparently correctly rated.

This paper will describe the main features of the "width effect," and also suggest that a further non-uniformity of current flow, across the thickness of the foils, is important at the higher frequencies.

2. WIDTH EFFECT

This term describes the tendency of the current in a foil coil to flow more densely near the edges of the foil than in the centre, as the frequency is raised. The detailed solution is given elsewhere,³ but the problem may be understood physically from the single-turn model of Figure 1. Let this turn be divided into equal strip-coils, such that in each one the current density is effectively uniform. The turn may now be represented by an equivalent circuit, Figure 2, of identical inductors, all in parallel and all mutually coupled. When the applied voltage amplitude is V , the currents are given by equations (one for each strip) of the form

$$V = I_{\alpha}R + j\omega L_{\alpha}I_{\alpha} + j\omega \sum_{\beta \neq \alpha} M_{\alpha\beta}I_{\beta} \quad (1)$$

So the current I_{α} in strip α is related to the currents I_{β} in all the others.

To understand the width effect physically, first suppose that all the currents are equal. Then any inductor representing a strip near the middle of the foil is subject to strong mutual couplings from near-neighbours on both sides, whereas a strip at the edge has near-neighbours on only one side, so the total inductive back-voltage there is less. Therefore

† This paper is based upon a lecture given by Dr. Murgatroyd to the Institution of Electrical Engineers colloquium on "Trends in Forced Commutation Components", on 12th January 1978.

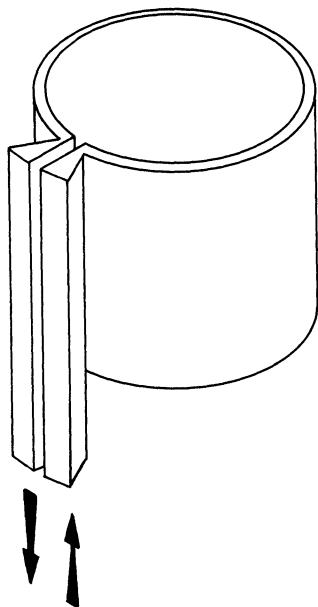


FIGURE 1 One-turn foil coil with torus-bars to maintain equipotential connections.

the currents cannot all be equal, and must therefore be larger near the edge than in the middle.

Our calculations used 40 equal strips to represent the turn. The current profile is clearly symmetrical, but this still leaves 20 complex unknowns, so requires the inversion of 40×40 matrices. For a given turn geometry the calculation is repeated for a range of frequencies. For increasing frequency, the following general results are obtained:

1) The current density at the centre decreases and lags in phase relative to that at the edge.

2) The turn inductance decreases, and the turn resistance increases somewhat faster, and the extra resistance is initially proportional to the square of frequency.

3) The axial magnetic field becomes increasingly uniform.

4) Within the model of "width effect," all the foregoing effects do not increase indefinitely with frequency, but approach a limit. Just as at d.c. the current profile across the width is uniform and is completely controlled by the foil resistances, there is in the high frequency limit another current profile that is completely controlled by the foil inductances. (This means that the known high-frequency properties of some coils cannot be due to "width effect" alone).

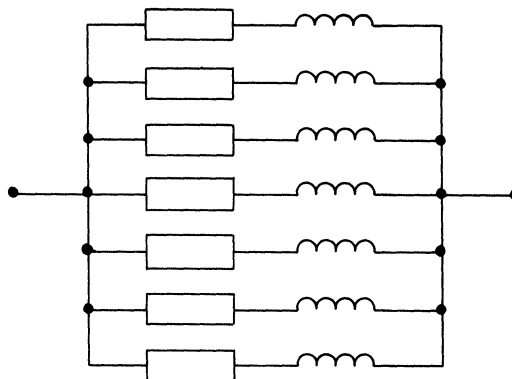


FIGURE 2 Equivalent circuit to represent "width effect" in a one-turn foil coil.

These results have been checked experimentally by bridge measurements over a range of frequencies.⁴ It was not possible to do this with a one-turn coil: we used sets of N -turn coils ($N = 25, 50, 100$) made with thin foil such that the buildup of turns was small compared with the internal diameter. In this instance the theory indicates a scaling rule for the "width effect": the proportional changes of inductance and resistance in a one-turn coil at frequency F will occur in an N -turn coil at frequency F/N . This rule is demonstrated in Figure 3: the coils were made from foil 60 mm wide, 50 μm thick, on an internal diameter of 64 mm. The rule predicts that with the abscissal scale of turns \times frequency, results for different coils must superpose, and this happens reasonably well. There is however a significant departure from the "width effect" theory at higher frequencies: theory predicts a levelling off, but in practice the resistance continues to rise with frequency. This is believed to be due to another effect, current non-uniformity across the foil thickness, and will be discussed.

Unlike our coils, which were specially made to test the theory, the commercial coils for use in thyristor circuits usually have a much larger build-up, to perhaps a square cross-section of windings, and foils perhaps as thick as 1.5 mm are used. Calculations for this type of coil have been done⁵ by an essentially similar method, except that the coil is divided into subsections along the foil length as well as axially. The equivalent circuit then has the form of Figure 4, and it allows for the possibility that the current density profile across the width may itself be dependent on radius. Small differences were calculated. The overall resistance increases calculated

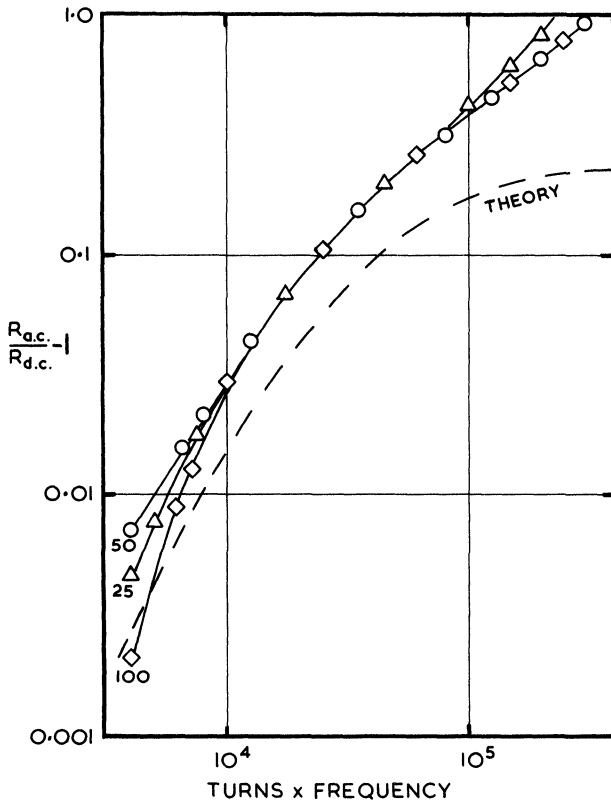


FIGURE 3 Resistance vs. Turns-frequency product in multiturn foil coils, demonstrating scaling.

were 19% and 23% for two particular cases, and most of the calculations were for a fixed frequency of 50 Hz. This is an important limitation, for thyristor chopper circuits at over 500 Hz are now in use, and the waveforms have a high harmonic content. Our measurements have shown resistance increases of over 300% in thin foil coils, and much more in thick foil coils. The conclusion drawn is that while calculations of "width effect" are moderately successful at low frequencies, they eventually fail,

and it is believed that the current density profile must also become non-uniform across the foil thickness.

3. THICKNESS LOSSES

It might perhaps be thought that provided the penetration depth δ , given by the standard formula, is large compared with the foil thickness λ , then no appreciable non-uniformity is possible. However, every foil is subject to the axial field of all the others: this collective field acts most strongly in the innermost and outermost turns. A simple theory suggests the formula

$$R_{a.c.} = R_{d.c.} \left(1 + \frac{N^2 \lambda^4}{72 \delta^4} \right) \quad (2)$$

This formula depends on rather sweeping assumptions and is not accurate. However the number 72 is an upper limit, so thickness losses will probably be worse than the formula suggests, and are additional to "width effect." The corresponding formula for skin effect in a single foil is

$$R_{a.c.} = R_{d.c.} \left(1 + \frac{1}{45} \frac{\lambda^4}{\delta^4} \right) \quad (3)$$

so the collective effect of N -turns is clear.

The formula may possibly explain a curious observation. In a particular inverter application, a coil ran hot, and was replaced by another with thicker foil conductors and presumably higher current rating. However the new coil ran even hotter. The decrease of d.c. resistance may well have been overtaken by the dependence on λ^4 in the thickness losses. The formula in Eq. (2) also may explain why our measurements, shown in Figure 3, still obeyed the turns-frequency rule at large departures from the "width effect" theory. Since N^2/δ^4 is proportional to $N^2 \omega^2$ the rule still holds.

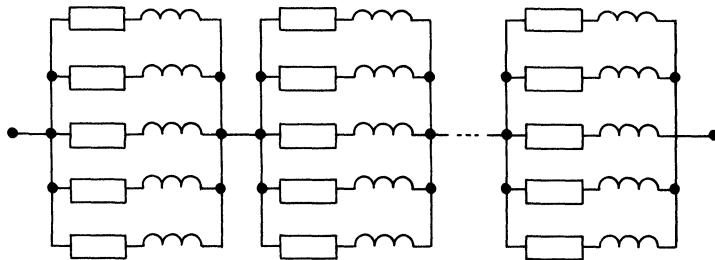


FIGURE 4 Equivalent circuit to represent "width effect" in a multiturn foil coil.

4. APPROACHES TO OPTIMUM DESIGN

There are a number of possible criteria for optimum design of foil coils, depending on their applications. The optimum shapes have been calculated⁵ for best Q , against "width effect" losses. However these shapes are only likely to be valid at quite low frequencies. In applications involving pulsed currents, hence high harmonics, it is likely that the thickness losses will be important, and ought to be considered in any situations where the effective a.c. resistance is more than 50% greater than the d.c. resistance.

A general principle for design is evident in Figure 2 and Figure 4. If it can be arranged that, for every inductor in each one-turn circuit, the sum of the mutual inductances with *all* other inductors in the circuit is a constant, then the current is equally shared and the "width effect" is eliminated.

A geometry that has this property is shown in

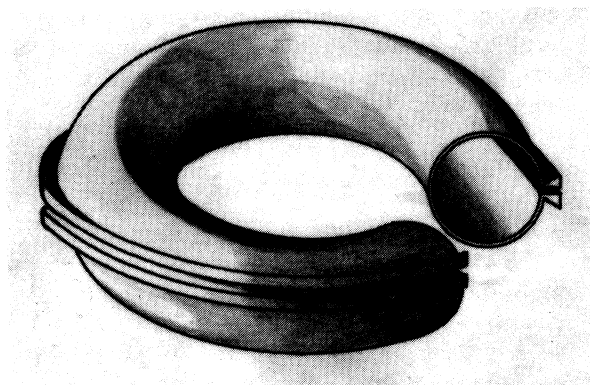


FIGURE 5 The one-turn foil torus (cut away drawing): an optimum design against "width effect."

Figure 5. The torus is to be considered as complete, the break is only for purposes of illustration.

Clearly in this one-turn foil coil all current paths round the minor circumference, from one bus-ring to the other, are geometrically identical to each other and in their relation to the coil as a whole.

The authors do not imagine that such a construction, especially with many turns, is a practical or commercial proposition. However the basic principle of current sharing, together with the fact that toroidal coils enclose their flux and so do not create eddy currents and interference elsewhere, suggest that for the most demanding applications an approach to toroidal design, perhaps in paralleled sub-sections, will be required.

ACKNOWLEDGEMENTS

We wish to thank Mr. B. Martindale of Aluminium Inductors Ltd. for supplying test devices, and Mr. P.G. LeGood of Loughborough University for his telepathic artwork.

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