Simple, Quasi-Transient Accurate Second Order 2-way Crossovers

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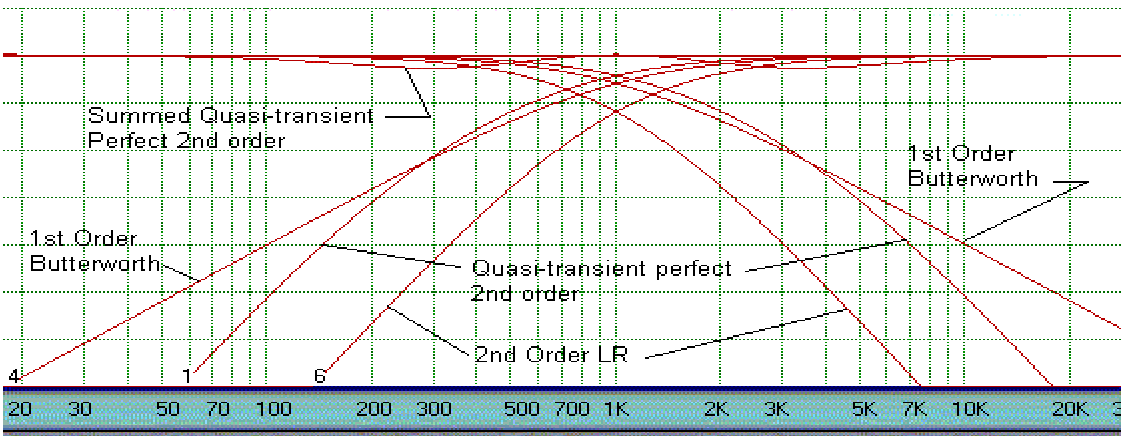
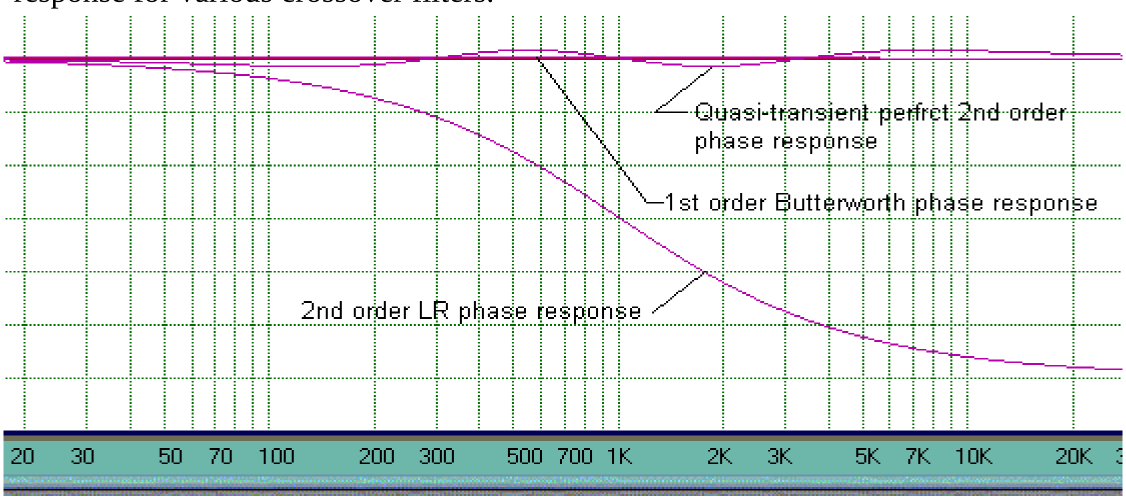
My obsession with crossover networks that yield transient perfect, or near transient perfect response began many years ago. In this document I present a discussion of a crossover network based on simple 2nd order filters. The high and low pass filters are over lapped in an effort to yield a response that is characterized by nearly flat magnitude response while having the necessary minimum phase characteristic which results in close to perfect impulse or transient response. The analysis is based on work I initially did back around 1978. Enjoy the discussion.



Comparison of individual filter magnitude response and summed Quasi-transient perfect response for various crossover filters.



Comparison of summed phase response for the filters shown above.

I was going through some old note on crossover design from back in 1978 the other day and I came across some early work I had done back then on 2nd order transient prefect crossovers, or at least a means of approximating a transient perfect crossover using 2nd order filters. Here I have revisited that effort and formalized the analysis somewhat. To begin, let's look at the simple 2-way 1st order Butterworth crossover which we all recognize as being transient perfect when the high pass and low pass filter outputs are summed in phase. We know that the response sums flat with zero phase for all frequencies. Looking at the relationship between the low pass and high pass output at the crossover point we can construct a diagram as shown in Figure 1.

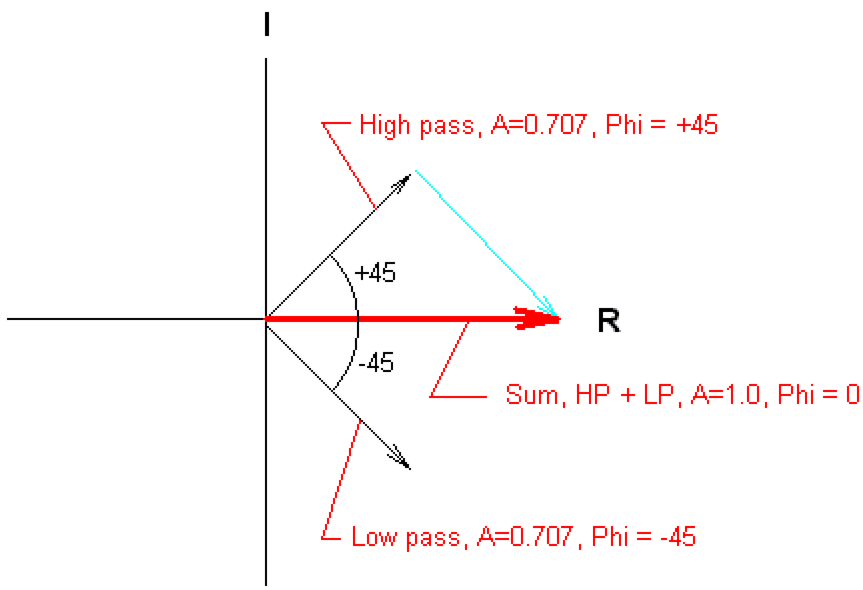


Figure 1. Amplitude vectors in complex plane for 1st order Butterworth high pass and low pass filters ans summed response. The high pass output has an amplitude of 0.707 (-3dB) at +45 degrees. The low pass output has an amplitude of 0.707 (-3dB) at -45 degrees. The summed response has an amplitude of 1.0 (0dB) at 0 degrees.

Figure 1 is a vector plot of the amplitude response in the complex plane. The horizontal axis is the real axis and the vertical axis is the imaginary axis. As can be seen, the high pass filter output has a magnitude of 0.707 at a phase of 45 degrees. The low pass filter also has a magnitude of 0.707 but at a phase of -45. Summing the amplitude vectors for the high pass and low pass results in the red summed response vector of magnitude 1.0 and phase zero.

Examining a 2nd order Linkwitz/Riley crossover at the crossover point using the same approach results in the diagram shown in Figure 2.

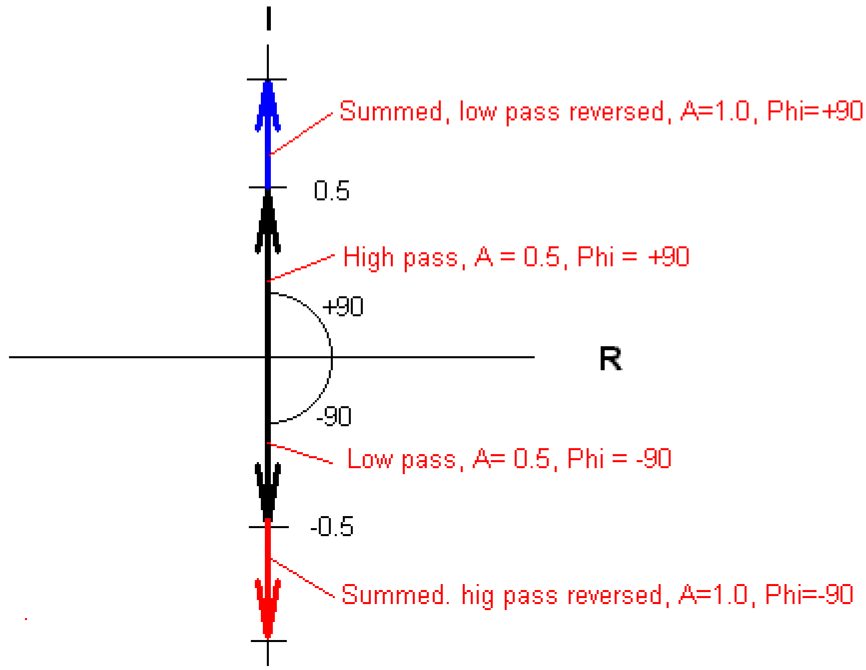


Figure 2. Amplitude vectors in complex plane for a 2nd order LR high lass and low pass filters and summed response. The high pass output has an amplitude of 0.5 (-6dB) at +90 degrees. The low pass output has an amplitude of 0.5 at -90 degrees. The summed response has an amplitude of zero with a phase of zero when the filters are summed without reversing the phase of one filter. If the low pass filter is reversed, the summed response has an amplitude of 1.0 (0dB) at +90 degrees (blue). If the high pass filter is reversed the summed response is 1.0 at -90 degrees (red).

In this case we see that the high pass filter output at the crossover point has magnitude of 0.5 at a phase of +90 degrees. The low pass filter has magnitude of 0.5 at a phase of -90 degrees. Note that this means the magnitude vectors point vertical, but in opposite direction. Thus, regardless of the magnitude, the summed result will be zero for any two filter sections connected with normal polarity, provided both have the same magnitude response. On the other hand, reversing one of the filter's polarity allows the output of both filter sections to simply be added algebraically. For the Linkwitz/Riley case the summed response is 1.0 at +/- 90 degrees, depending on which filter has its polarity reversed. Unlike the 1st order filter case, there is no component of the response of either filter section in the horizontal direction, or along the real axis. As a result, it is never possible to obtain a summed response with zero phase as long as the filters are crossed over at the point where the phase is +/-90 degrees.

However, the phase of a 2nd order filter, either high pass or low pass, varies continuously with frequency. This begs the question, "What would happen if the high pass and low pass filter sections were designed such that they crossed over at a point where the phase was less than +/- 90 degrees, but greater than the +/-45 degrees of a 1st order filter. This is shown conceptually in Figure 3. The idea is to design the filter sections in such a manner that they crossover symmetrically with the phase in the shaded area.

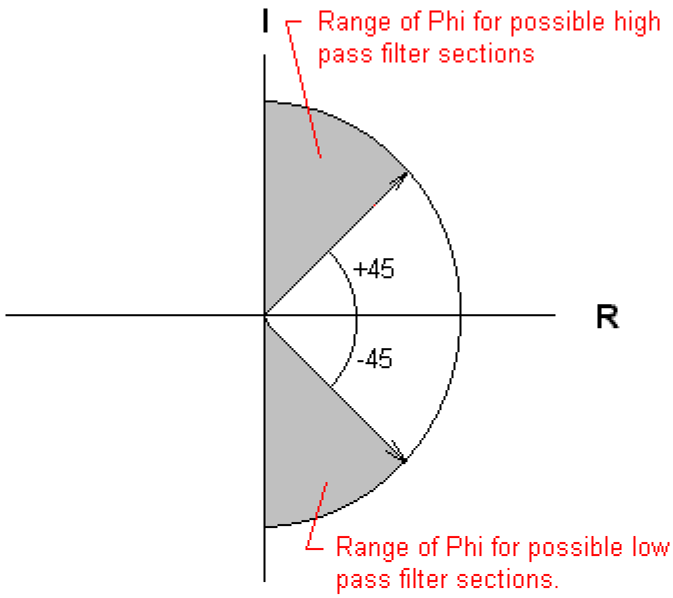


Figure 3. Regions where symmetric high pass and low pass filters could be constructed such that the summed response at the crossover point is 1.0 with a phass of 0 degrees. The high pass filter would have +45 < Phi <90 degrees. Th low pass filter would have -90 < Phi < -45.

By symmetric we mean that the magnitude of the high pass and low pass sections are the same at the crossover point and that the phase of the low pass section is the negative of the phase of the high pass section. As long as the phase is not 90 or -90 degrees, a summed response with finite magnitude and zero phase can be obtained at the crossover point. However, we still have to be concerned with how the magnitude and phase response varies to either side of the crossover point. For example, can a flat amplitude response with zero phase be obtained for all frequencies? Those of us who are familiar with 2nd order filter theory know that without additional response shaping the 2nd order filter transfer functions obtainable from standard RLC circuits can not yield a flat magnitude response with zero phase. But how close can we come? Can a crossover that has acceptable magnitude response and close to zero phase response be obtained? This is examines in the following paragraphs.

We start be writing the transfer functions for the standard 2nd order low pass and high pass filters. The transfer function for a second order low pass filter can be expressed as

LP section:

$$T_{LP}(s) = \frac{1}{(1 - s^2) + j\, s/Q}, \qquad (1)$$

where s is the reduced frequency. ω/ω_0, and ω_0 is the crossover frequency. Similarly, the transfer function for a second order high pass filters can be expressed as,

HP section:

$$T_{HP}(s) = \frac{-s^2}{(1 - s^2) + j\, s/Q}, \qquad (2)$$

At the crossover point, s = 1.0, the phase of the LP function of Eq(1) is -90 degrees and that of the HP section is +90 degrees. What we want to do is to be able to change the phase at the crossover point. We can do this by overlapping the filters. We can generalize the transfer functions of Eqs. (1&2) as,

LP section:

$$T_{LP}(s) = \frac{1}{(1 - (s/\gamma)^2) + j\, s/(\gamma Q)}, \qquad (3)$$

and

HP section:

$$T_{HP}(s) = \frac{-\gamma s^2}{(1 - (\gamma s)^2) + j\, s\gamma/Q}, \qquad (4)$$

where γ is an "overlap" parameter. For Eq. (3) to yield a phase of -90 degrees, as would be the case at the crossover point of a standard 2nd LP filter, the first term in the denominator, $(1 - (s/\gamma)^2)$, must be zero, or $s = \gamma$. Thus, if $\gamma > 1$, s must be greater than 1 at the point where the phase is -90 degrees. Hence, that phase at the crossover point (s=1) will be greater than -90 (that is a smaller negative value). Without further discussion, the inverse holds for Eq. (4); with $\gamma > 1$ the frequency at which the phase is 90 degrees for the high pass sections will be below the crossover point and the phase at the crossover point will therefore be less than 90 degrees.

Focusing on the low pass section, the amplitude of the transfer function is given as,

$$|T_{LP}(s)| = \frac{1}{\sqrt{(1-(s/\gamma)^2)^2 + (s/(\gamma Q))^2}}. \qquad (5)$$

and the phase as.

$$\tan(|\phi|) = \frac{s/\gamma}{Q(1-(s/\gamma)^2)}, \qquad (6)$$

We then ask the question, "For a specified value of ϕ at the crossover point, s=1, within the limits of $45 < \phi < 90$ degrees for the high pass section and $-90 < \phi < -45$ degrees for the low pass section, what are the values of γ and Q for the HP and LP sections of a crossover that would yield a summed amplitude of 1.0 with a phase of zero degrees at the crossover point?" To determine this, first assume that symmetric filters are to be implemented. Under this constraint the values of $|\phi|$, γ and Q will be the same for both the LP and HP filter sections. If the amplitude is to sum to 1.0, then with $\phi_{HP} = -\phi_{LP}$, the amplitude of the individual filters' transfer function is given as,

$$|T(s=1)| = 0.5 / \cos(\phi) \qquad (7)$$

As an example, consider the 1st order filter where ϕ is +/-45 degrees at the crossover point. The expression above tells us the amplitude of the filters sections should be 0.707 (or -3dB) at the crossover point as was shown in Figure 1. For a conventional 2nd order crossover, with $\gamma = 1.0$, the phase is +/- 90 degrees at the crossover point and the expression above tells us that the amplitude is undefined for the desired condition. This is correct and simply tells us what we already know about standard 2nd order crossovers, namely that when wired in phase the will always sum to zero. To have them sum to finite amplitude one filter section must be connected with the polarity reversed. The need to reverse the polarity on one filter section is removed as soon as we restrict the phase to be less (greater) than 90 (-90) degrees at the crossover point. This is what is accomplished by overlapping the filters to some degree, which is the reason the overlap parameter has been introduced.

Returning to the evaluation of γ and Q, we first define

$$a = \frac{1}{\mathrm{Tan}(|\phi|)}, \qquad (8)$$

and

$$b = \frac{1}{|\,T(s=1)\,|}, \qquad (9)$$

Combining these equations with equations 5 and 6 we obtain the following expressions for γ and Q.

$$\gamma = \frac{1}{\{1 - [a^2b^2/(1+a^2)]^{1/2}\}^{1/2}}, \qquad (10)$$

and,

$$Q = \frac{1}{\gamma}\,[\,(1 + a^2)/\,b^2]^{1/2} \qquad (11)$$

Thus to find the values of γ and Q we first choose the desired phase angle, compute "a" from Eq (8), then compute "b" from Eqs (7&9), and then compute γ and Q directly from Eqs (10&11). We can then go back to Eqs (3&4), and completely specify the transfer function characteristics.

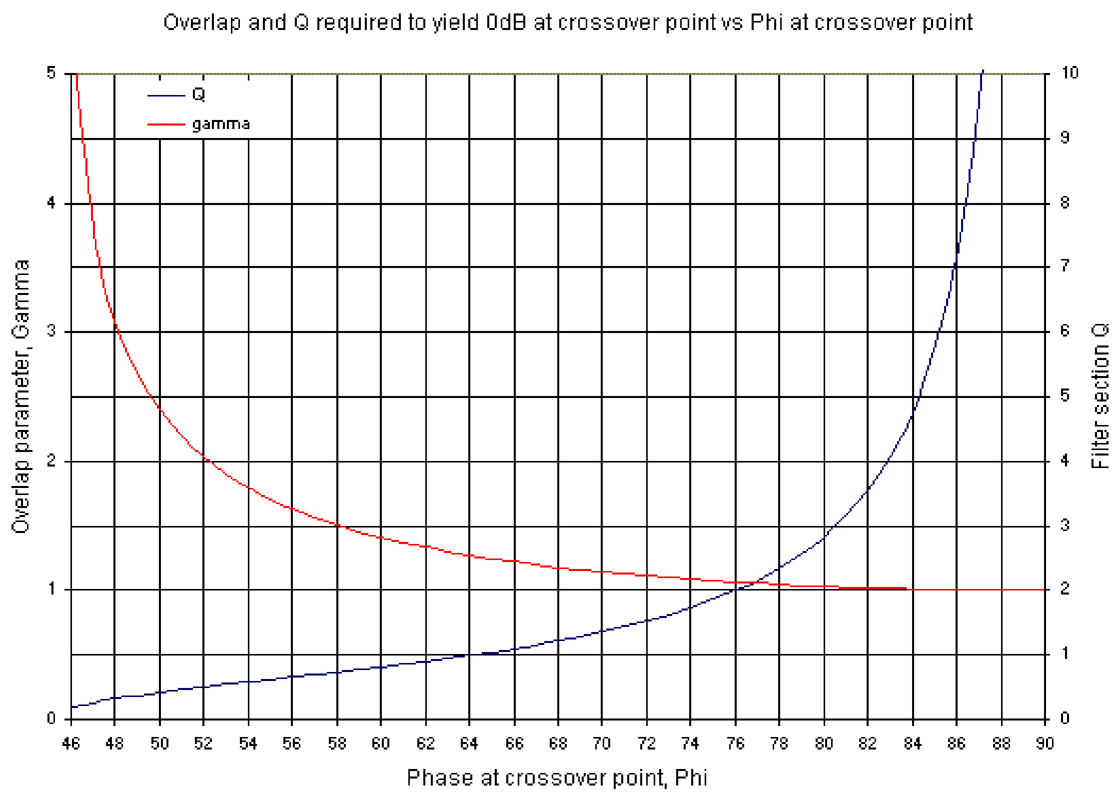


Figure 4. Variation of γ and Q with phase at the crossover point.

Figure 4 shows the variation of γ and Q with the phase specified at the crossover point. A couple of points should be made with regard to the behavior as the phase approaches 45 and 90 degrees. As 45 degrees is approached Q goes to zero and γ goes to infinity. This is interpreted as the filter sections reducing to true 1st order filters. As 90 degrees is approached, γ goes to 1.0 and Q goes to infinity. Between 45 and 90 degrees we have a continuous range of candidates for a potential crossover. But will any of them provide the desired response; reasonably flat, minimum phase and acceptable lobing. Without offering proof, it is noted that any crossover constructed in accordance with the relationship between γ and Q given in Figure 4 will have a minimum phase summed response when the filters are connected with the same polarity. We also know that at the frequency extremes and at the crossover point, the magnitude response will be 1.0. Thus we need only be concerned with possible ripples in the response to either side of the crossover point. (Minimum phase does not guarantee flat response.)

As an example of such a crossover, I simulated a 2-way speaker system where the phase at the crossover point was +50 and -50 degrees for the high and low pass filters, respectively, at the crossover point. For the simulation the drivers were assumed to be perfect, point sources. The simulations were made using LspCAD and SoundEasy. The on axis frequency response is shown below in Figure 5

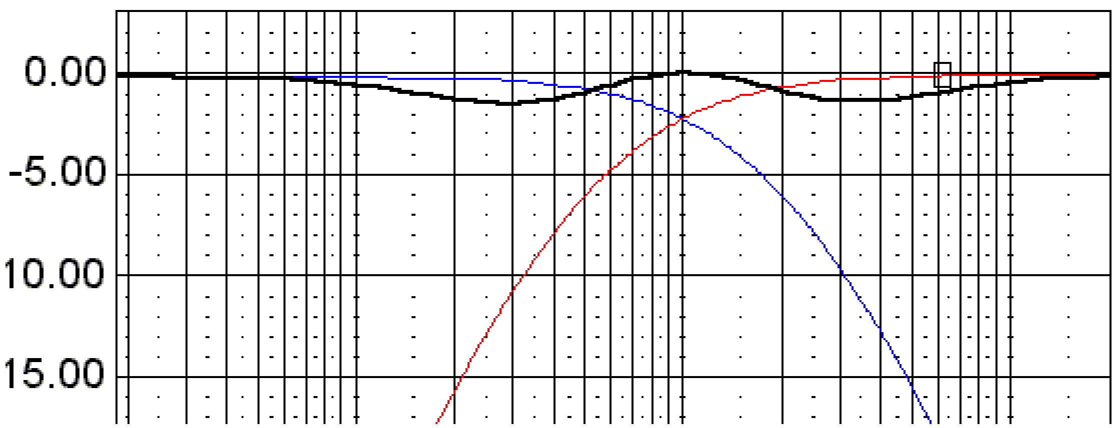


Figure 5. High pass, low pass and summed magnitude response for a crossover constructed such that the individual filter phase at the crossover point is +/- 50 degrees for the high pass and low pass sections, respectively.

The result shows the individual filter response and the summed response. The dips on either side of the crossover point (1 K Hz) are not greater than -1.5dB. This would seem to be acceptable, particularly when the variations in SPL level of real drivers in entered into the picture. The phase response is better than +/-5 degrees. Increasing the phase at the crossover point to a values greater than 50 degrees results in deeper dips to each side of the crossover and greater deviation from zero phase, reducing the phase reduces the dips.

Figures 6 and 7 show the off axis response for the crossover of Figure 5 used for a simple MT system and for an MTM system.

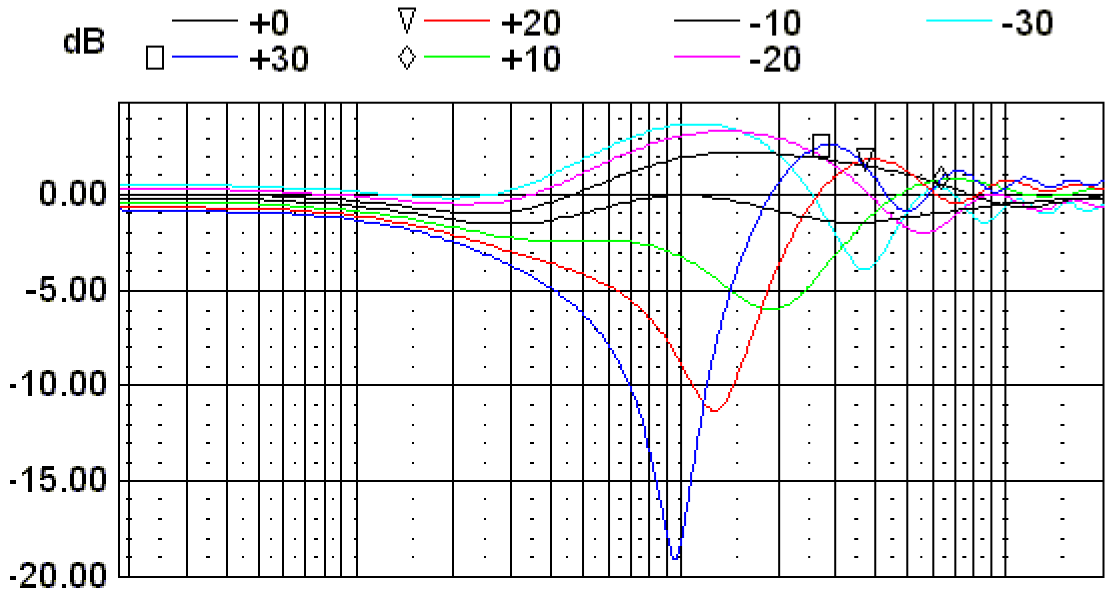


Figure 6. Vertical off axis response of an MT speaker system using the crossover of Figure 5. 15 cm driver spacing.

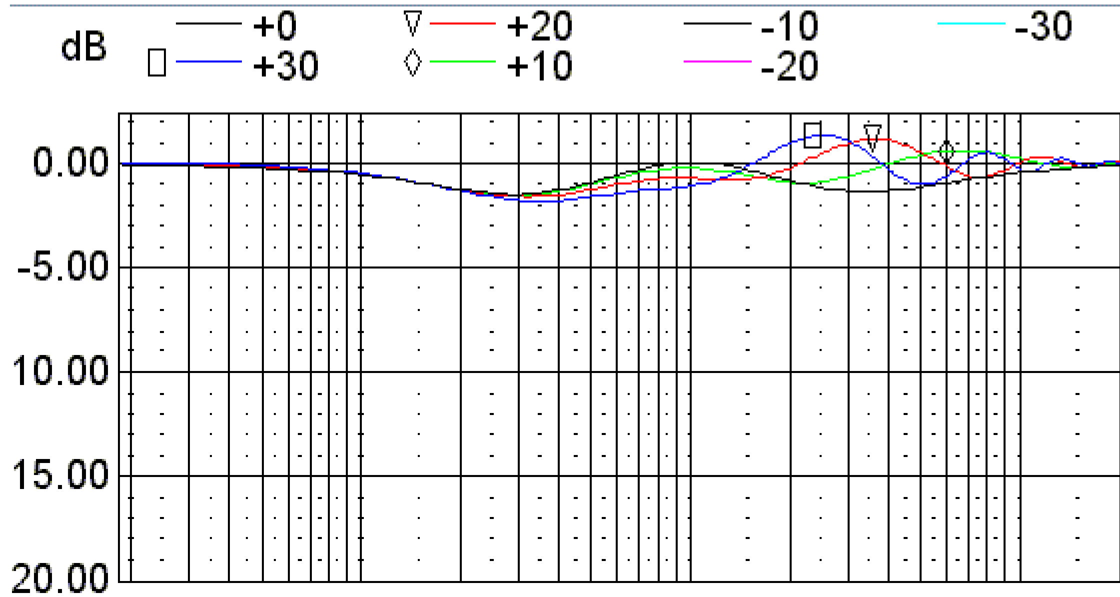


Figure 7. Vertical off axis response of an MTM speaker system using the crossover of Figure 4. 15 cm driver spacing

The results shown for the MT speaker in Figure 6 is similar to that of a 1st order crossover, however the variations in magnitude response are slightly greater. This may prove unacceptable in practice. However, the MTM system response, shown in Figure 7, is really very good with off axis variations in the +/- 2dB range. Thus this type of crossover may provide more than suitable performance when used in an MTM design.

But simply having reasonably flat response is not the goal of this design approach. The bottom line is how good is the transient response? After all, there are many crossovers that yield perfectly flat magnitude response but at the expense of poor transient response. Clearly if we must give up some flatness in the magnitude response we would hope that our goal of obtaining better transient response is realized. Figures 8, 9 and 10 below show the impulse response of the commonly used 2nd and 4th order L/R crossovers and the present design.

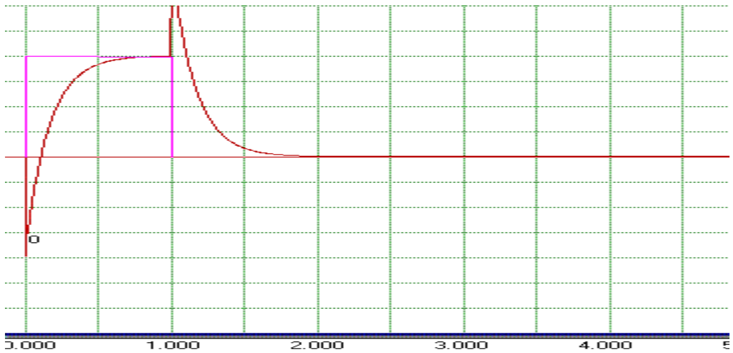


Figure 8. 1.0 msec Impulse response for 2nd order L/R crossover

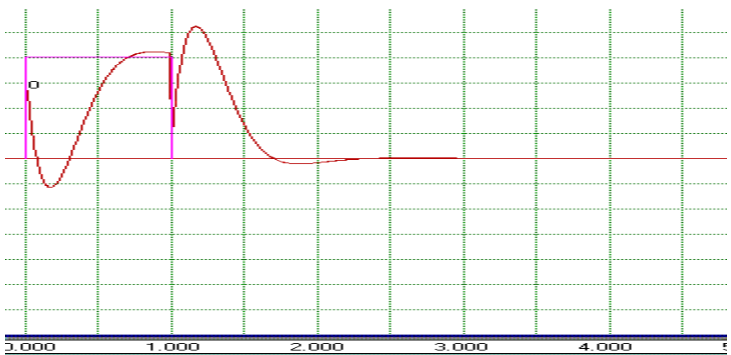


Figure 9. 1.0 msec Impulse response for 4th order L/R crossover

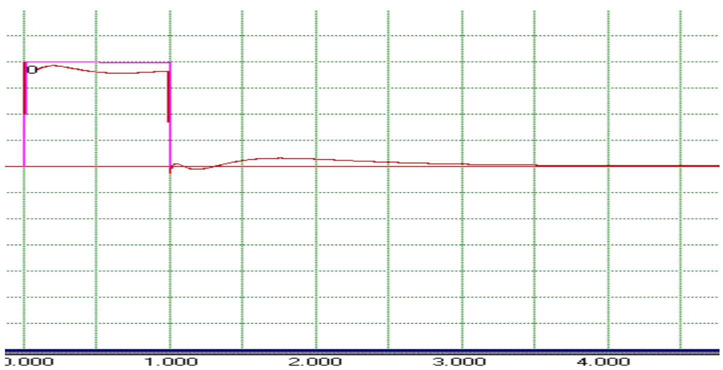


Figure 10. 1.0 msec Impulse response for the present crossover design

As is evident, the 2nd order LR crossover, shown in Figure 8, has very poor impulse response. Due to the need to connect the tweeter with reversed polarity, the initial response to the impulse input is actually in the wrong direction. Then the woofer component brings the response back positive. On the decay side of the impulse the reverse is true. The tweeter first fires positively, followed by the woofer negative pulse bringing the response back to zero amplitude. The 4th order LR crossover, shown in Figure 9, is not much better. In this case at least the woofer and tweeter are connected with the same polarity and their respective contributions to the impulse response are both in the same direction. However, due to the significant phase shift across the frequency band, there is a considerable delay between the tweeter and the woofer and some ringing is evident. In fact, the tweeter response is decaying before the woofer even gets started.

Finally the present overlapped design is shown in Figure 10. The response is not perfect, but it is surely an obvious improvement to either of the LR crossovers. The rise and fall of the impulse is very accurate with only slight deformation in the flat, constant amplitude part of the impulse. Clearly the trade off of perfectly flat magnitude response for improved impulse response has been achieved. However, is this trade off warranted? We know there are any number of other crossover configurations that yield both perfectly flat magnitude response ***and*** perfect impulse (or transient) response. The simplest of these is the 1st order crossover. The benefit of the present crossover design over the 1st order crossover is s that it affords greater driver protection in the stop bands than the 1st order crossover due to the greater roll off rate (see figures on front cover). When compared to higher order transient perfect crossovers we note that those crossovers usually require additional response shaping elements in the filter circuit or equalization networks. The advantage of the present design over those types of crossovers is one of simplicity. The present design also has several advantages over time-aligned system using 2nd order Bessel low pass filters and 1st order high pass filters. First, the Bessel time aligned approach is an approximation to a linear phase or constant delay system, and the tweeter acoustic center must be mounted well behind the woofer's acoustic center to make high frequency delay consistent with the delay of the Bessel low pass filter. This leads to highly offset or steeply sloped baffles such, as those of the older Spica systems. The present design requires the more conventional alignment of the drivers' acoustic centers since it approximates a minimum phase, zero delay system. Additionally, for the same crossover point the present design again provides greater tweeter protection.

In closing I would like to make a few comments about why I believe that transient perfect crossovers are superior to standard crossovers, be they the even ordered LR type of odd order Butterworth. The common argument is that we can not hear phase differences in musical tones. While this continues to be an area of argument between audiophiles and theorists, it remains an unresolved issue. However, perhaps a more relevant issue, and one that may be easier to grasp is that associated with the over shoot and ringing of the impulse response as shown in Figures 8, 9 and 10. For example, ignoring the observation that the tweeter is firing in the wrong direction in Figure 8, clearly there is still the issue of overshoot. The initial response to the impulse is significantly greater than the signal. In the real case there would be some limitation on this due to the limited band width of the tweeter, but never the less, this over shoot would result in excessive SPL levels for the initial rise to a sharp musical tone. This may be perceived as coloration to the sound of such instruments as a piano or other percussion instruments. Ultimately this may be perceived as unnatural or even fatiguing. Obviously, this depends on the sensitivity of the listener to such effects, but I believe that the issue goes beyond simply arguments on the audibility of phase shifts. From my personal experience I have found that among speakers that are relatively equal on all other factors, those with superior transient response prove to sound more natural in the long term when listened to on their design axis.