7 - 12 SEPTEMBER 2014, KRAKÓW
FORUM ACUSTICUM

A short-term analysis of a sigma-delta modulator with nonstationary audio signals

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Summary
Signal conversion quality of analog-to-digital (ADC) and digital-to-analog (DAC) audio converters with structurally embedded sigma-delta ($\Sigma\Delta$) modulators mainly depends on the $\Sigma\Delta$ modulator's parameters. Conventional quality examination of audio ADCs and DACs has been performed in the frequency domain and can be considered indicative for quality only in the case of linear and stationary data conversion systems. However, highly nonlinear and nonstationary $\Sigma\Delta$ modulators create errors which depend on the input signal. In this study, a new method for evaluating $\Sigma\Delta$ modulators in the time domain is proposed. Simulations and analysis were performed with the use of music signals at the input fed to 1-, 3- or 5-bit, 1^{st}, 3^{rd}, 5^{th} and 7^{th} order $\Sigma\Delta$ modulators. Results showed that the short-term performance of $\Sigma\Delta$ modulators is highly correlated with the variation of the input signal. This is particularly important as $\Sigma\Delta$ modulators are commonly used in ADCs and DACs of both consumer and professional audio equipment.

PACS no. 43.60.Dh, 43.60.Wy

1. Introduction

Audio systems used for recording, processing, storage, transmission and playback of sound must meet specific technical requirements, since audio signals display large dynamic changes both in time and amplitude. This applies in particular to the high-fidelity (hi-fi) audio systems equipped with analog-to-digital (ADC) and digital-to-analog (DAC) audio converters.

Audio converters can be classified into two groups: multi-bit, conventional PCM audio converters and one- or multi-bit ADCs and DACs equipped with low-pass, analogue and digital sigma-delta ($\Sigma\Delta$) modulators, respectively. Unlike conventional PCM audio converters, the latter are produced from elements of average parameter tolerances and their production costs are relatively low. Therefore, $\Sigma\Delta$ audio converters are now commonly used in both consumer and professional audio equipment.

Signal conversion quality of ADC and DAC audio converters with structurally embedded $\Sigma\Delta$ modulators depends mainly on $\Sigma\Delta$ modulator parameters. Conventional quality examination of audio ADCs and DACs has been performed in the frequency domain with the use of stationary input signals by measuring

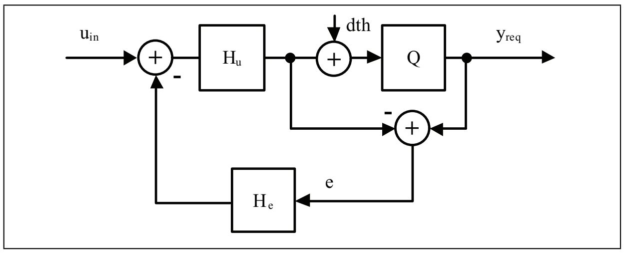

Figure 1. Block diagram of digital $\Sigma\Delta$ modulator with LSB's error feedback structure and optional dither signal.

such parameters as signal-to-noise ratio (SNR), total harmonic distortion (THD), spurious-free dynamic range (SFDR) or signal-to-noise and distortion ratio (SINAD), and can be considered indicative for quality only in the case of linear and stationary data conversion systems. The $\Sigma\Delta$ modulator, which is presented with the block diagram in Figure 1, can only be considered as a linear and stationary system when its parameters do not change over time and the requantizer Q is replaced by an additive white noise source. However, highly nonlinear and nonstationary $\Sigma\Delta$ modulators create errors which depend on the input signal. Both for one- or multi-bit requantizers enclosed in the noise-shaping loop of $\Sigma\Delta$ modulators the output depends on the variation in the input signal and is severely influenced by the past history of signal modulation and requantization error of conversion. As a result, the signal quality of ADC and DAC audio sys-

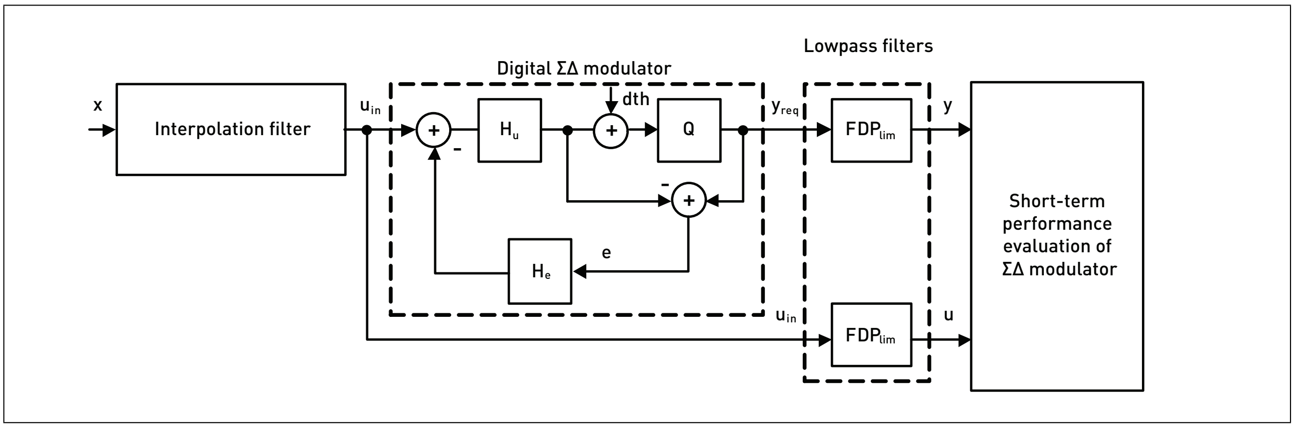

Figure 2. Block diagram of the proposed time domain method for evaluation of ΣΔ modulators.

tems with ΣΔ modulators varies over time for such signals as speech and music. Thus, conventional quality examination carried out in the frequency domain fails to explain the performance of the ΣΔ modulator with the use of nonstationary input signals. As others have highlighted [1, 2, 3, 4, 5, 6] investigations into short-term performance evaluation in the time domain are essential for describing ΣΔ modulators performance entirely.

2. Time domain evaluation method

In this study, a new method of the evaluation of ΣΔ modulators in the time domain is proposed. Its main processing blocks, namely an interpolation filter, a digital ΣΔ modulator, lowpass filters FDP_{lim}, and the main block of time domain evaluation are presented in Figure 2. This method introduces the estimation of nonlinear modulation error (NME) signal with the use of nonstationary signals at the input of the ΣΔ modulator. The NME signal is represented by its mean value and equivalent power calculated for input signal time intervals corresponding to the specific input signal parameters.

2.1. Interpolation filter and digital ΣΔ modulator

Nonstationary input signal x (see Figure 2) of a given sampling frequency and a resolution of 16 to 24 bits is oversampled 64 times and processed in the digital ΣΔ modulator. The interpolation filter has finite impulse response, stopband attenuation from 23 kHz and above at a level greater than -300 dB and a passband ripple of $\pm 0.5 \times 10^{-14}$ dB. The interpolation filter is designed with modified an 11-term cosine-sum window, which was first introduced in [7]. Next, the interpolated signal u_{in} is fed to the input of the digital ΣΔ modulator which has an LSB's error feedback structure. Its signal transfer function (STF) and noise transfer function (NTF) are defined as $STF(z) = H_u(z) = 1$ and $NTF(z) = 1 - H_e(z)$.

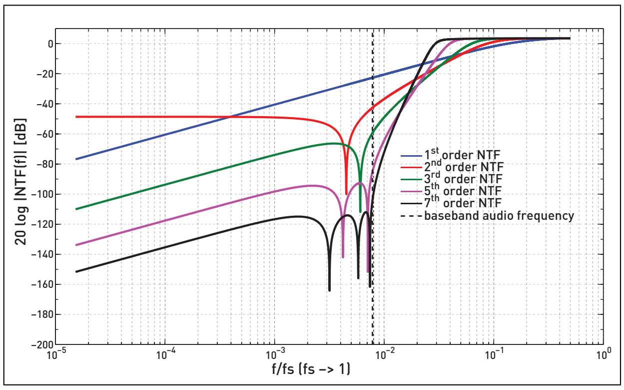

Figure 3. Noise transfer functions used in digital ΣΔ modulator simulations [8].

Simulations have been performed with 1st, 2nd, 3rd, 5th and 7th order NTFs as presented in Figure 3.

Wideband signal y_{req} at the output of the digital ΣΔ modulator is then limited to the audio band by the lowpass filter FDP_{lim}, which has the same amplitude characteristic as the interpolation filter. At the same time, the interpolated signal u_{in} is filtered by the same FDP_{lim} filter to provide equivalent conditions for subsequent signal analysis.

2.2. Estimation of the NME signal

Short-term performance examination of ΣΔ modulators is done by the NME signal analysis (see Figure 4). The NME signal represents jointly nonlinear distortions, noise modulation and requantization error of the ΣΔ modulator and cannot be estimated as a difference between ΣΔ modulator output and input signals as there is no direct time correlation between them. For that reason, the estimation of the NME signal is calculated by subtracting the right and left side of a differential equation corresponding to the modeled STF of the ΣΔ modulator (see equation 1 and 2). ΣΔ modulator STF is modeled in the s-plane by a general form, second-order transfer function with

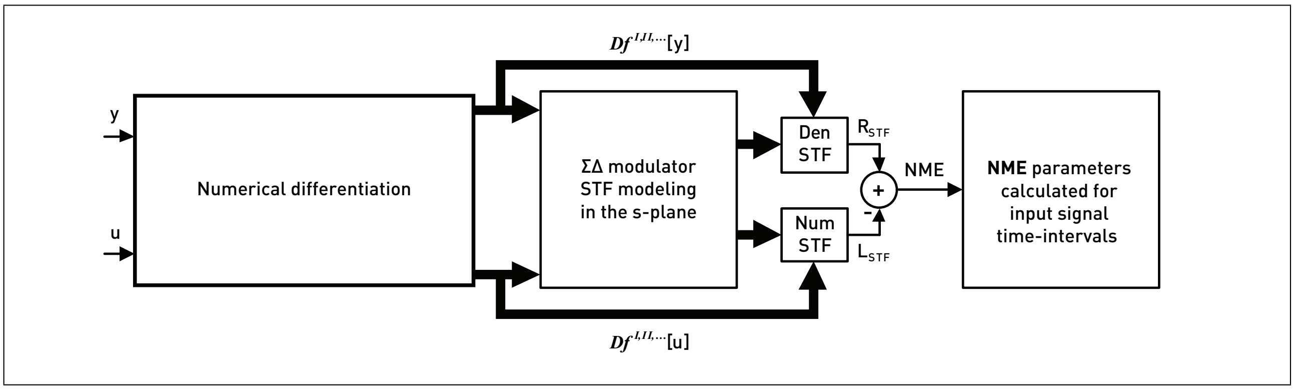

Figure 4. Block diagram of the signal analysis in the time domain.

the use of band-limited ΣΔ modulator output y and filtered input u signals (see Figure 4).

$$STF(s) = \frac{b_2 s^2 + b_1 s + 1}{a_2 s^2 + a_1 s + a_0}, \quad (1)$$

$$NME(t) = \left[a_2 \frac{d^2 y(t)}{dt^2} + a_1 \frac{dy(t)}{dt} + a_0 y(t)\right] - \\ - \left[b_2 \frac{d^2 u(t)}{dt^2} + b_1 \frac{du(t)}{dt} + u(t)\right]. \quad (2)$$

Coefficients b_2, b_1, a_2, a_1 and a_0 in equation 1 and 2 are estimated by solving a system of linear equations with the use of an algorithm based on the Gaussian elimination with partial pivoting. Coefficients are estimated with numerical precision of 10^{-10}. First and second derivatives of ΣΔ modulator input (u) and output (y) signals are estimated as described in the algorithm proposed in the author's Ph.D. thesis [9]. The algorithm is based on high-order finite difference formula calculations and shifting ΣΔ modulator input and output signals by a fractional number of samples. Shifting operation is performed by the modified low-pass filters FDP_{lim}. The numerical precision of the algorithm is 10^{-12} and 10^{-10} for approximation of first and second derivative, respectively.

The estimated NME signal is then represented by its mean value and equivalent power calculated for input signal time intervals corresponding to input signal amplitude ranges and rates of amplitude change measured by first and second derivatives. This allows short-term performance of the ΣΔ modulator to be examined. Each time interval range is determined by the number of simulation intervals and is different for each ΣΔ modulator input signal. Figure 5 shows an example of the signal marked with nine time intervals corresponding to rates of amplitude change measured by the first derivative, where time intervals numbered 1 to 9 correspond to the largest positive and largest negative slope of the input signal.

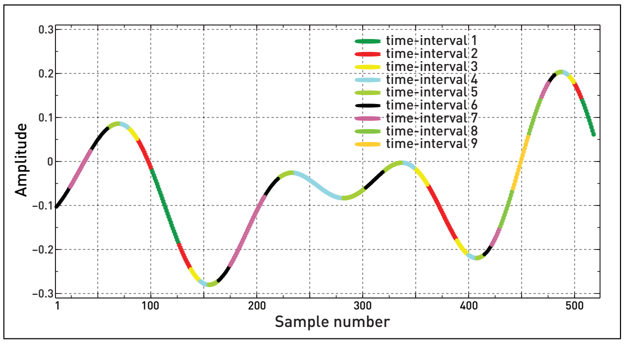

Figure 5. Time intervals corresponding to input signal rates of amplitude change measured by the first derivative.

3. Simulations

Simulations have been performed with six different music signals at the input fed to 1-, 3- and 5-bit, 1^{st}, 3^{rd}, 5^{th} and 7^{th} order ΣΔ modulators with LSB's error feedback structure and optional dither signal. The oversampling ratio was set to 64 and input signal amplitudes were limited to ±0.5. The mean value and equivalent power of the NME signal were calculated for 130 time intervals of input signal parameters. Simulation procedures were written in the MATLAB environment using its native programming language. Computationally demanding numerical procedures, such as the solving system of linear equations and calculating approximations of derivatives, were optimized and rewritten in the C-language.

Since the simulation results obtained for all six input signals have proved practically the same, this study only covers the results for one input signal. The remaining subsections examine the short-term performance of the undithered and dithered ΣΔ modulator depending on its bit resolution and NTF order.

3.1. Change of bit resolution

Short-term performance of the ΣΔ modulator depending on its bit resolution is presented in Figure 6 as a variation of the NME signal power calculated

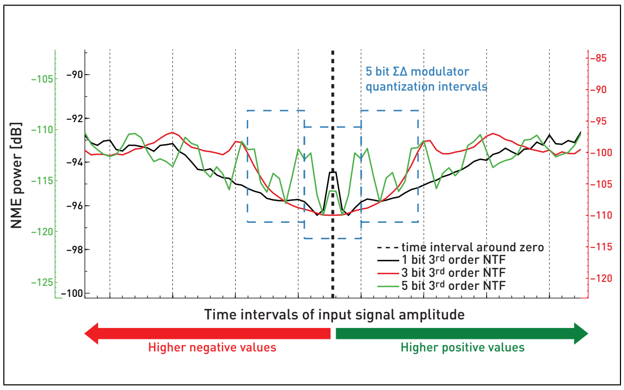

Figure 6. The power of NME signal calculated for time intervals corresponding to input signal amplitude ranges.

for time intervals corresponding to the input signal amplitude change. The vertical dashed line represents the time interval that corresponds to input signal amplitudes around zero. On its left side there are time intervals that correspond to higher negative amplitudes and on the right - higher positive amplitudes. As illustrated in Figure 6 for the 1-bit ΣΔ modulator, the calculated NME power is rising as absolute values of the input signal amplitude increase and is higher for input signal amplitudes around zero. Similar results are confirmed for 3- and 5-bit ΣΔ modulators, but additionally the calculated NME signal power changes over each quantization interval.

3.2. Change of NTF order

Short-term performance of the ΣΔ modulator depending on its NTF order is presented in Figure 7 as a variation of NME signal mean values calculated for time intervals corresponding to input signal amplitude rates of change measured by the second derivative. The vertical dashed line represents time intervals that correspond to amplitude rates of change around zero. On its left side there are time intervals that correspond to higher negative values and on the right side - higher positive values, which represent an increase in input signal amplitude rates of change. As illustrated in Figure 7 for a 5-bit ΣΔ modulator with 1st and 3rd order NTF, the mean values of the calculated NME signal do not change significantly along with fast rates of input signal amplitude change. However, these correlations are different for ΣΔ modulators with 5th and 7th order NTF. They are changing at the lower level, but as illustrated in Figure 7, the more memory elements the ΣΔ modulator feedback path contains, the less accurate the ΣΔ modulator ability to follow fast rates of input signal amplitude change becomes.

3.3. Adding dither

Short-term performance of undithered and dithered 5-bit ΣΔ modulators with 3rd order NTF is presented in Figure 8 as a variation of NME signal mean values calculated for time intervals corresponding to input signal amplitude changes. The vertical dashed line represents the same time intervals as in Subsection 3.1. As illustrated in Figure 8, the use of dithering technique partially decorrelates data conversion errors with input signal parameters, but does not eliminate these correlations. Moreover, further studies, which are not described here, have shown that dithering has no effect on the short-term performance of the ΣΔ modulator as a function of rates of input signal amplitude change measured by the first and second derivative.

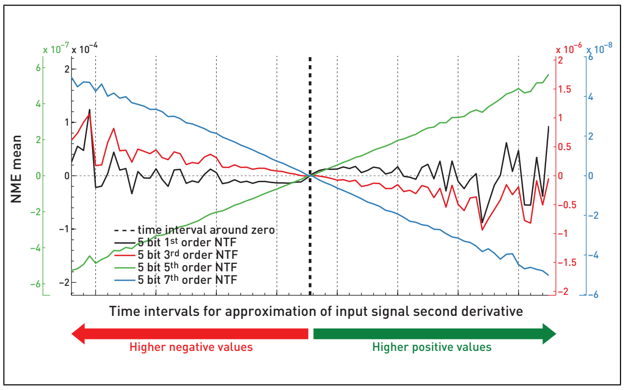

Figure 7. The mean value of the NME signal calculated for time intervals corresponding to rates of input signal amplitude change measured by the second derivative.

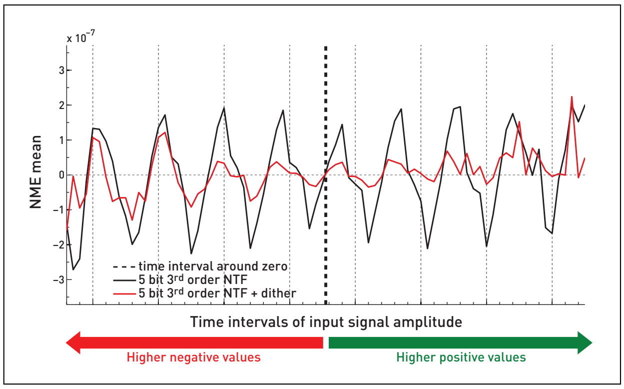

Figure 8. The mean value of the NME signal calculated for time intervals corresponding to input signal amplitude ranges.

4. Conclusions

Simulation results have showed that the short-term performance of ΣΔ modulators is highly correlated with the variation of input signal, specifically:

- data conversion errors in ΣΔ modulators increase along with the amplitude of the input signal;
- ΣΔ modulator output signal is not able to follow rapid changes of the input signal, thus creating significant errors as rates of input signal amplitude change;
- as bit resolution or NTF order of the ΣΔ modulator increases, the overall level of the NME signal

mean value and its equivalent power are reduced, but the correlation character remains the same over the same time intervals;

- dithering technique used in the ΣΔ modulator can lead to partial decorrelation of data conversion errors with the input signal parameters, but does not eliminate correlations entirely;
- increase of the ΣΔ modulator's NTF order reduces conversion errors, but adversely affects ΣΔ modulator ability to exactly follow rapid changes in the input signal over time.

This study has highlighted the importance of ΣΔ modulator performance examination in the time domain as ΣΔ modulators are now commonly used in ADCs and DACs of both consumer and professional audio equipment. The short-term performance examination of ΣΔ modulators has been conducted by analyzing the proposed NME signal, which jointly represents nonlinear distortions, noise modulation and requantization error in ΣΔ modulators. The NME signal is represented by its mean value and equivalent power calculated for input signal time intervals corresponding to the input signal amplitude ranges and rates of amplitude change measured by first and second derivatives. Evaluation of the NME signal is performed by modeling ΣΔ modulator STF in the s-plane with the use of algorithms proposed in the author's Ph.D. thesis [9].

Future work should concentrate on optimizing the short-term performance of ΣΔ modulators in the time domain. Further data collection from all memory elements in the feedback path of ΣΔ modulators would be needed to determine exactly how data conversion errors affect the output signal quality. Additional investigations should cover the analysis of a ΣΔ modulator model, the parameters of which would change adaptively not only as a function of input signal amplitude, but also as a function of rates of input signal amplitude change.

The proposed method for evaluating nonlinear systems with the use of nonstationary input signals is applicable not only in the field of electroacoustics, but also in electronics and telecommunications, where nonlinear systems with structurally embedded feedback loops are widely used. Existing simulation results examined in the frequency domain can be supplemented with results in the time domain, thus allowing a complete analysis of nonlinear systems with nonstationary input signals to be carried out.

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