A macroscopic-scale wave-particle duality :

With :

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Can any of the phenomena characteristic of the quantum wave-particle duality be observed in a non quantum system?

We were drawn into investigating this question by the, almost accidental, finding of a wave-particle association at macroscopic scale.

Introduction

A massive particle driven by the wave it generates

The bouncing droplet and its coupling to surface waves.

Part I : Walking straight

The wave-field structure and its "path-memory": a non locality in time.

Part II: Walking in circles

The orbits of walkers when submitted to a transverse force

Part III : Walking when confined

(a) In corrals

(b) Through slits (diffraction and interference)

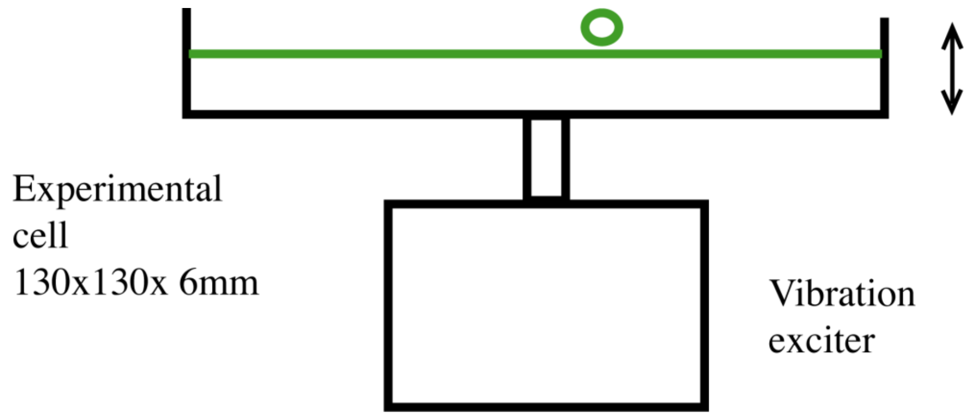
(c) By a single barrier: a tunnel effect of a kind

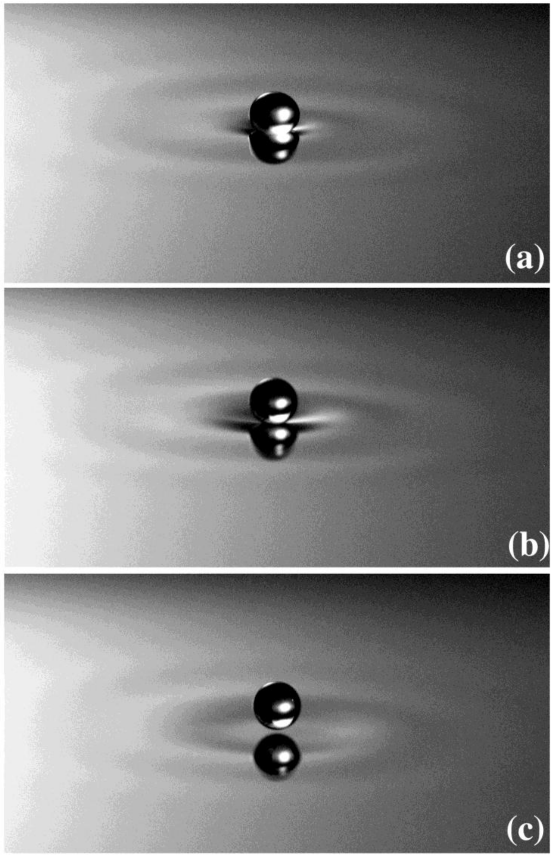
Discussion

The relation to de Broglie's pilot waves

Trajectories and probability of visit

The basic experimental set-up





Silicon oil of viscosity

μ = 50 10^{-3} Pa.s

Vertical acceleration

$$\gamma = \gamma_m \cos(\omega t)$$

with $\omega/2\pi = 80$ Hz

and $0 < \gamma_m < 5$

There is always an air film between the drop and the substrate

The drop can bounce for days!

Part I

An experiment where a particle is driven by the wave it generates

Couder Y., Protière S., Fort E. & Boudaoud A., *Nature* 437, 208. (2005)

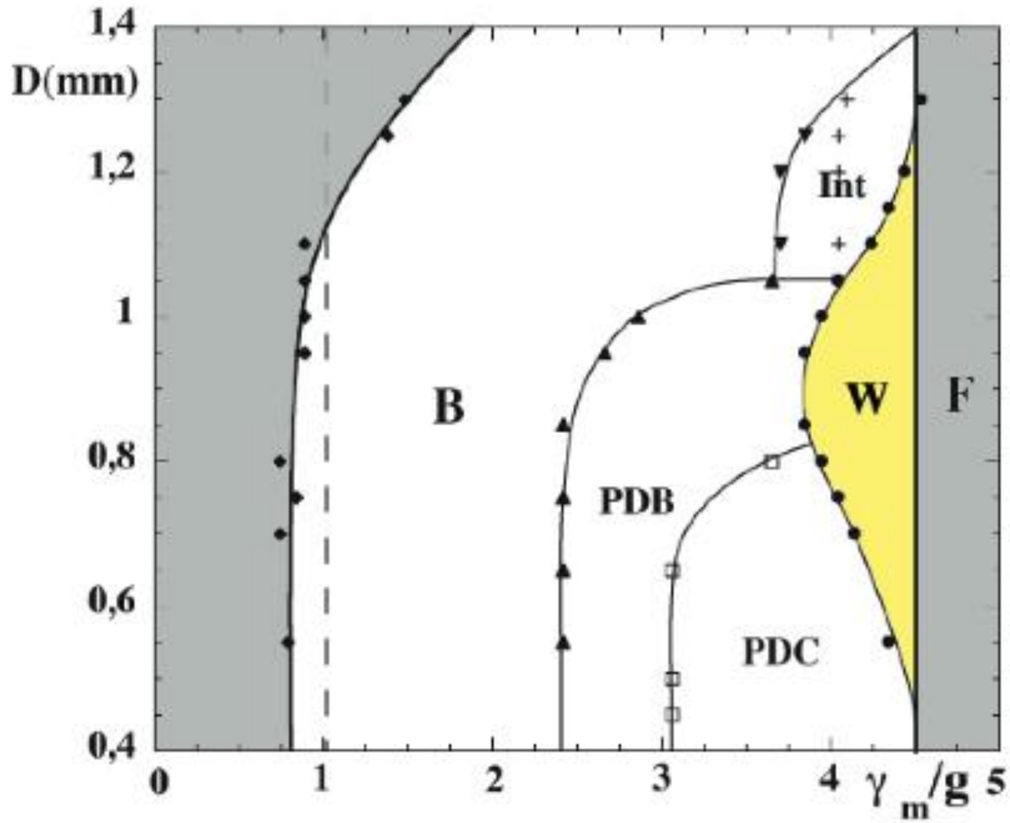
Protière S., Boudaoud A. & Couder Y. *J. Fluid Mech*. 554, 85-108, (2006)

Phase diagram of the different types of bouncing

(for $\mu = 50\ 10^{-3}$ Pa s   and $\omega/2\pi = 80$ Hz)

as a function of the droplet size and the amplitude of the forcing

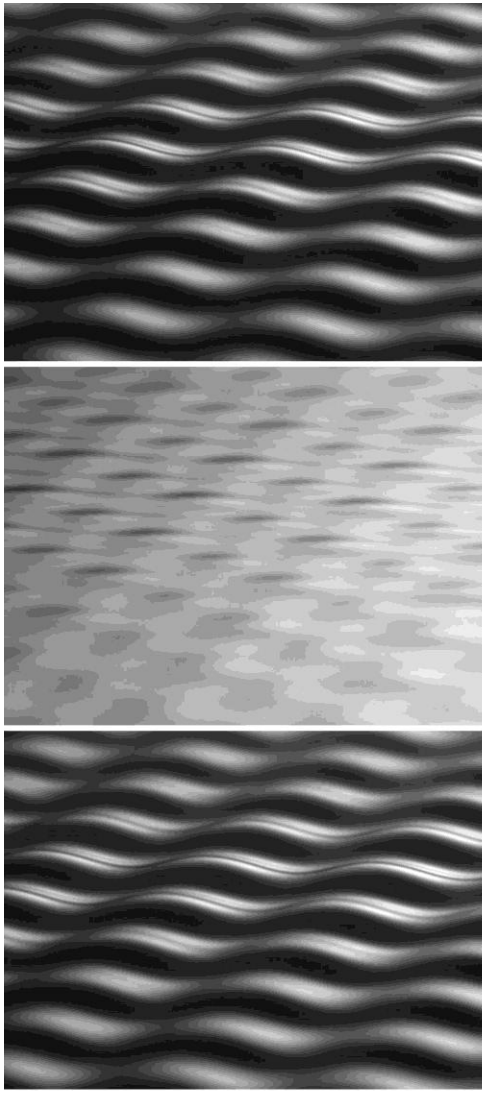
- •B : simple bouncing at the forcing frequency
- •PDB : Period doubling
- •PDC: Temporal chaos
- •W : “walkers“
- •F : Faraday instability

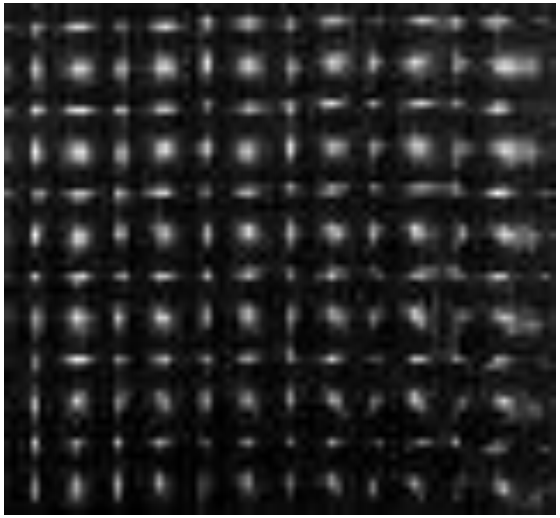


Bouncing threshold

Faraday Instability threshold

The Faraday instability of a vertically vibrated liquid surface





Vertical oscillations

$$\gamma = \gamma_m \cos(\omega t)$$

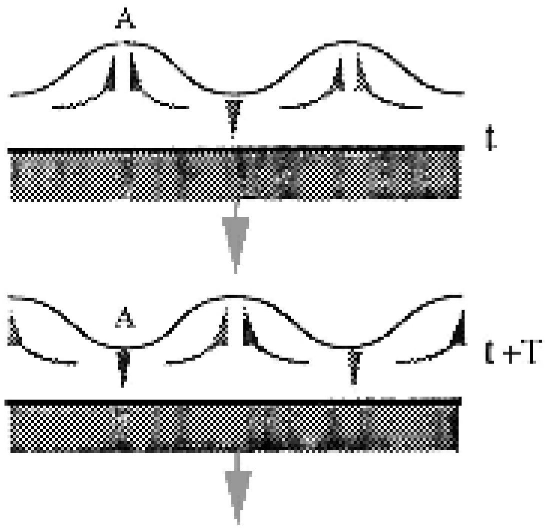
When $\gamma_m \geq \gamma_m^{Far}$
the surface becomes covered
with standing waves of frequency ω/2

In the present experiment

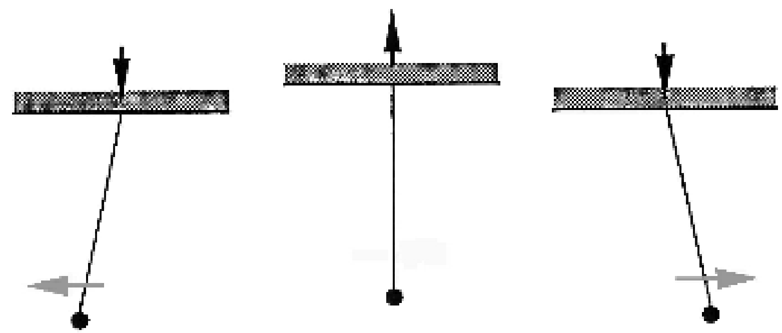
$$\omega/2\pi = 80Hz \quad \text{and} \quad \gamma_m^{Far} = 4.5\, g$$

The Faraday instability results from the parametric forcing of the surface waves

cf e.g. Stéphane Douady, *Thesis* (1989)



Analogous to the parametric forcing of a rigid pendulum



The motion is given by Mathieu's equation:

$$\frac{\partial^2 a}{\partial t^2} + 2f\frac{\partial a}{\partial t} + \omega_0^2\left(1 + 2\varepsilon\cos\omega t\right) = 0$$

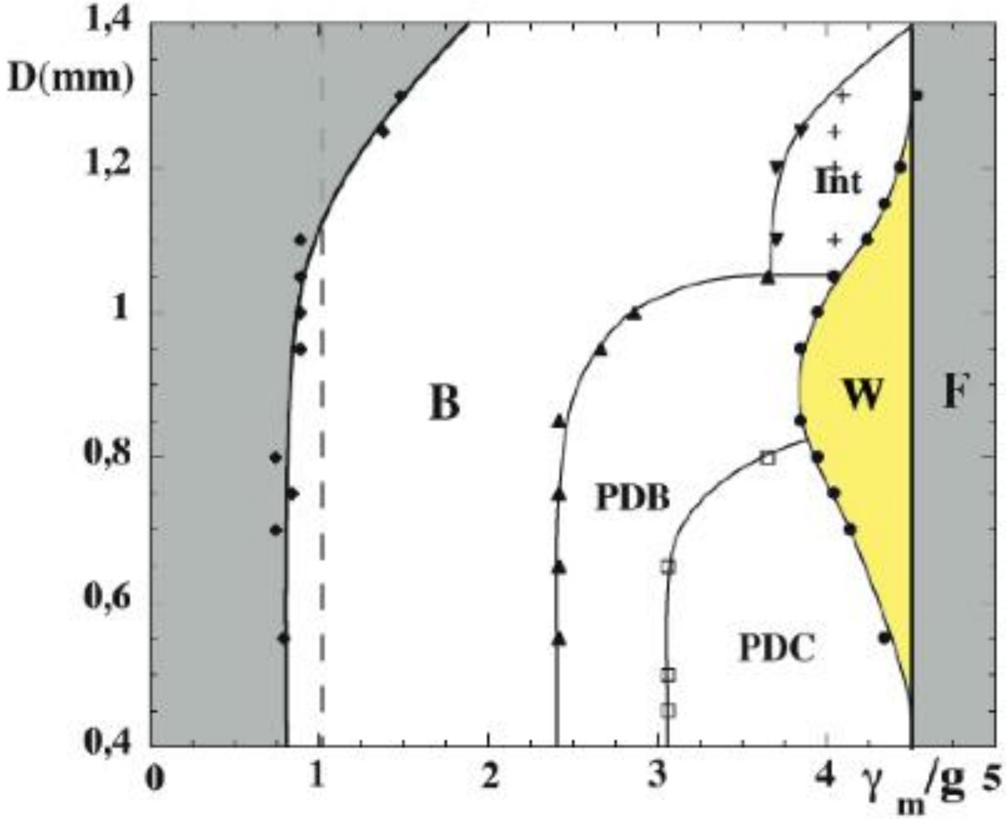
and its frequency is half that of the forcing

Phase diagram of the different types of bouncing

(for μ=50 10^{-3} Pa s   and $\omega/2\pi$=80 Hz)

as a function of the droplet size and the amplitude of the forcing

- B : simple bouncing at the forcing frequency
- PDB : Period doubling
- PDC: Temporal chaos
- W : "walkers"
- F : Faraday instability



Bouncing threshold

Faraday Instability threshold

The drop's bouncing: (for 0.6 < D < 1.1mm)
Spatio-temporal diagrams of the vertical motion

γ_m=1.5g

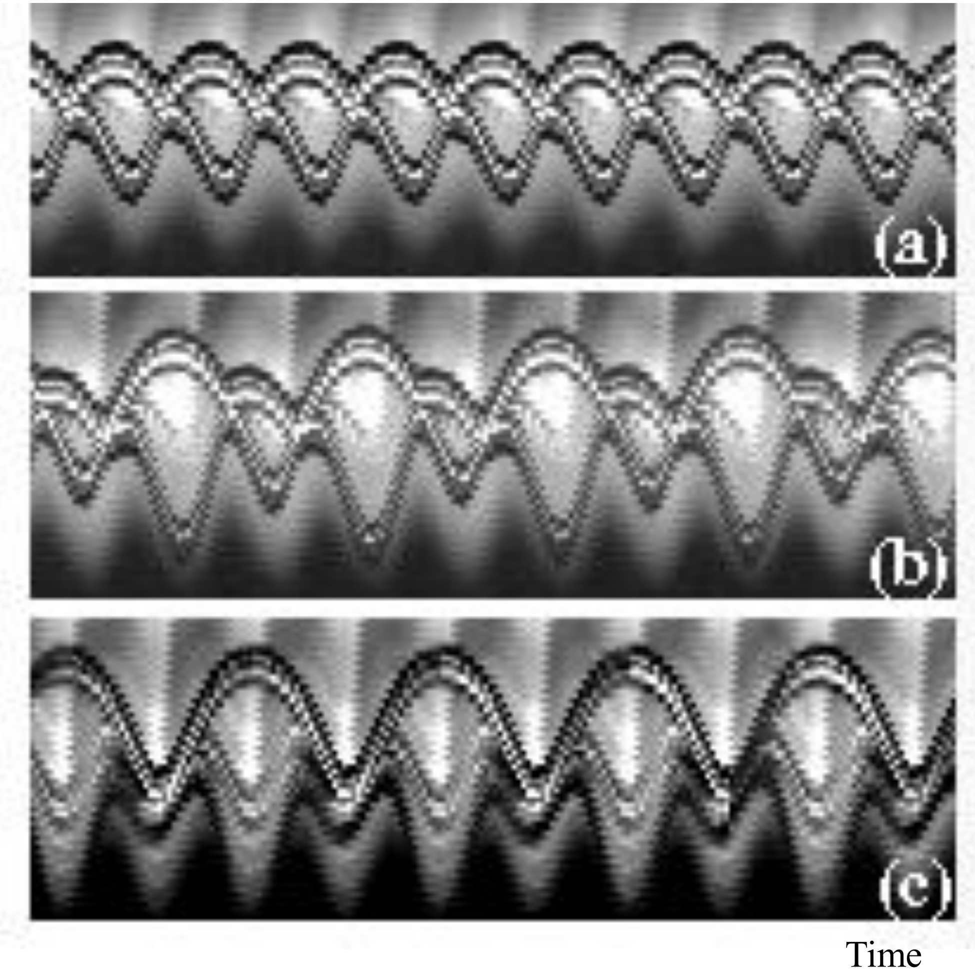
Bouncing at the forcing frequency

γ_m= 3g

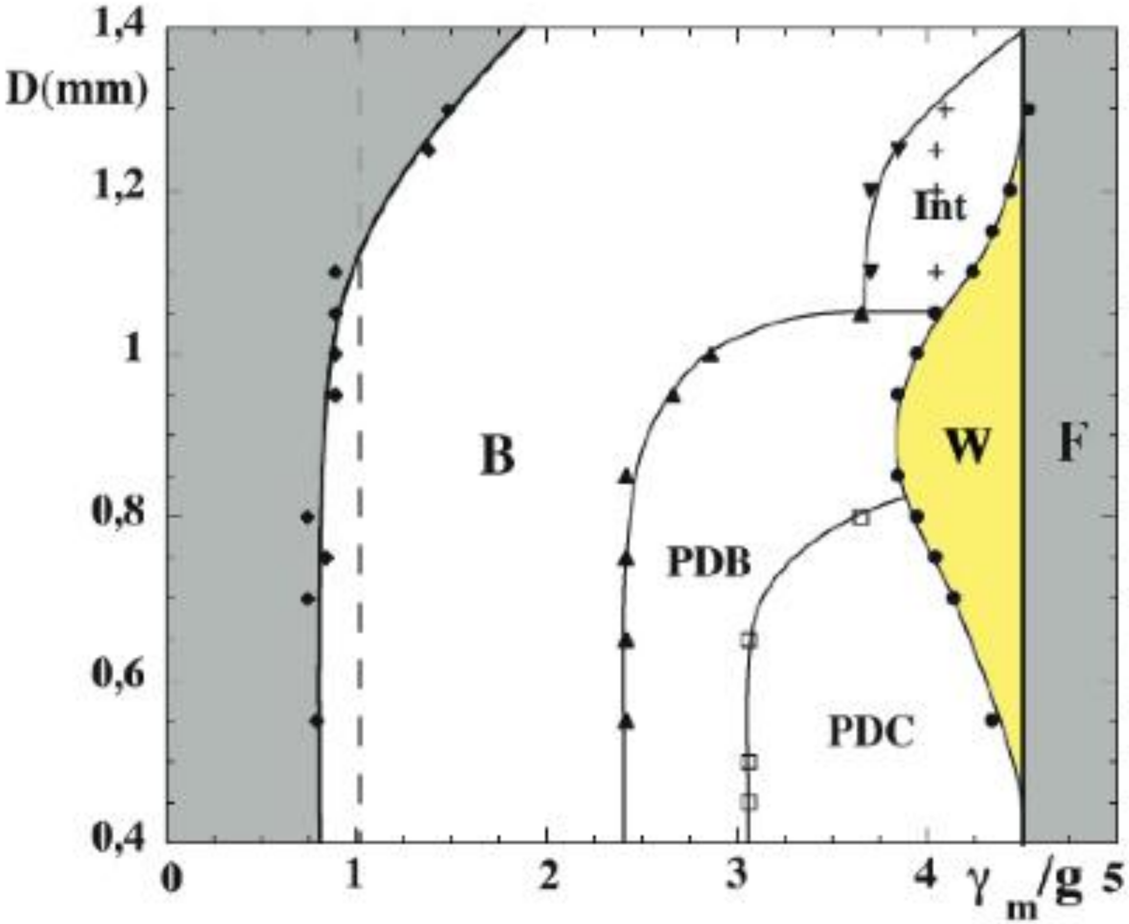
Period doubling

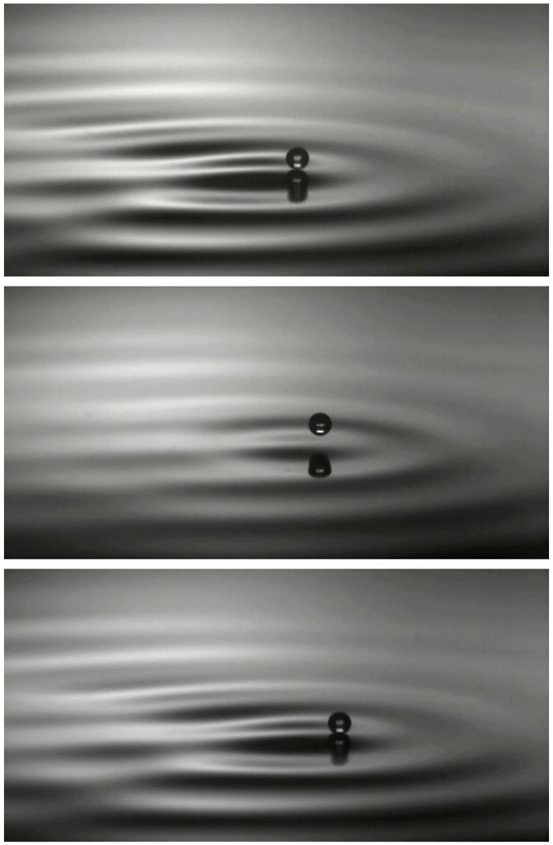
γ_m= 4g

no chaos and complete period doubling



Near the Faraday instability onset, the droplets become local sources of Faraday waves, correlatively they become "walkers"





There is a simple model of the walking bifurcation

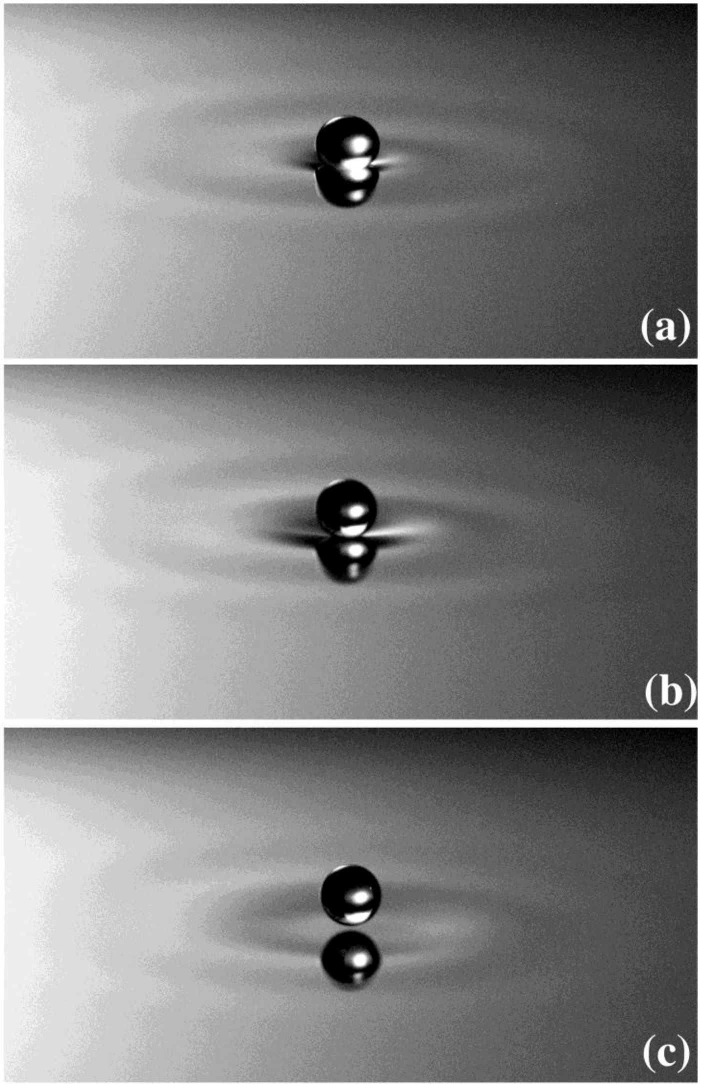
Part I (b)

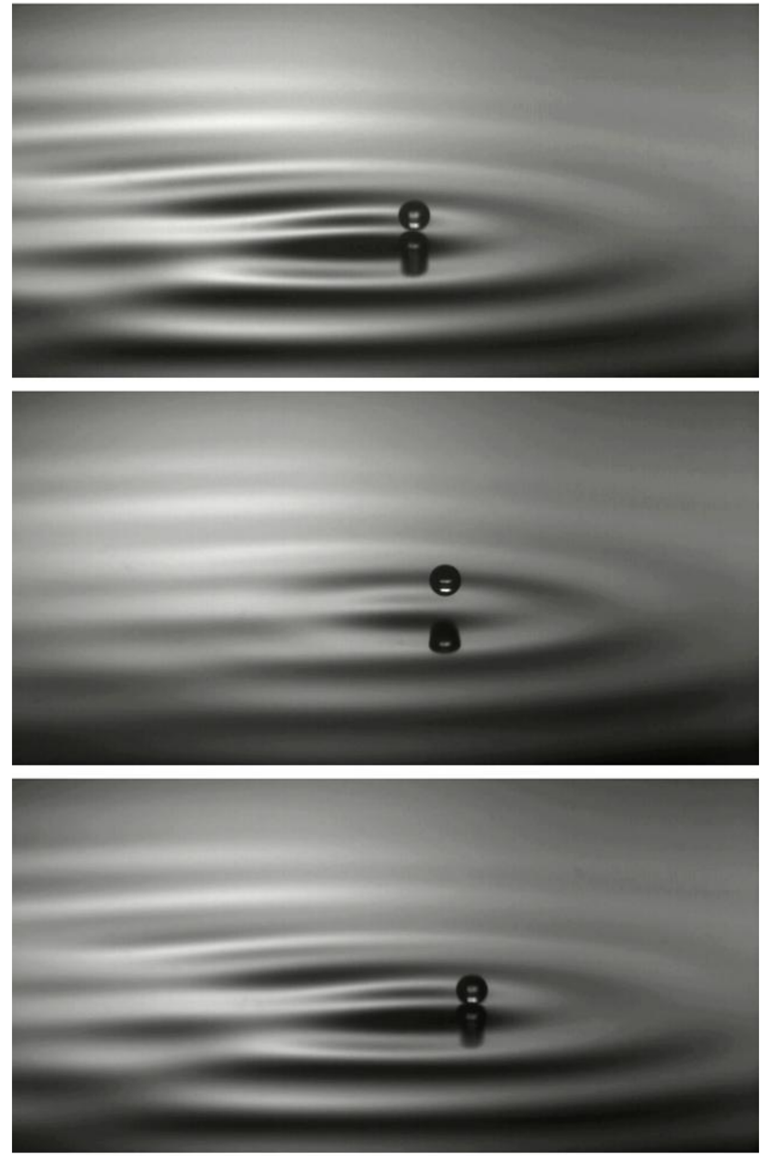
A simple model of the walking bifurcation

(Arezki Boudaoud)

Protière S., Boudaoud A. & Couder Y. *J. Fluid Mech*. 554, 85-108, (2006)

How can the coupling to waves generate a bifurcation to walking?





The first model for the walking transition

(Arezki Boudaoud)

Newton's equation, the fast vertical motion being averaged over one period

$$\boxed{m\frac{d^2x}{dt^2} = F^b \sin\left(2\pi\frac{dx/dt}{V_F^{\varphi}}\right) - f^v\, dx/dt}$$

- $m \sim 10^{-6}$ kg mass of the droplet

- F^b effective force due to the bouncing on an inclined surface

$$F^b \approx m\gamma_b \frac{A_w}{\lambda}\left(\frac{\tau}{T_F}\right) \approx 10^{-6}\ N$$

γ_b Vertical acceleration,
A_w/λ slope of the surface
τ duration of the collision

- f^v : damping due to the shearing of the air film

$$f^v \approx \frac{\mu_a S}{b}\left(\frac{\tau}{T_F}\right) \approx 10^{-6}\ N$$

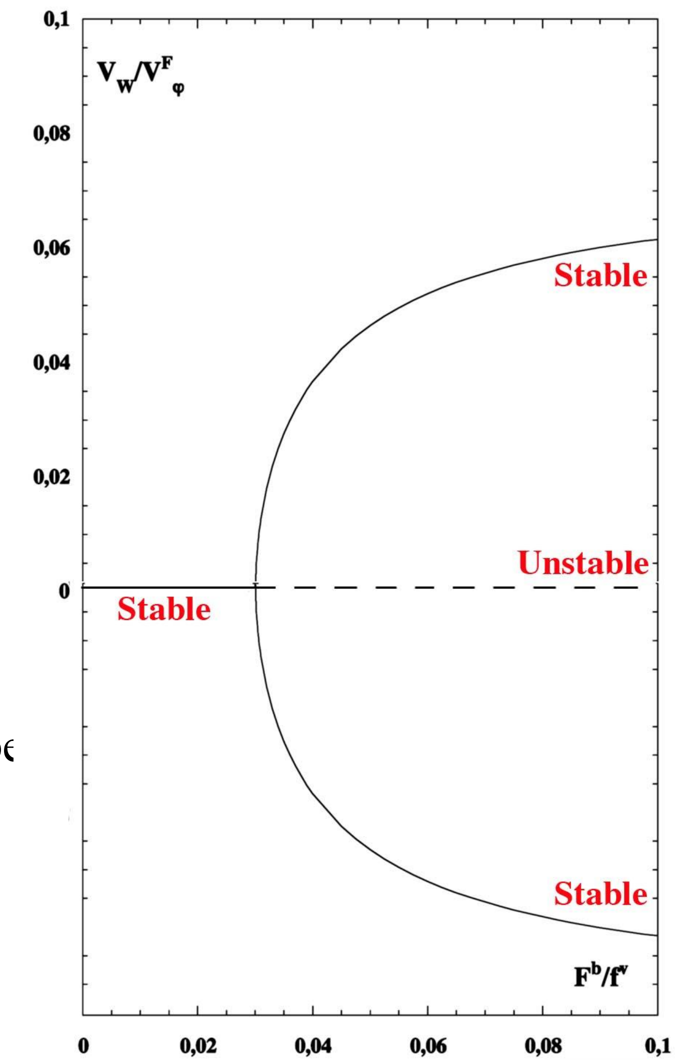
The “walking“ bifurcation

$$m\ddot{x} = F^b \sin\left(\frac{2\pi \dot{x}}{V_F^{\varphi}}\right) - f^v \dot{x}$$

Seeking steady solutions
(and in the limit of small velocities)

$$f^v \dot{x} = F^b \left[\frac{2\pi \dot{x}}{V_F^{\varphi}} - \frac{1}{6}\left(\frac{2\pi \dot{x}}{V_F^{\varphi}}\right)^3 \right]$$

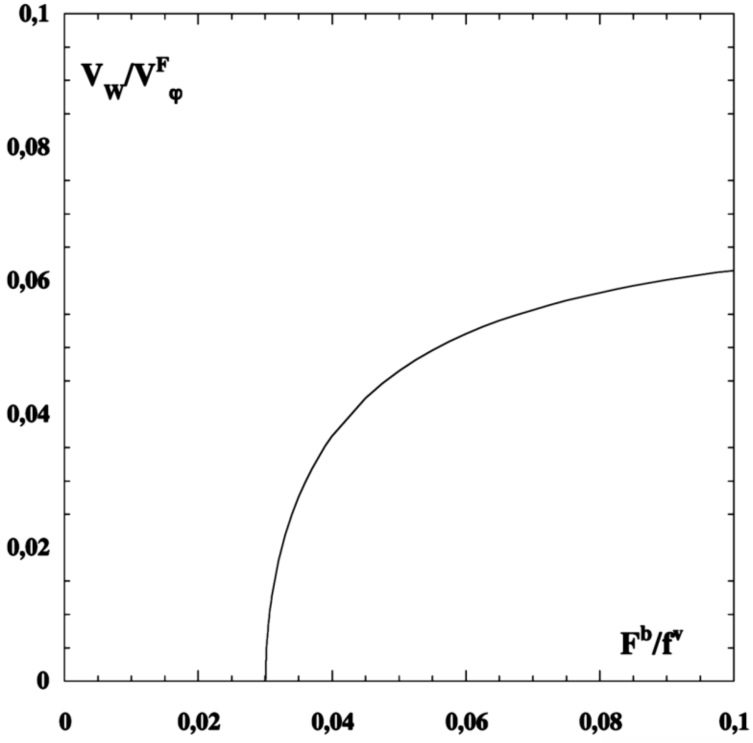
for small values of F^b the only solution is $\dot{x} = 0$

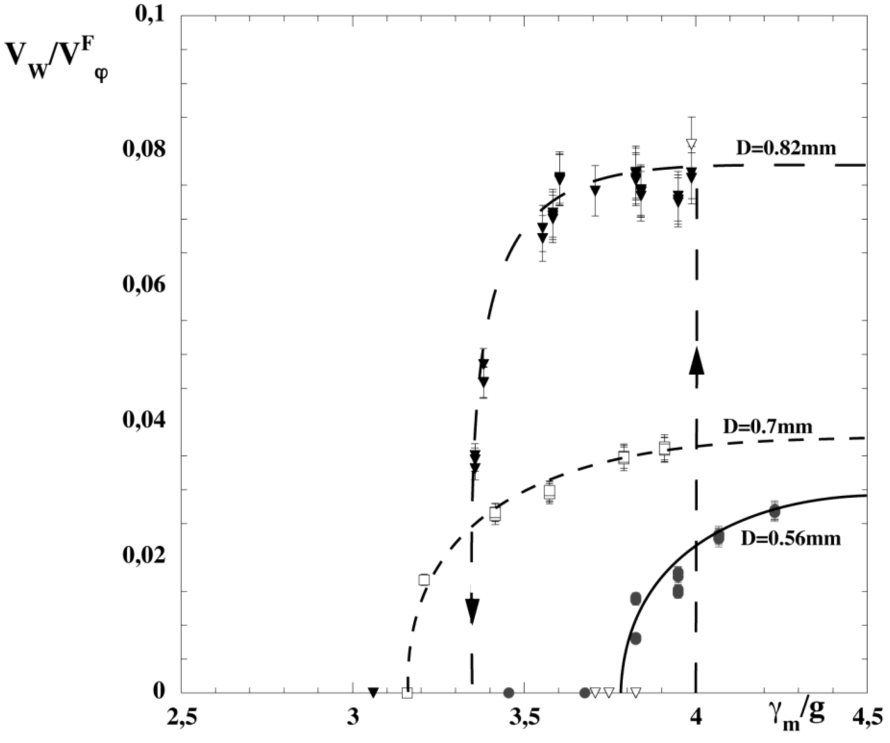


Above a threshold,the motionless solution becomes unstable
And two self propagative solutions of opposite velocities appe

$$\boxed{\dot{x} / V_F^{\varphi} = \pm\left(\sqrt{6}/2\pi\right)\sqrt{\left(F^b - F_c^b\right)/F^b}}$$

The computed and observed bifurcation





A more complete model based on the same principle has been developed recently by Jan Molacek and John Bush

Part I (c)

The energy balance : the steady regimes of a dissipative structure

The energy balance

The system is dissipative : viscous friction damps the droplet motion and the wave

However steady regimes are obtained because energy is provided by the forcing to both the droplet and the wave.

- The droplet is kicked up at each of its collision with the interface.

(similar to the escapement mechanism of mechanical clocks)

- The wave being a Faraday wave is almost sustained by parametric forcing

(in the vicinity of the instability threshold).

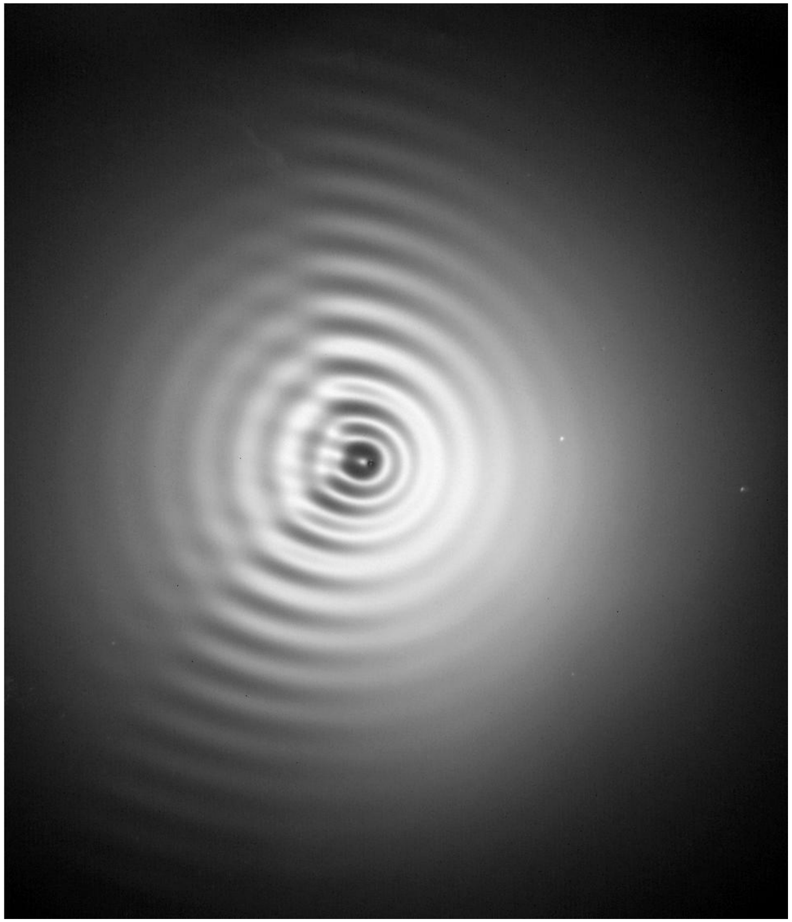
The main limitation of this experiment is that the forcing imposes a fixed frequency: the energy is fixed

A walker is formed of:

- A spread-out and continuous wave

- A discrete and localized droplet

How can they have a common dynamics?



I Walking straight

II Walking in circles

III Walking when confined

Part I

Walking straight

The wave field structure and its "path-memory",

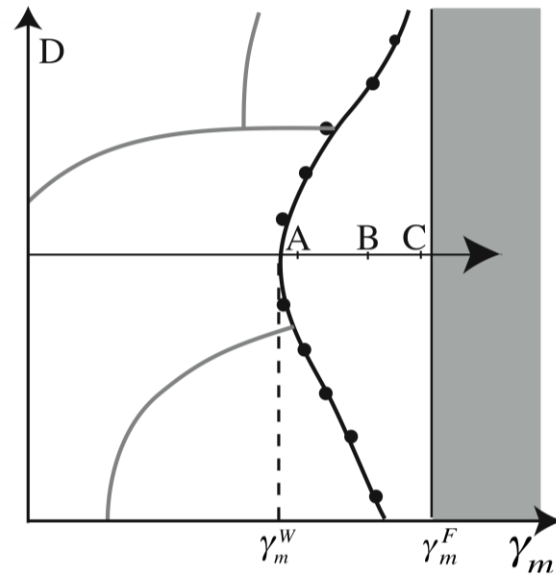
A. Eddi, E. Sultan, J. Moukhtar, E. Fort, M. Rossi, and Y. Couder, ***J. Fluid Mech.*****, 674, 433- 464, (2011).**

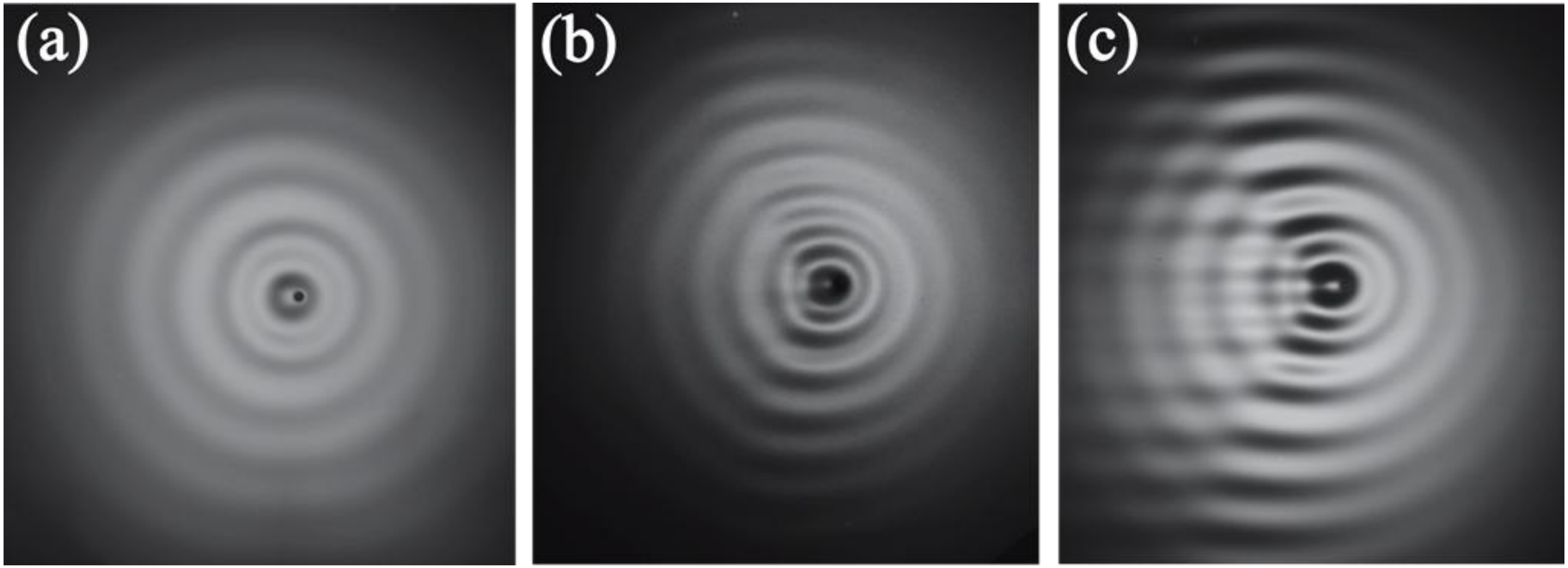
Evolution of the wave field as a function of the distance to the Faraday instability threshold

Γ the non-dimensional distance to threshold tends to zero

$$\Gamma = \left(\gamma_m^F - \gamma_m\right) / \gamma_m^F$$

A detail of the phase diagram

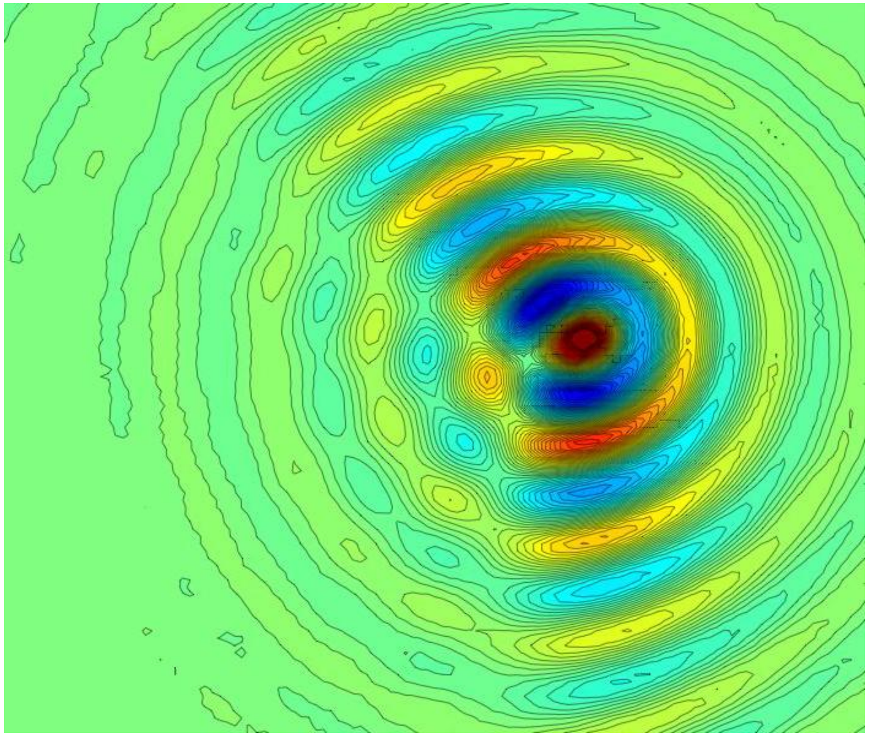


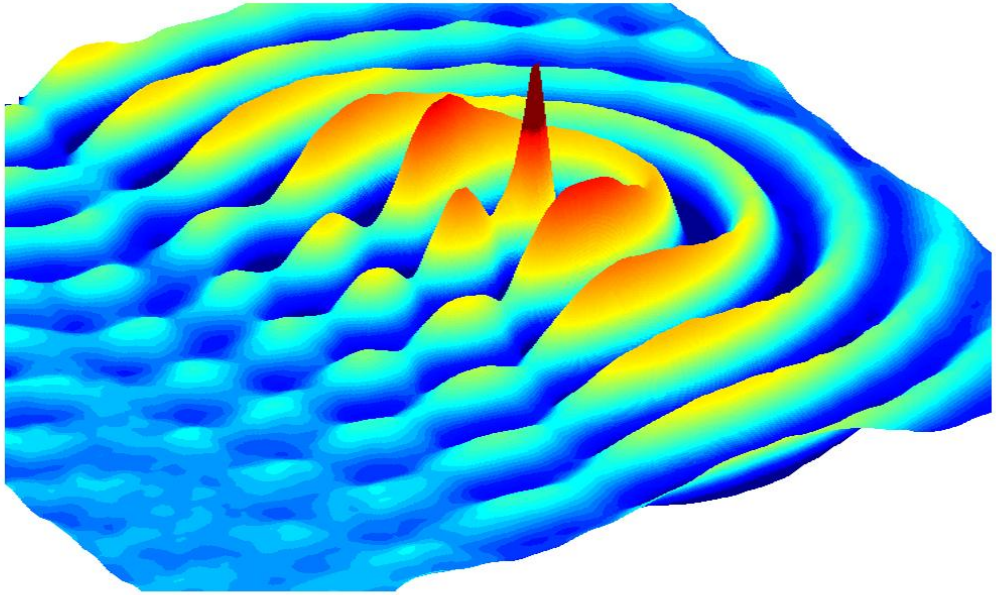


The measured wave field

Obtained by an adaptation of a particle image velocimetry technique (PIV) to measure the shape of the interface, a technique due to

Frédéric Moisy and Marc Rabaud (FAST Orsay)



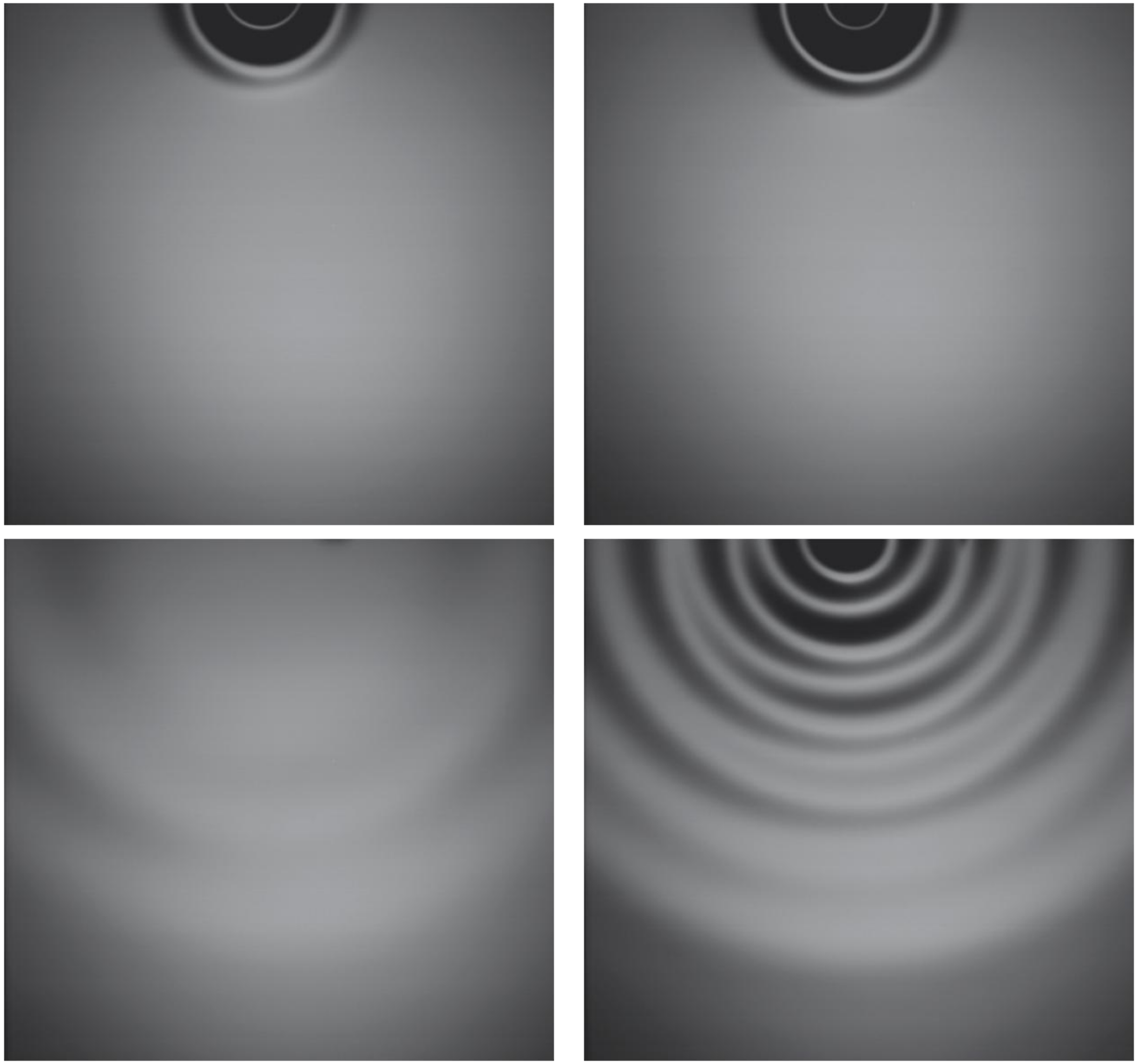


The interface is disturbed by the repeated impacts of the droplet

What type of wave is generated by one single collision?

The wave-field produced by one single collision

(a 1 mm steel ball dropped in the bath)



t=10 ms

t=300 ms

Without periodic forcing

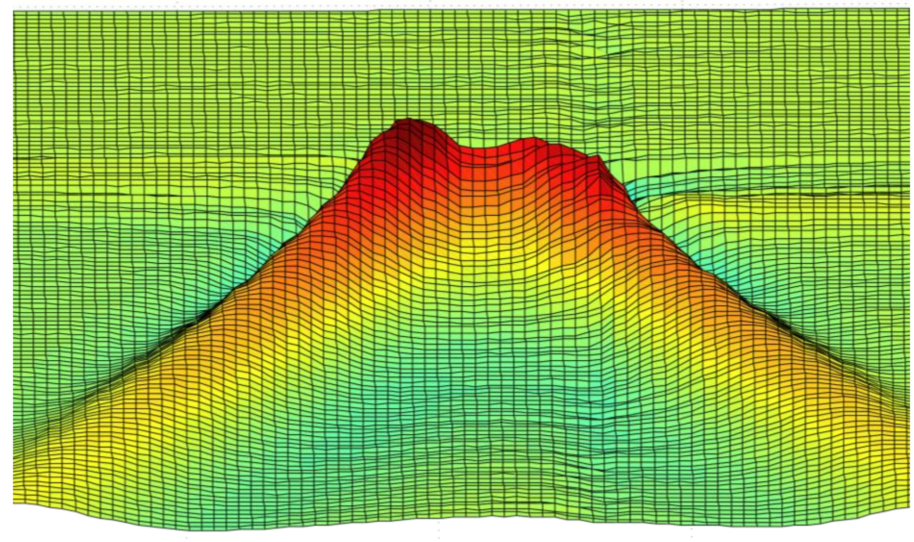
With a periodic forcing near the Faraday instability threshold

The Faraday waves decay with a characteristic time:

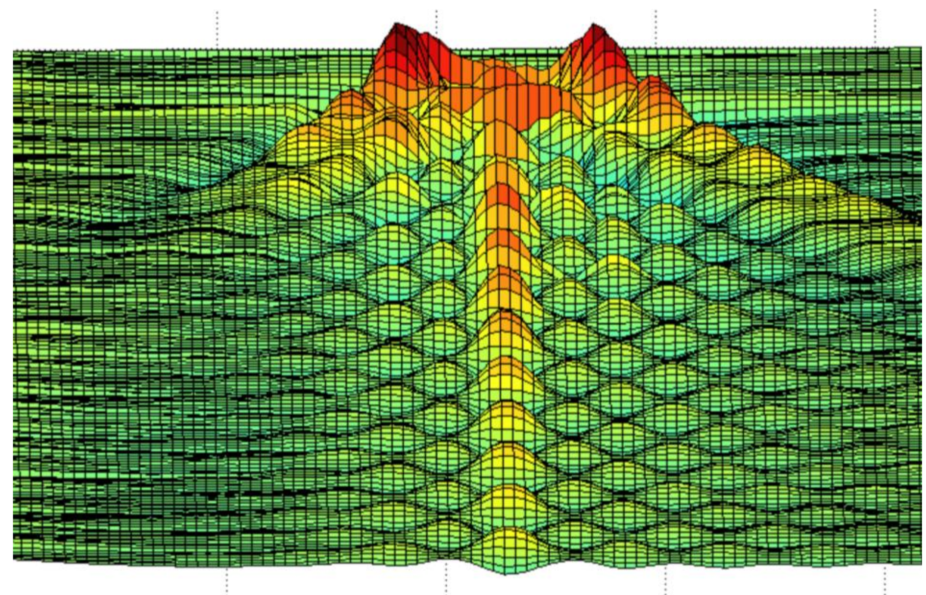
$$\tau \propto \left| \gamma_m - \gamma_m^F \right|^{-1}$$

Spatio-temporal evolution of the radial profile of the wave emitted by one bounce

Without periodic forcing



With a periodic forcing near the Faraday instability threshold

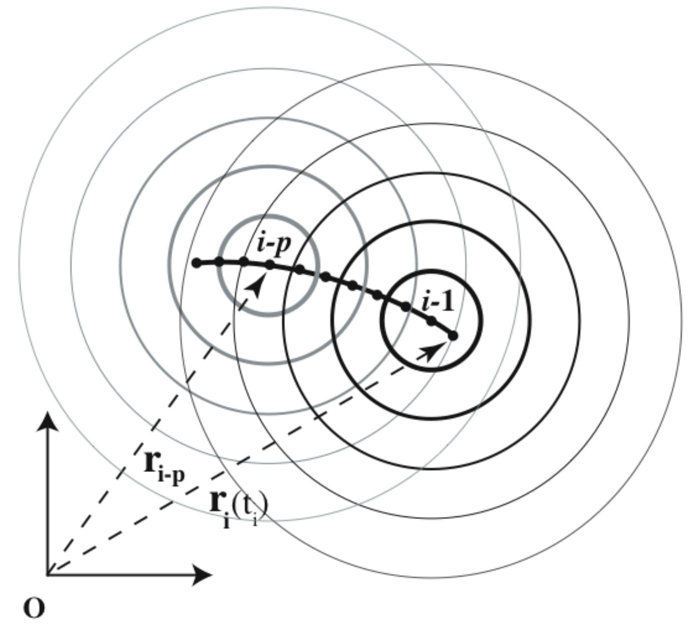


Conclusion: near the Faraday threshold, a point which has been disturbed remains the centre of a localized state of almost sustained Faraday waves

The numerical model of walkers

(Emmanuel Fort)

The numerical model of walkers
(Emmanuel Fort)



A/ Motion of the droplet :

(1) Take-off and landing times are determined by the forcing oscillations only.

(2) The walk result from successive displacements δr_n due to the kicks. The direction and modulus of δr_n are determined by the surface slope at the point of landing.

(3) This slope results of the interfering waves due to the previous bounces

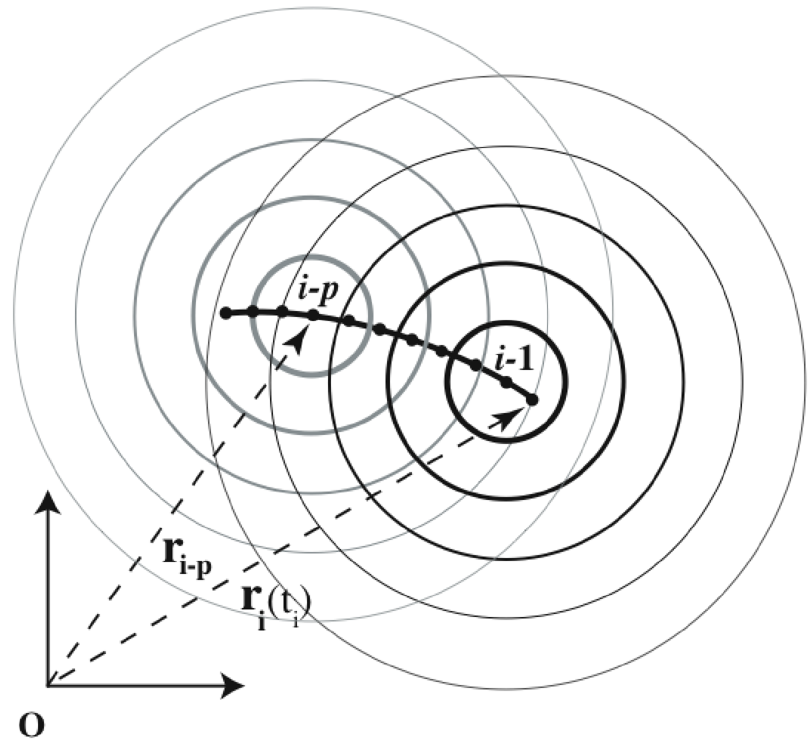
B/ Computation of the wave-field

1) At each bounce, a circular localized mode of Faraday waves is generated.

(2) The points of the surface visited by the droplet in the past remain the centres of such a localized mode.

(3) The wave field results from the superposition of all these waves, and thus contains a memory of the path followed by the droplet

B/ The computation of the wave-field



$$h(\mathbf{r},t_i)=\sum_{p=i-1}^{-\infty}\mathrm{Re}\left[\frac{A}{\left|\mathbf{r}-\mathbf{r}_p\right|^{1/2}}\ \exp-\left(\frac{t_i-t_p}{\tau}\right)\exp-\left(\frac{\left|\mathbf{r}\ -\mathbf{r}_p\right|}{\delta}\right)\exp i\left(\frac{2\pi\left|\mathbf{r}-\mathbf{r}_p\right|}{\lambda_F}+\varphi\right)\right]$$

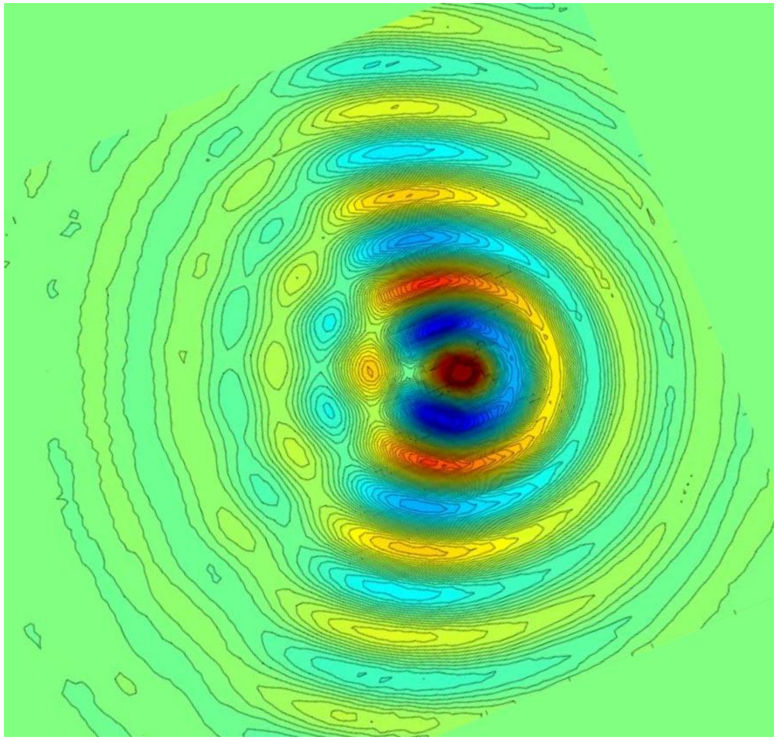
$\mathbf{r}_{\mathrm{p}}$ position of the droplet at time $t_p=t_i-(i-p)T_F$

the damping time is related to the distance to Faraday instability onset: $\tau\propto\left|\gamma_m-\gamma_m^F\right|^{-1}$

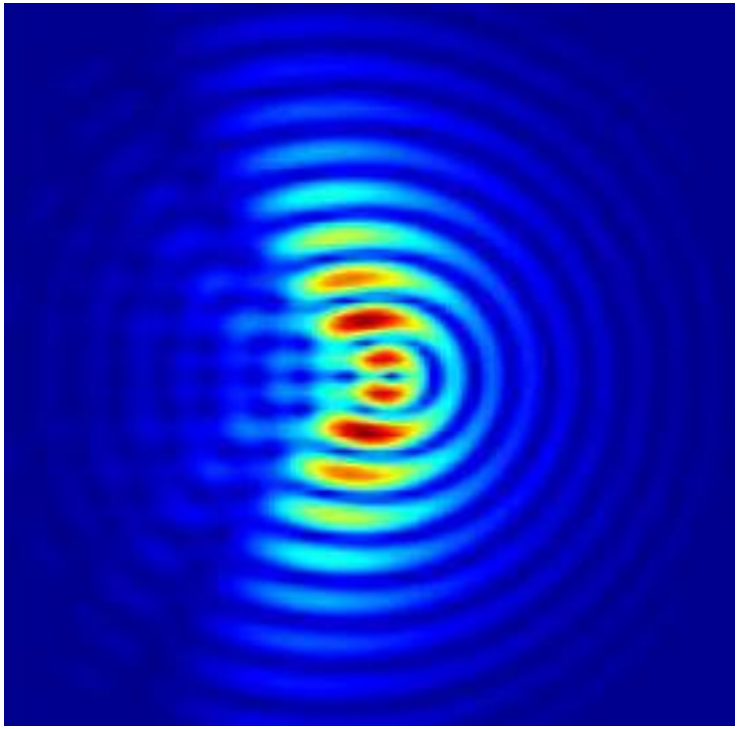
First results of the numerical simulation

(1) The walking bifurcation is recovered

(2) A realistic structure of the wave field is obtained for a rectilinearly moving walker



The measured field

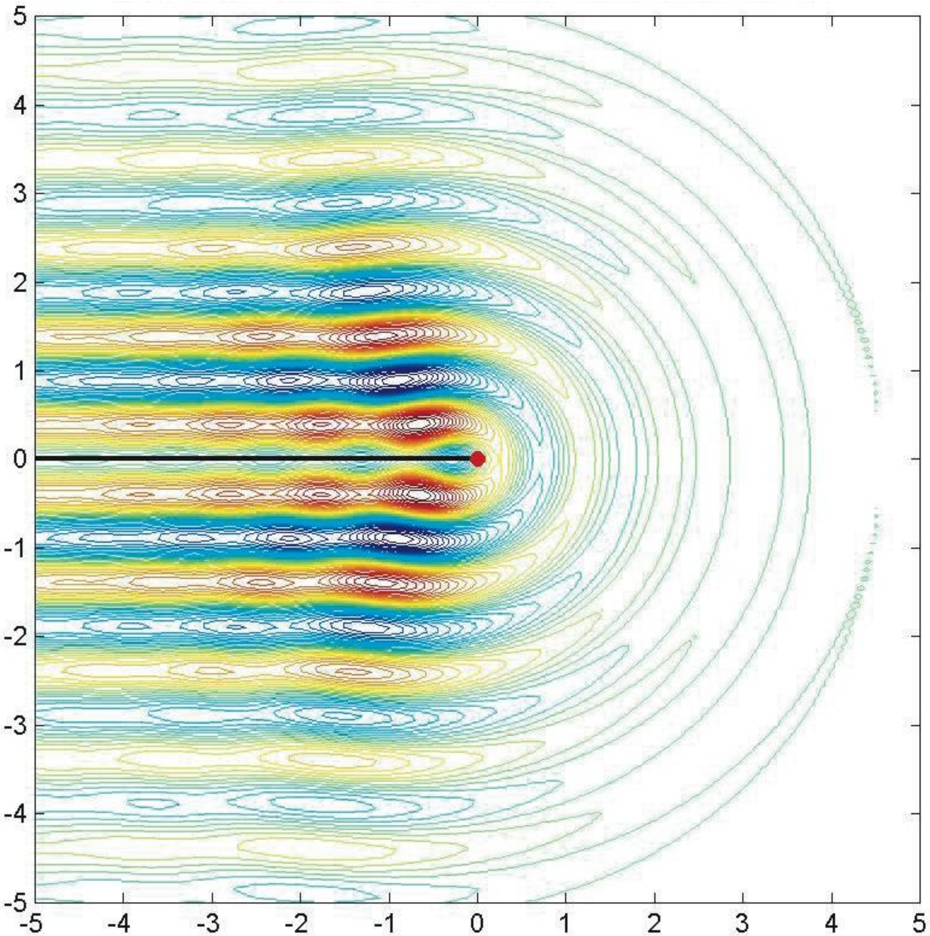


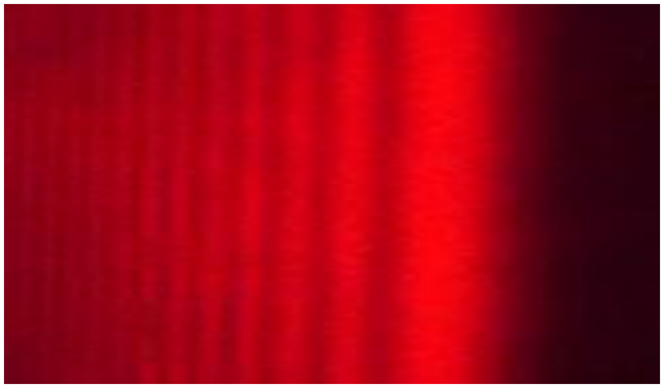
Its simulation

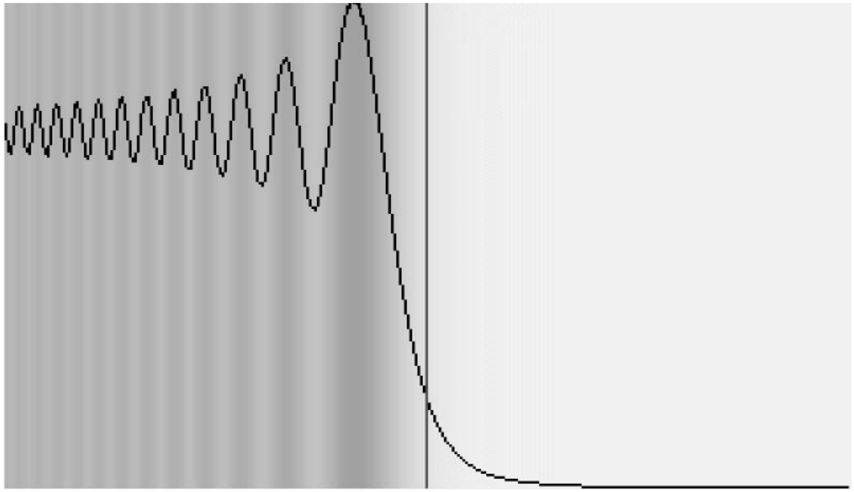
The structure of the wave field exhibits Fresnel interference fringes

In the limit of weak decay times:

Fresnel fringes



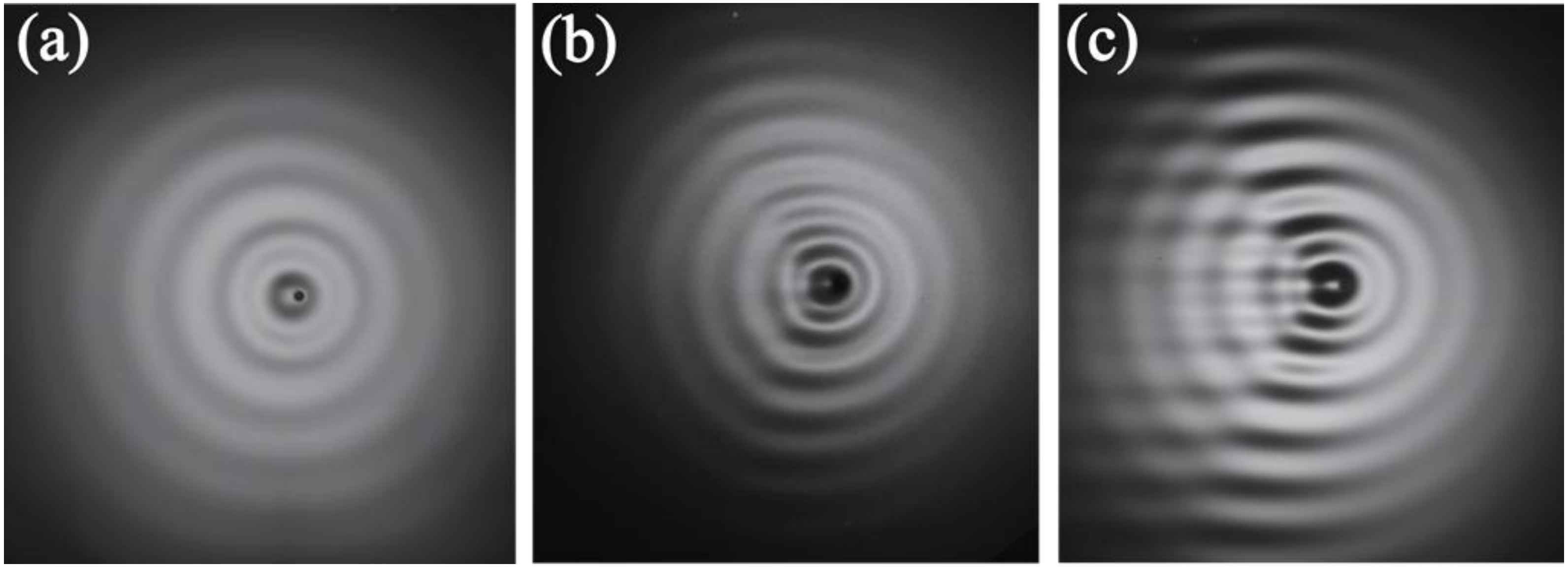




Fresnel diffraction behind an edge

(simulation John Talbot 1997)

M=5 M=10 M=40



- At each bounce, a circular localized mode of standing waves is generated

- The Faraday instability is a super critical bifurcation : below its threshold a perturbation is damped on a typical time :

$$\tau \propto \left| \gamma_m - \gamma_m^F \right|^{-1}$$

The wave field structure of a walker is dominated by a "path memory" effect

- A memory parameter can be defined as : $M = \tau / T_F$

- M is the number of bounces that contribute to the wave field

Part II

Walking in circles

Orbiting due to an external force

E. Fort, A. Eddi, A. Boudaoud, J. Moukhtar, and Y. Couder,
***PNAS*, 107, 17515-17520, (2010)**

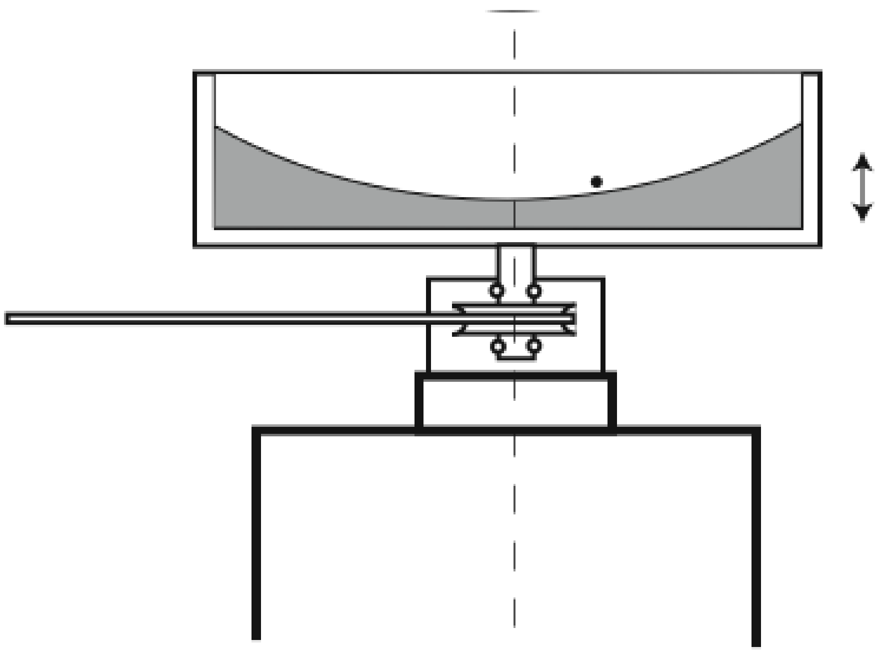
How to obtain circular trajectories?

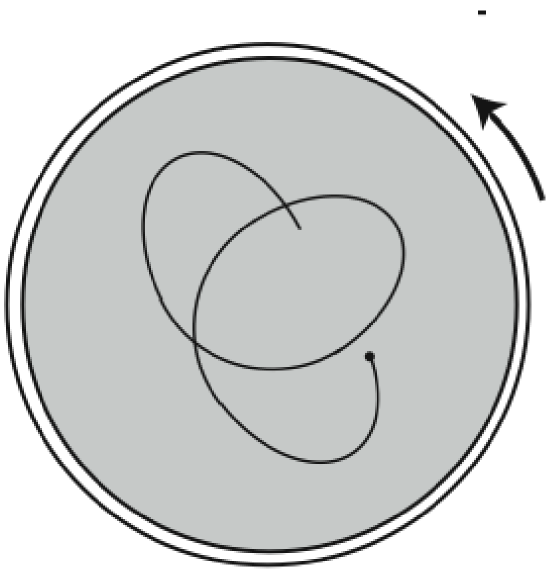
Use either a magnetic field or a rotating system,

(an analogy used by Michael Berry
to obtain a fluid mechanics analog of the Aharonov-Bohm effect)

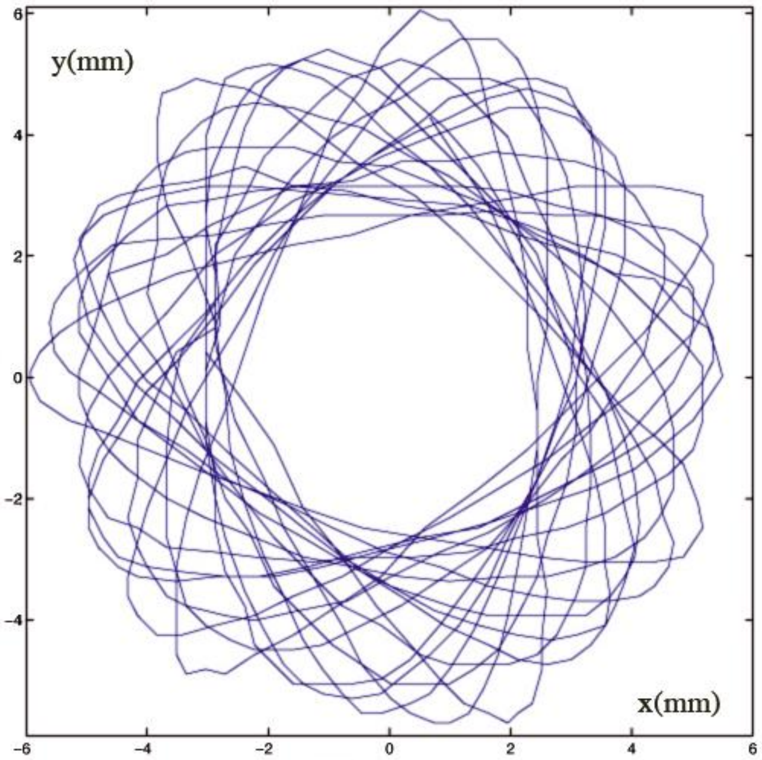
In a magnetic field **B** $$\vec{F} = q\left(\vec{V} \wedge \vec{B}\right)$$ Orbital motion	On a surface rotating with angular velocity Ω $$\vec{F}_c = -2m\left(\vec{V} \wedge \vec{\Omega}\right)$$ Orbital motion in the rotating frame
Larmor angular velocity $\omega_L = qB/m$	Orbital motion angular velocity 2Ω
Orbit radius $\rho_L = V/\omega_L$	Orbit radius $R = V/2\Omega$

The rotating Faraday experiment

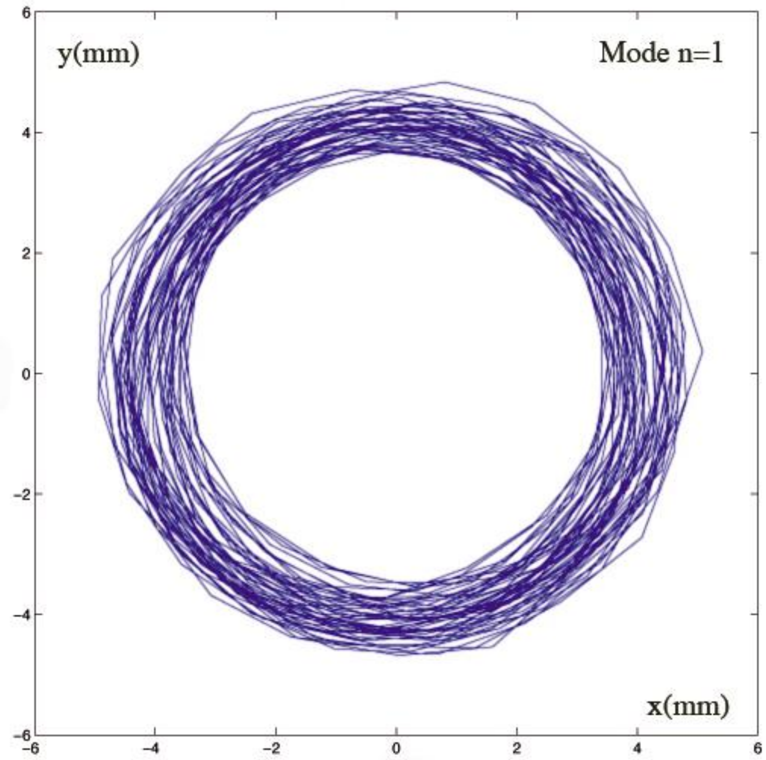




Measured trajectories



Trajectory in the laboratory frame of reference

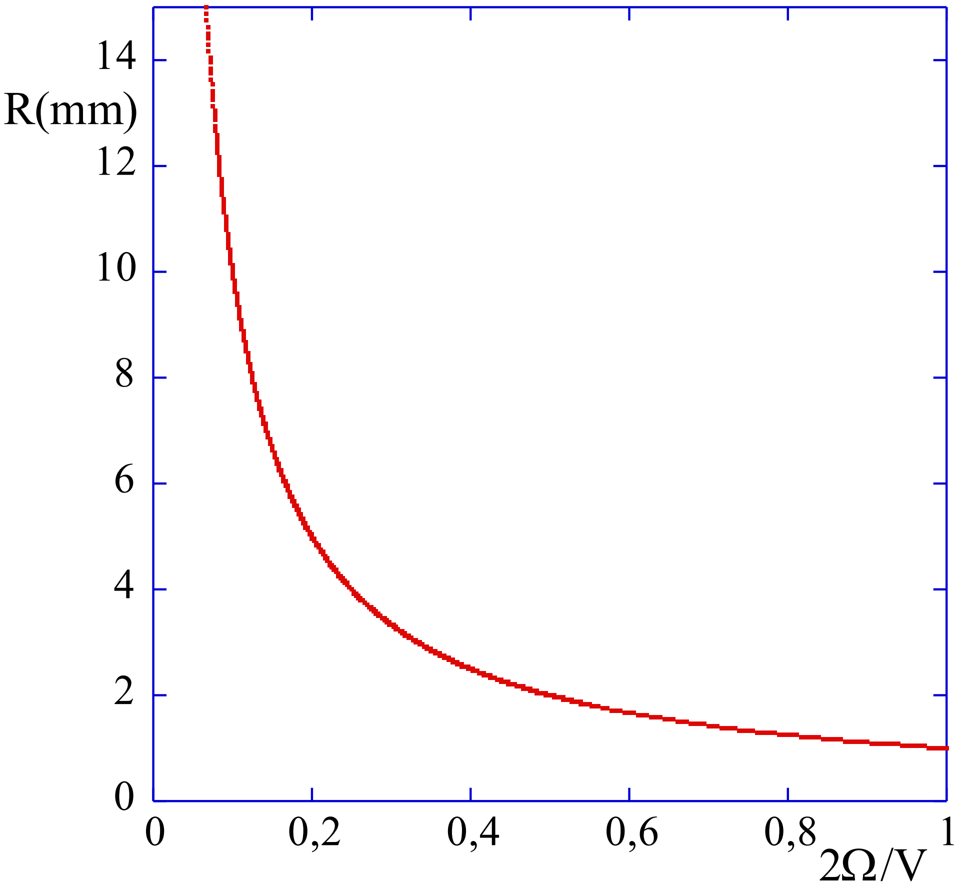


Trajectory in the rotating frame of reference

Classical radius of the orbits

The "classical" radius of the orbit observed in the rotating frame and due to Coriolis effect should be:

$$R=V/2\Omega$$



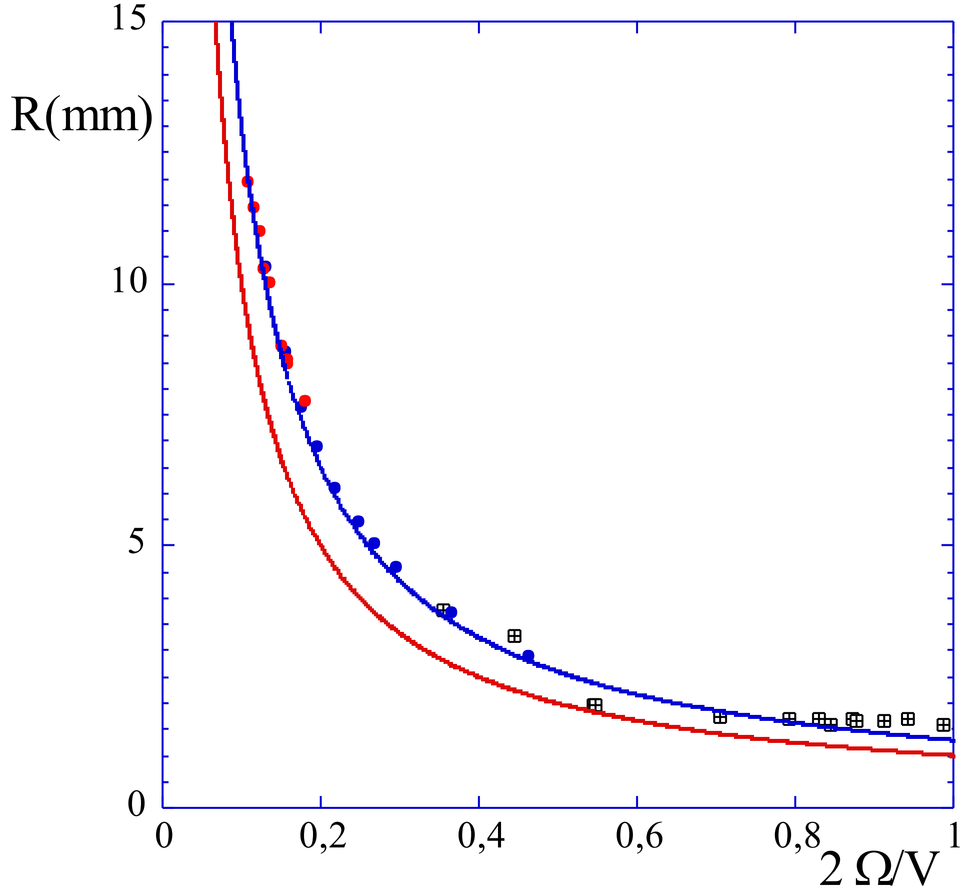
Measured radius of the orbits for walkers with weak path memory (far from the Faraday threshold)

The radius of the orbit observed in the rotating frame has a “classical“ dependence, but slightly shifted

R=a (V/2Ω)

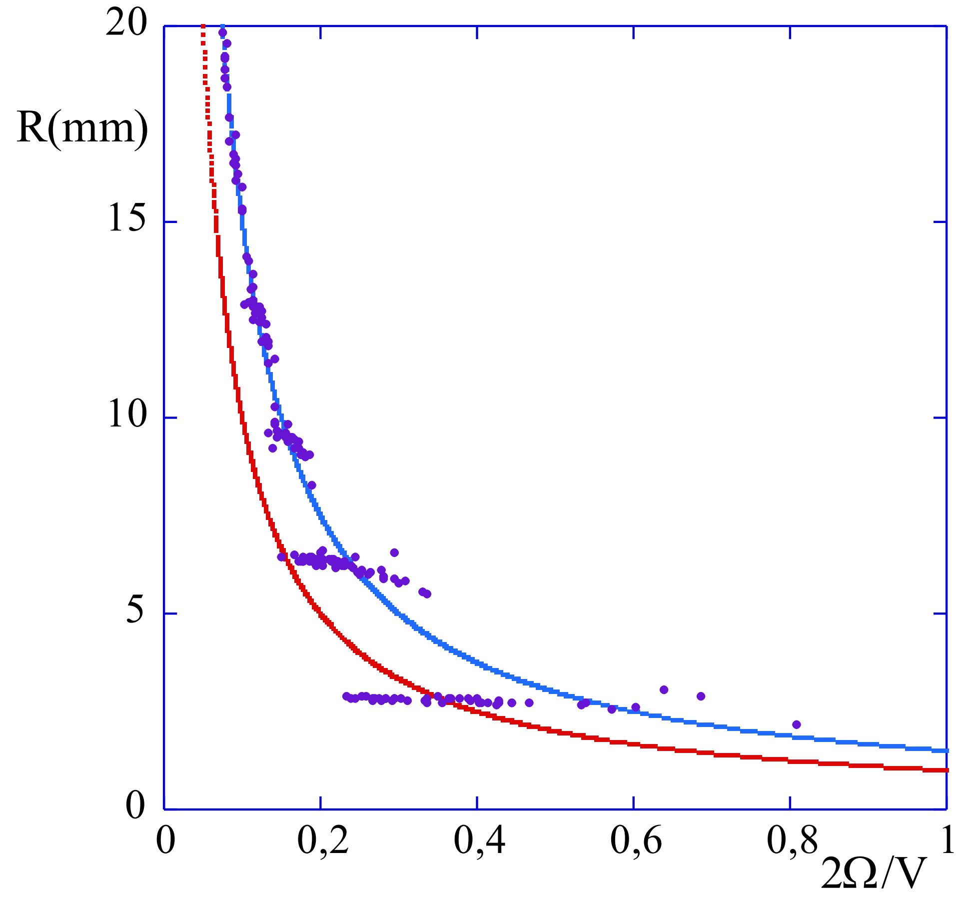
With

$a \approx 1.3$



Measured radius of the orbits of walkers having long term path memory (near the Faraday threshold)

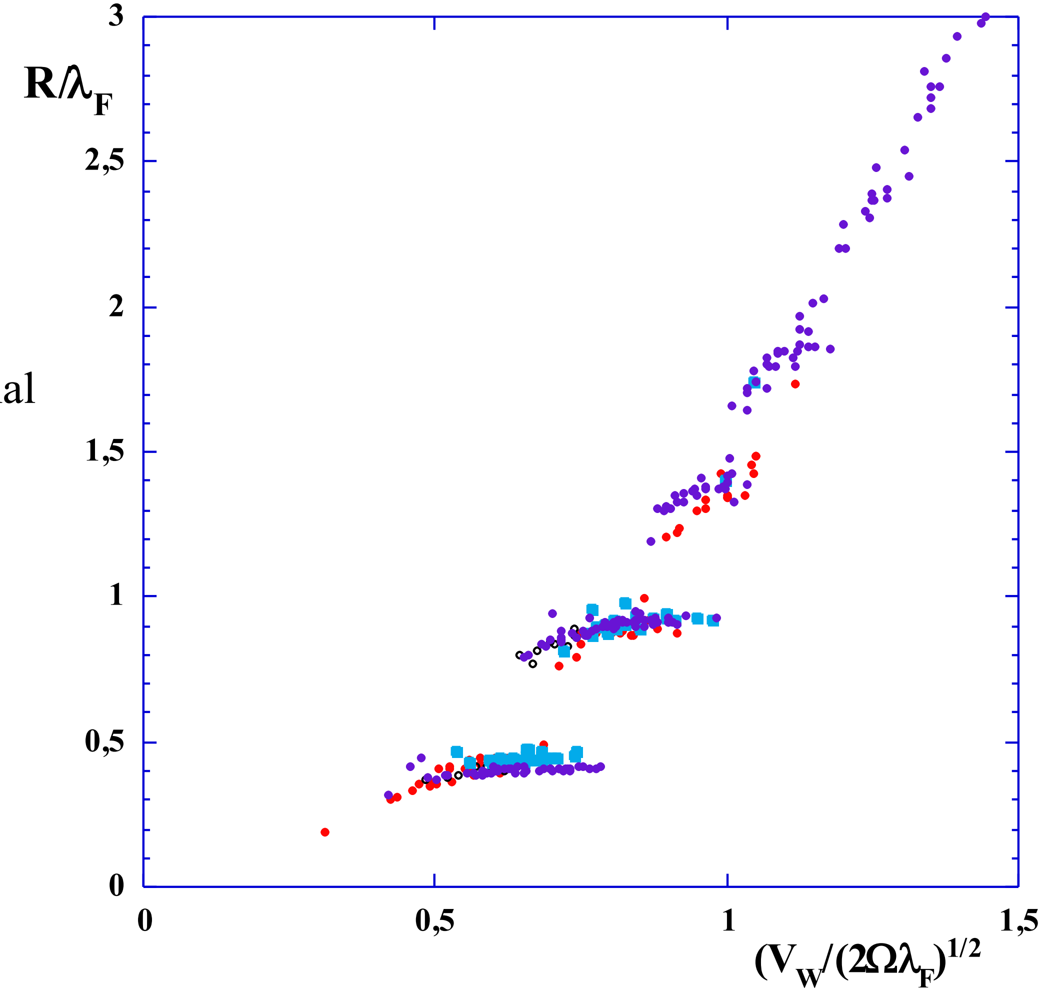
Near the Faraday instability threshold, the radius of the orbit evolves by discrete jumps when Ω is increased



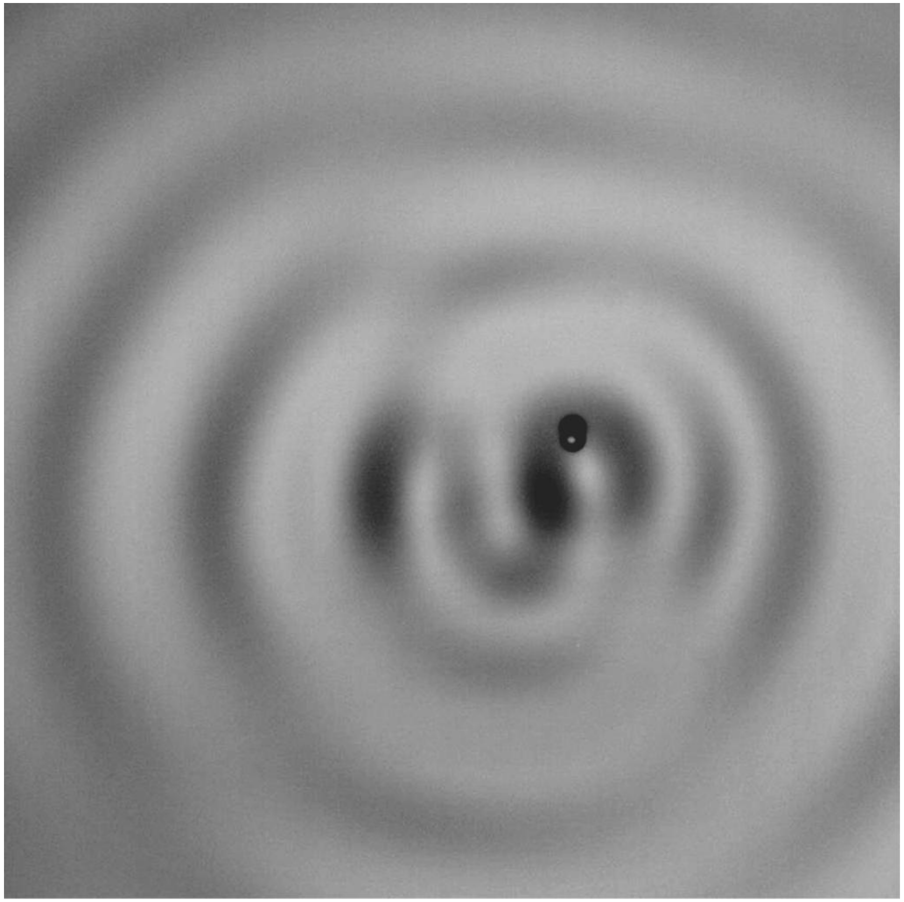
Dimensionless radius of the orbits

The radii of the orbits obtained at various frequencies and for walkers of various velocities are all rescaled by expressing:

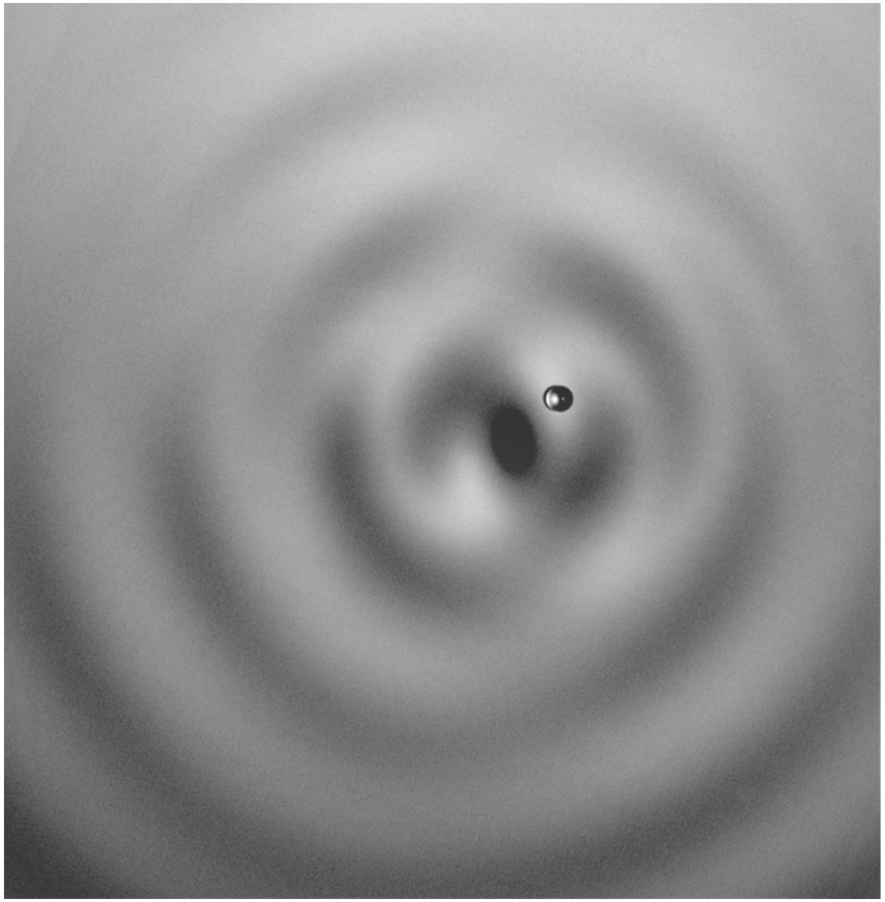
$\frac{R_n}{\lambda_F}$ as a function of the non-dimensionnal parameter $\sqrt{\frac{V_W}{2\Omega\lambda_F}}$



The first two modes



$d_n^{orb}=2\ \lambda_F$



$d_n^{orb}=\lambda_F$

Landau levels:

the Bohr-Sommerfeld quantization in a magnetic field

The Bohr Sommerfeld condition imposes:

$$\oint (\mathbf{p} - e\mathbf{A}) \cdot d\mathbf{l} = (n + \gamma)h$$

The Larmor radius can only take discrete values

$$\rho_L^n = \frac{1}{\sqrt{\pi}} \sqrt{\frac{h}{qB}\left(n + \frac{1}{2}\right)}$$

The radii of the Landau orbits in a magnetic field

The quantized Larmor radius:

$$\rho_L^n = \frac{1}{\sqrt{\pi}}\sqrt{\left(n+\frac{1}{2}\right)\frac{h}{qB}}$$

can be expressed as a function of the de Broglie wavelength $\lambda_{dB} = \dfrac{h}{mV}$

$$\boxed{\frac{\rho_n}{\lambda_{dB}} = \sqrt{1/\pi}\sqrt{\left(n+\frac{1}{2}\right)\frac{m}{qB}\frac{V}{\lambda_{dB}}}}$$

$$\boxed{\begin{array}{l} \dfrac{m}{qB} \Leftrightarrow \dfrac{1}{2\Omega} \\ \lambda_{dB} \Leftrightarrow \lambda_F \end{array}}$$

The analogy suggests: $\dfrac{R_n}{\lambda_F} = \sqrt{1/\pi}\sqrt{\left(n+\dfrac{1}{2}\right)\dfrac{V_W}{2\Omega\lambda_F}}$

Dimensionless radius of the orbits

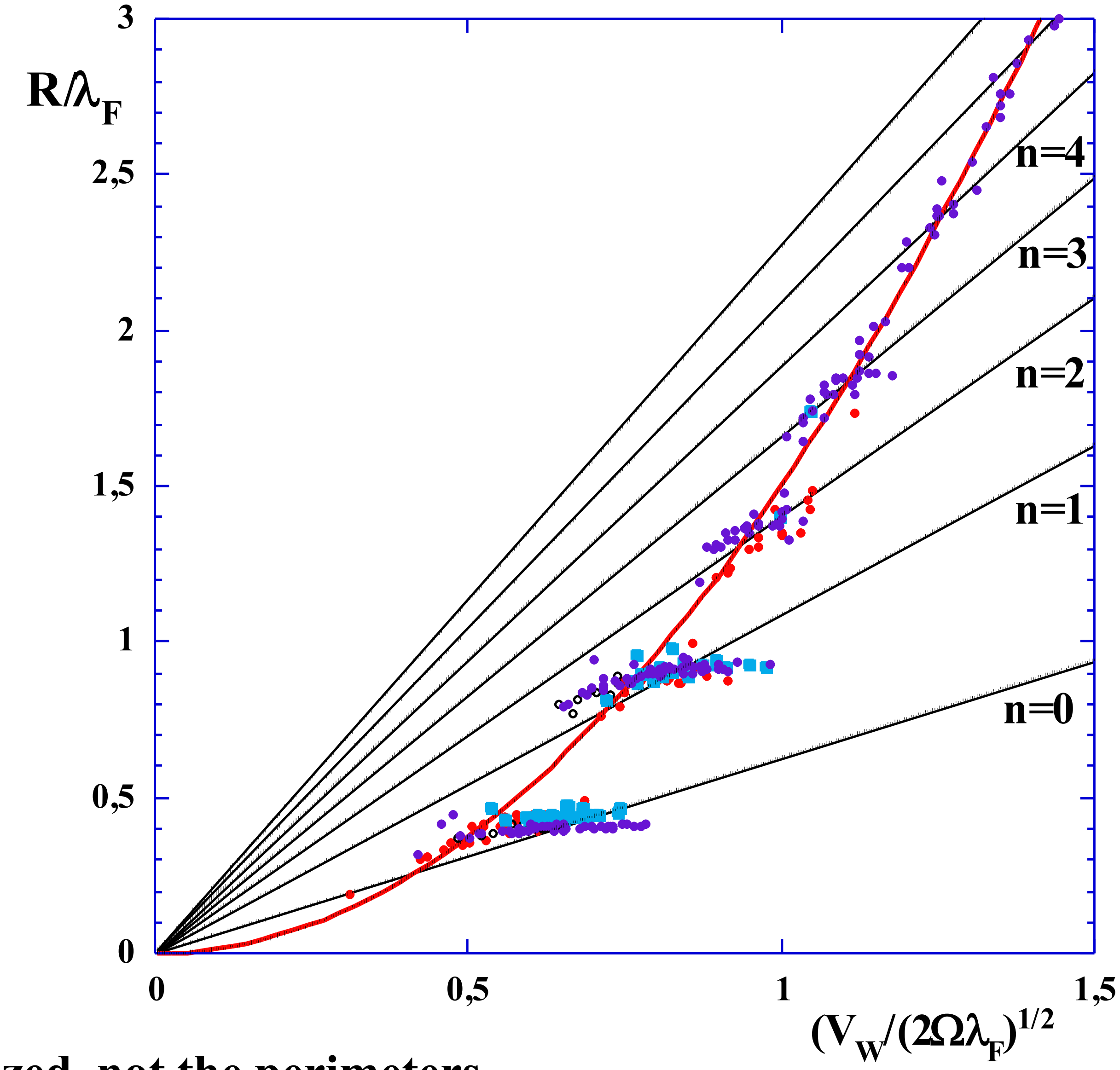
The analogy suggests:

$$\frac{R_n}{\lambda_F} = \sqrt{1/\pi}\sqrt{\left(n+\frac{1}{2}\right)\frac{V_W}{2\Omega\lambda_F}}$$

$$\sqrt{1/\pi} = 0.564$$

We find

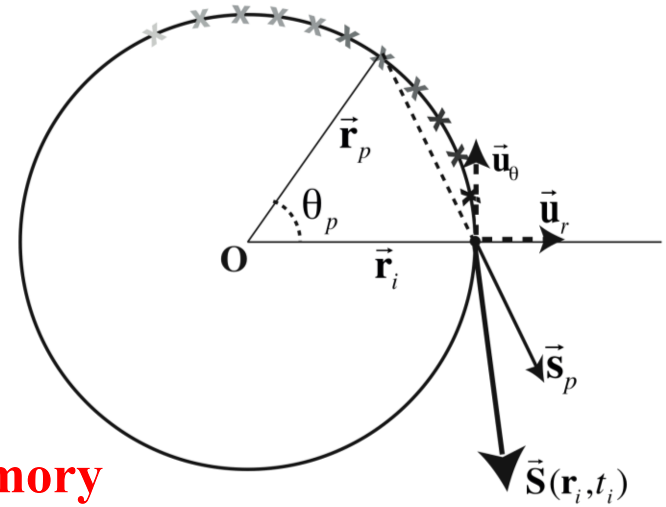
$$\frac{R_n}{\lambda_F} = 0.89\sqrt{\left(n+\frac{1}{2}\right)\frac{V_W}{2\Omega\lambda_F}}$$



The diameters are quantized, not the perimeters

The path memory effect in the case of circular trajectories

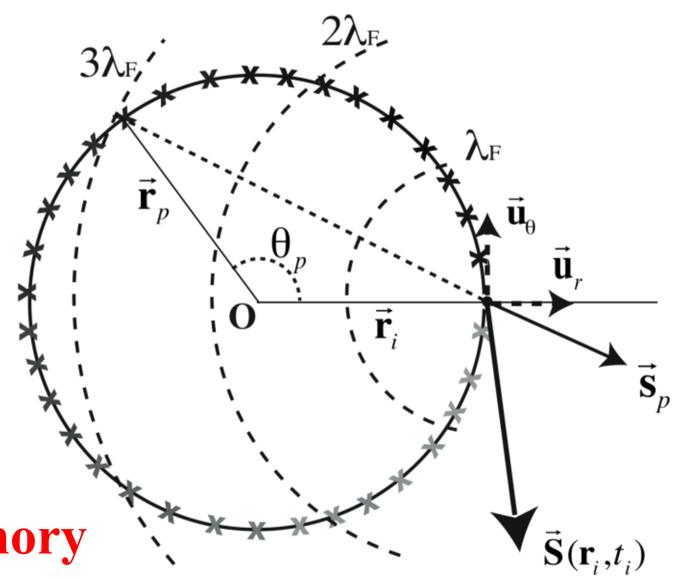
How to compute the local slope induced at the point of impact by the superposition of the waves due to “sources” distributed on a circular orbit



$$\vec{S}(\vec{r}_i,t_i)=\vec{\nabla}h(\vec{r}_i,t_i)$$

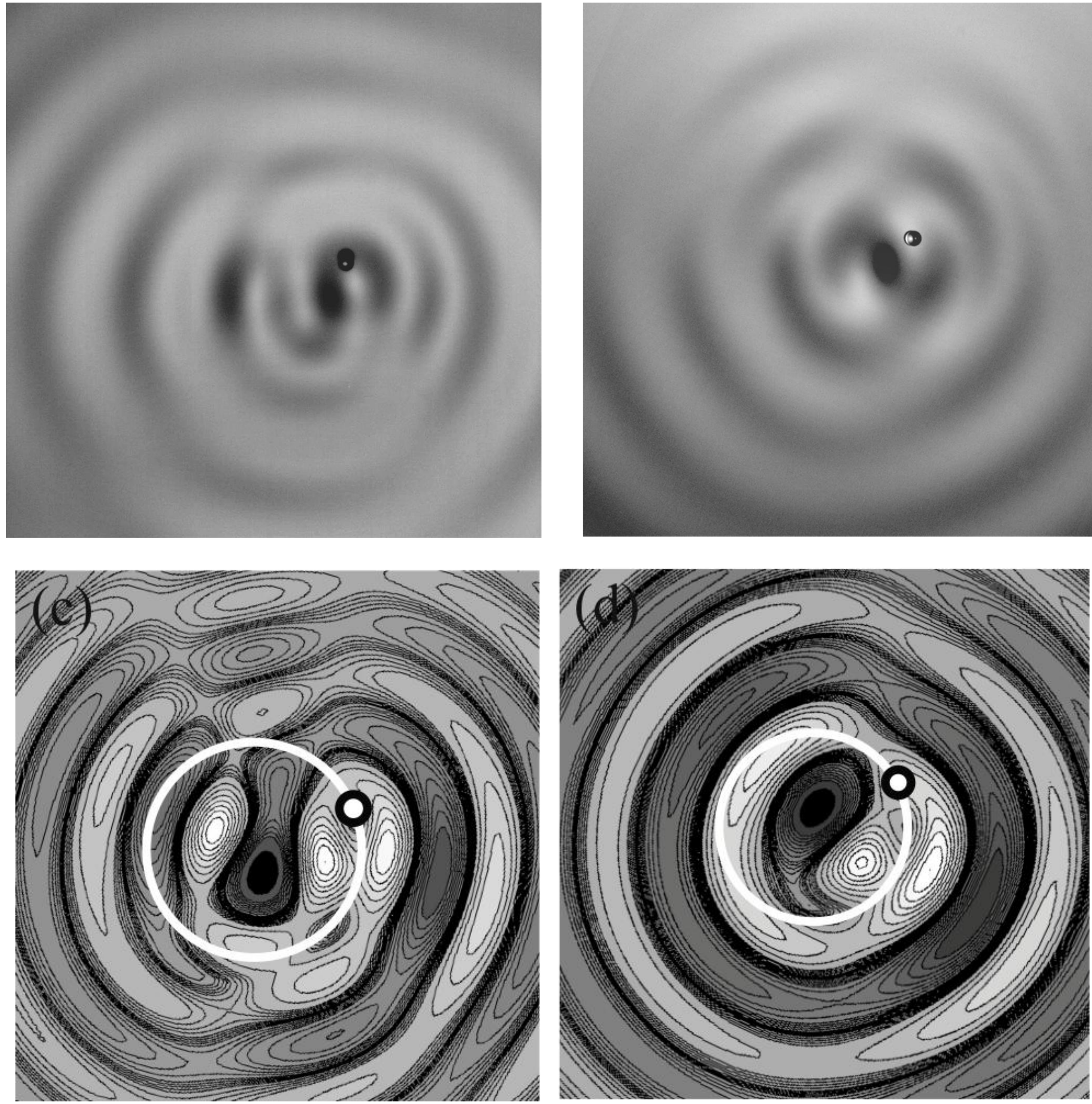
Short-term memory

$$\vec{S}(\vec{r}_i,t_i)=\sum_{p=i-1}^{\infty}\vec{s}\,(\vec{r}_i-\vec{r}_p,t_i-t_p)$$



Long-term memory

The first two modes as observed and simulated

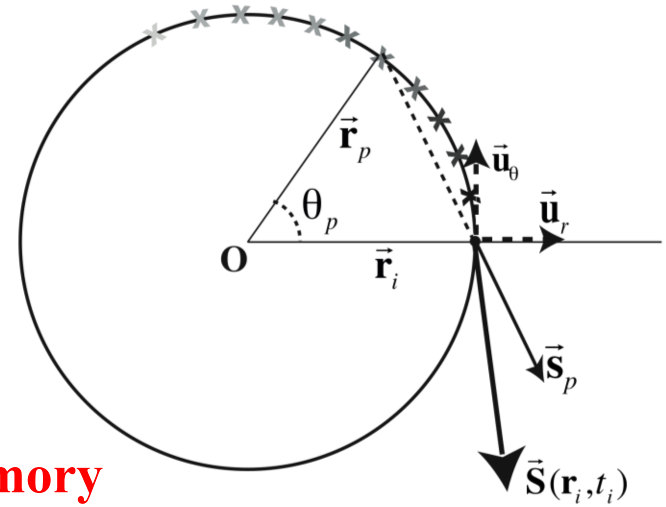


The path memory effect in the case of circular trajectories

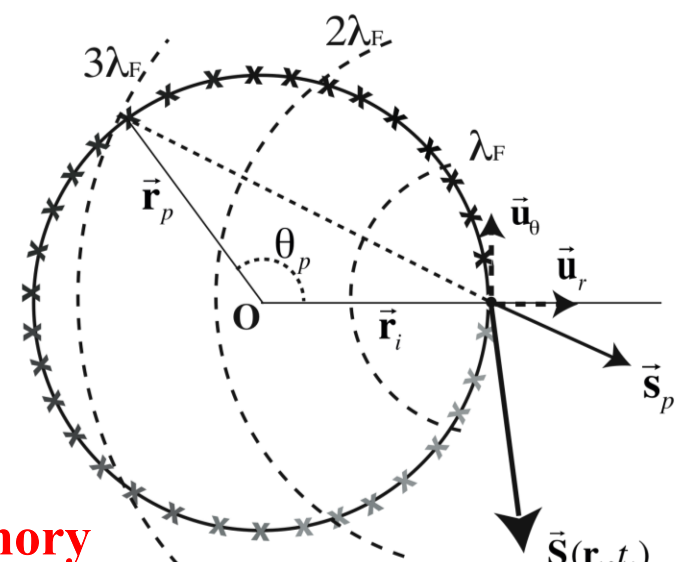
How to compute the local slope induced at the point of impact by the superposition of the waves due to "sources" distributed on a circular orbit

$$\vec{S}(\vec{r}_i, t_i) = \vec{\nabla} h(\vec{r}_i, t_i)$$

$$\vec{S}(\vec{r}_i, t_i) = \sum_{p=i-1}^{\infty} \vec{s}\,(\vec{r}_i - \vec{r}_p, t_i - t_p)$$

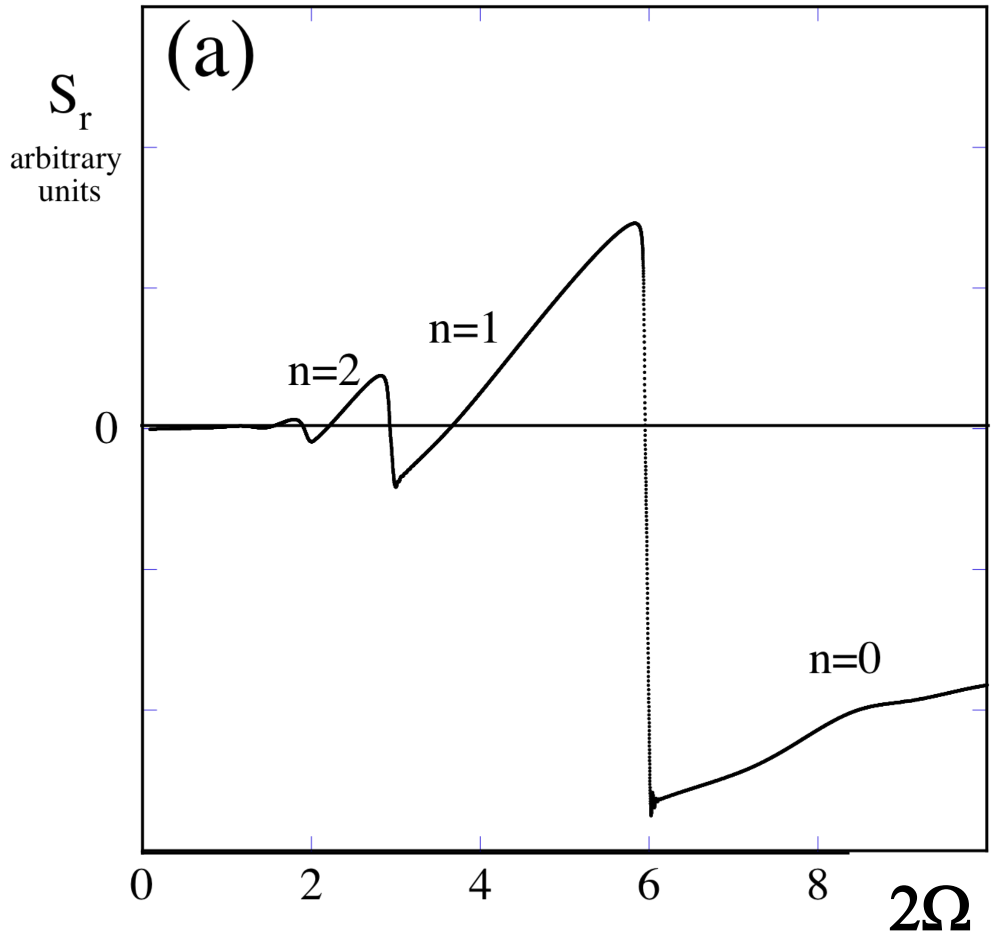


Short-term memory

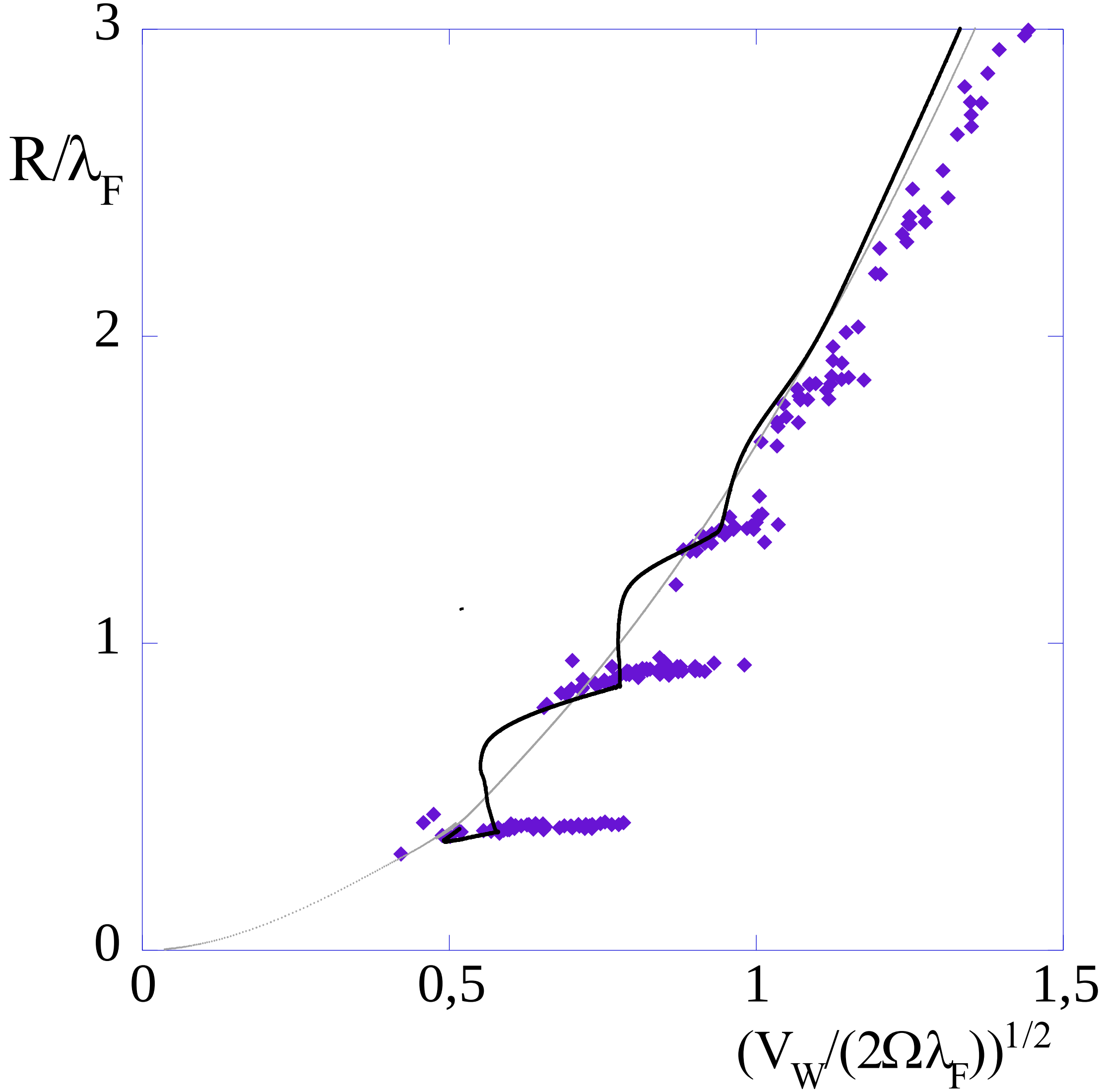


Long-term memory

The radial slope S_r at the point of bouncing generates an additional centripetal (or centrifugal) force.
It is responsible for the formation of plateaus



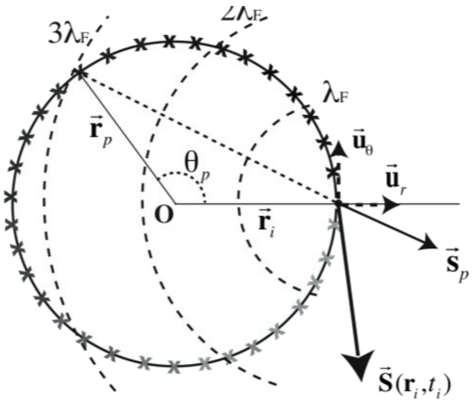
The results of numerical simulations (Emmanuel Fort's model)

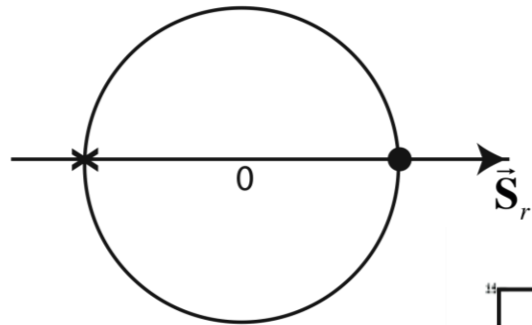


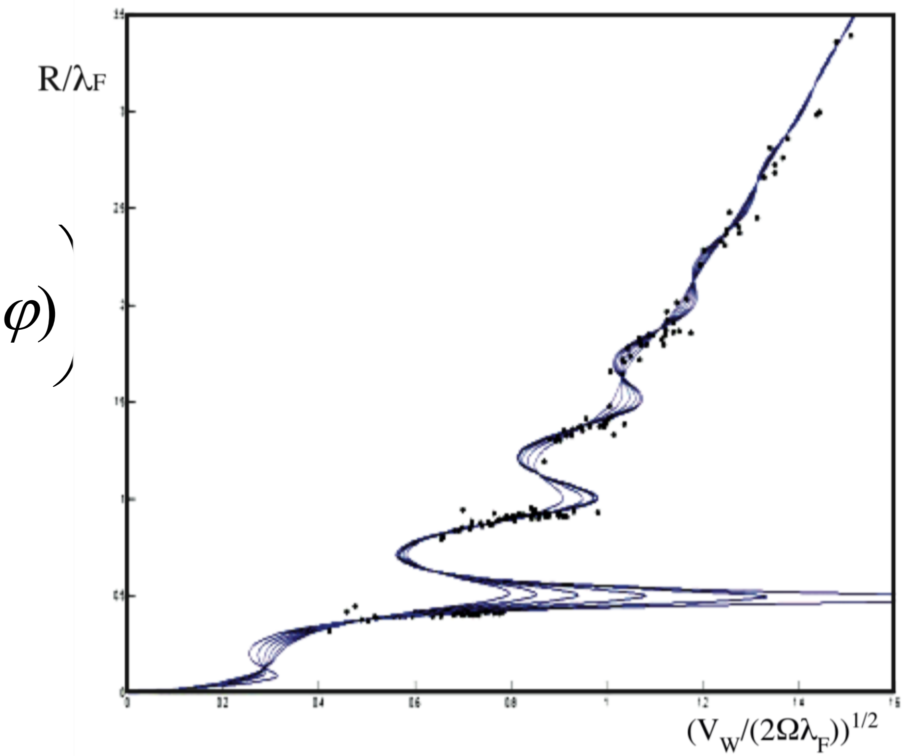
In light grey:
short term path memory

In black:
long-term memory

The evolution of the orbits diameters can be recovered analytically by assuming that with increasing memory only one source diametrically opposed to the droplet generates the additional force







$$\frac{mV_W^2}{R} = -m\left(2\Omega V_W + A\sin(2\pi\frac{2R}{\lambda_F} + \varphi)\right)$$

with

$$A = A_0\, e^{-\frac{r}{n_s\lambda_F}}\ e^{-\frac{t}{n_t T_F}}$$

Part III

Walking when spatially confined

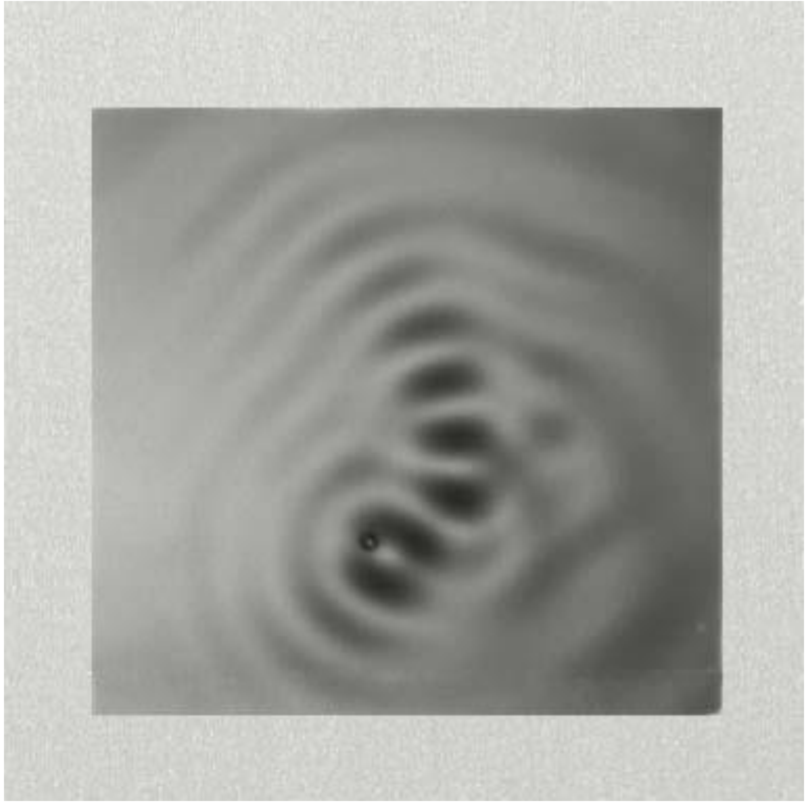
What is the effect of path memory when the trajectory is confined by boundaries?

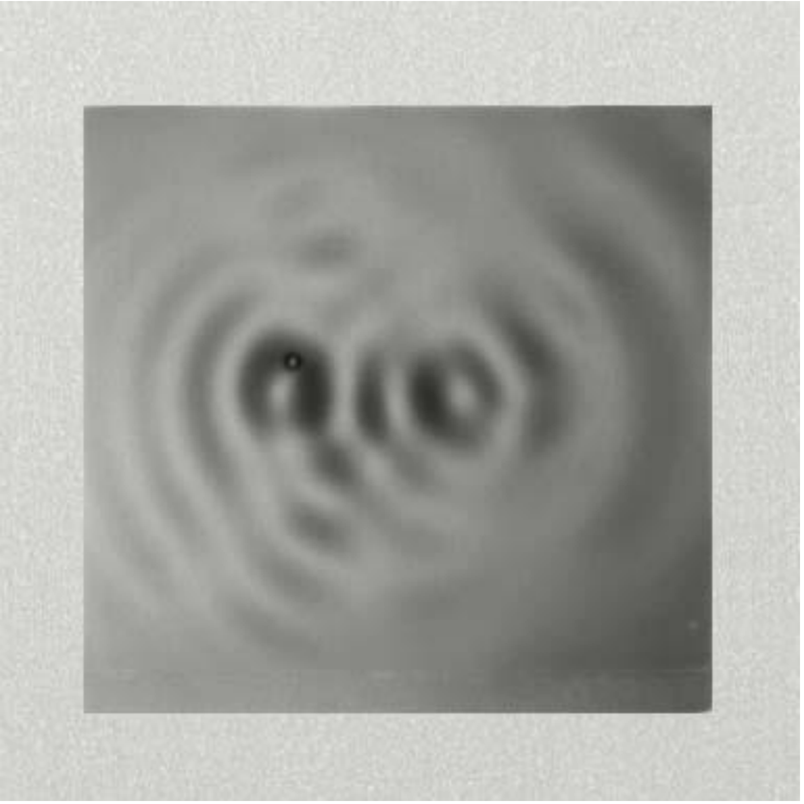
(a) Trajectories in closed cavities

(b) Trajectories through slits : diffraction and interferences

(a) Effect of the confinement in a corral

Chaotic trajectories in a square corral





Has the probability of visit a relation to the eigenmodes of the cavity?

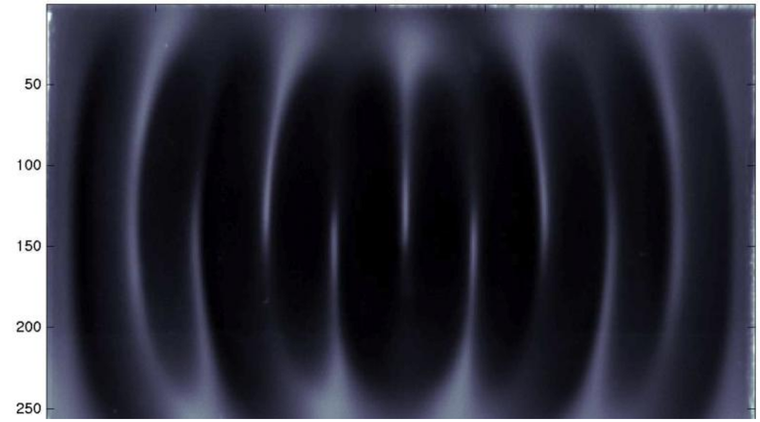
We chose to study rectangular cavities tuned with one dominant resonant mode,

Dan Harris and John Bush (MIT) chose circular cavities (John's talk)

In a rectangular cavity

with *Julien Moukhtar*

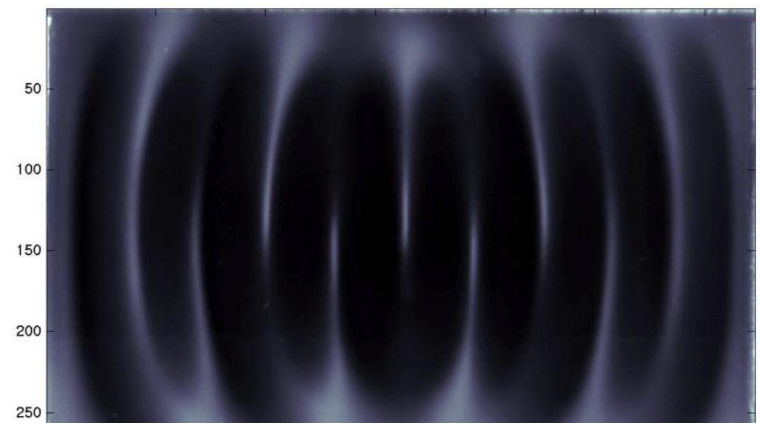
The Faraday eigenmode of the rectangular cavity



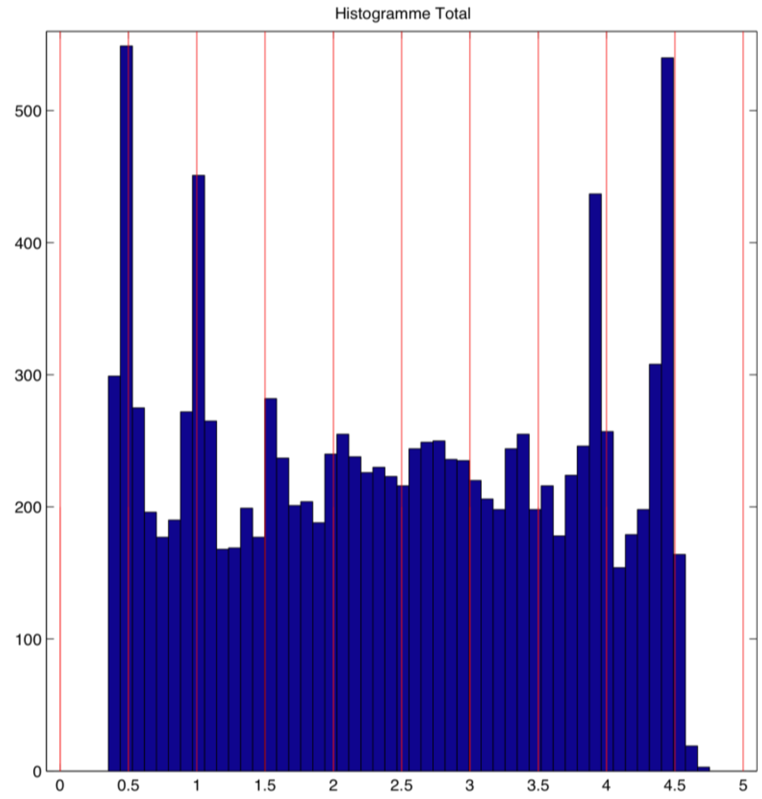
In a rectangular cavity

with *Julien Moukhtar*

The Faraday eigenmode of the rectangular cavity

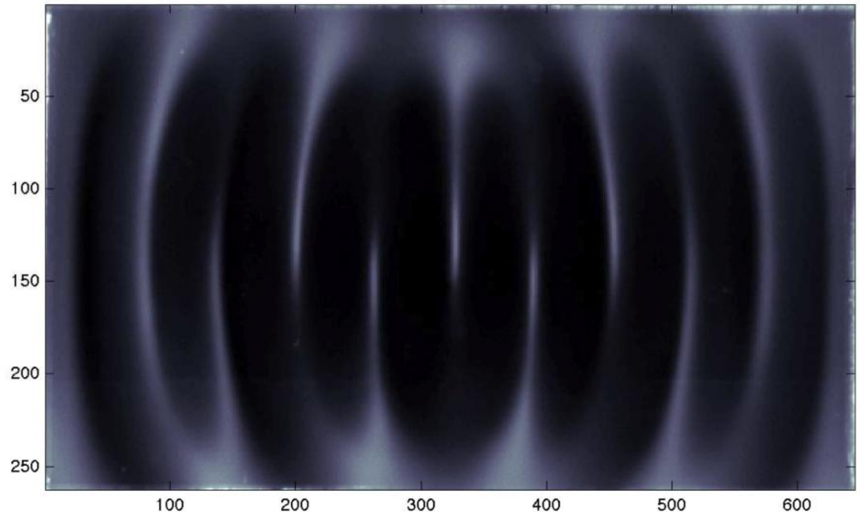


The probability of presence of a walker along the main axis of the cavity

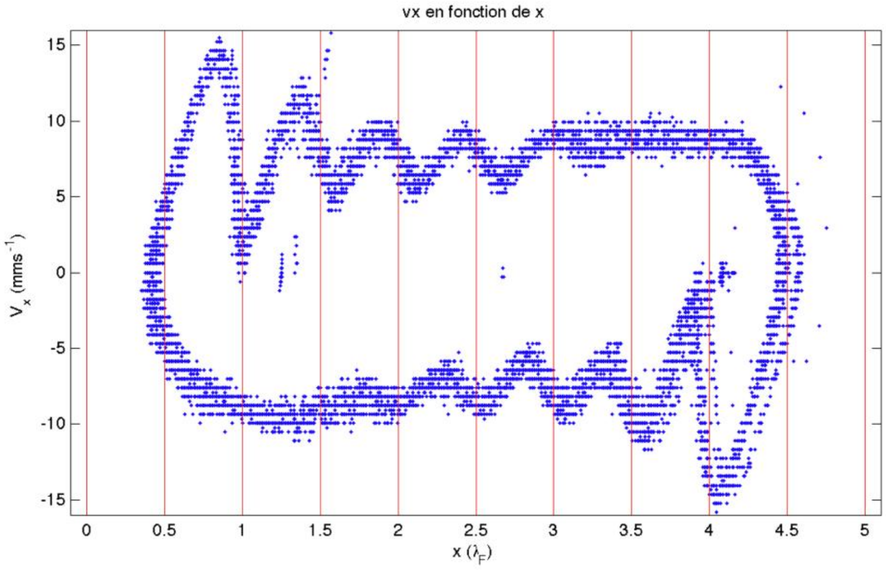


In a rectangular cavity

The Faraday eigenmode of the rectangular cavity

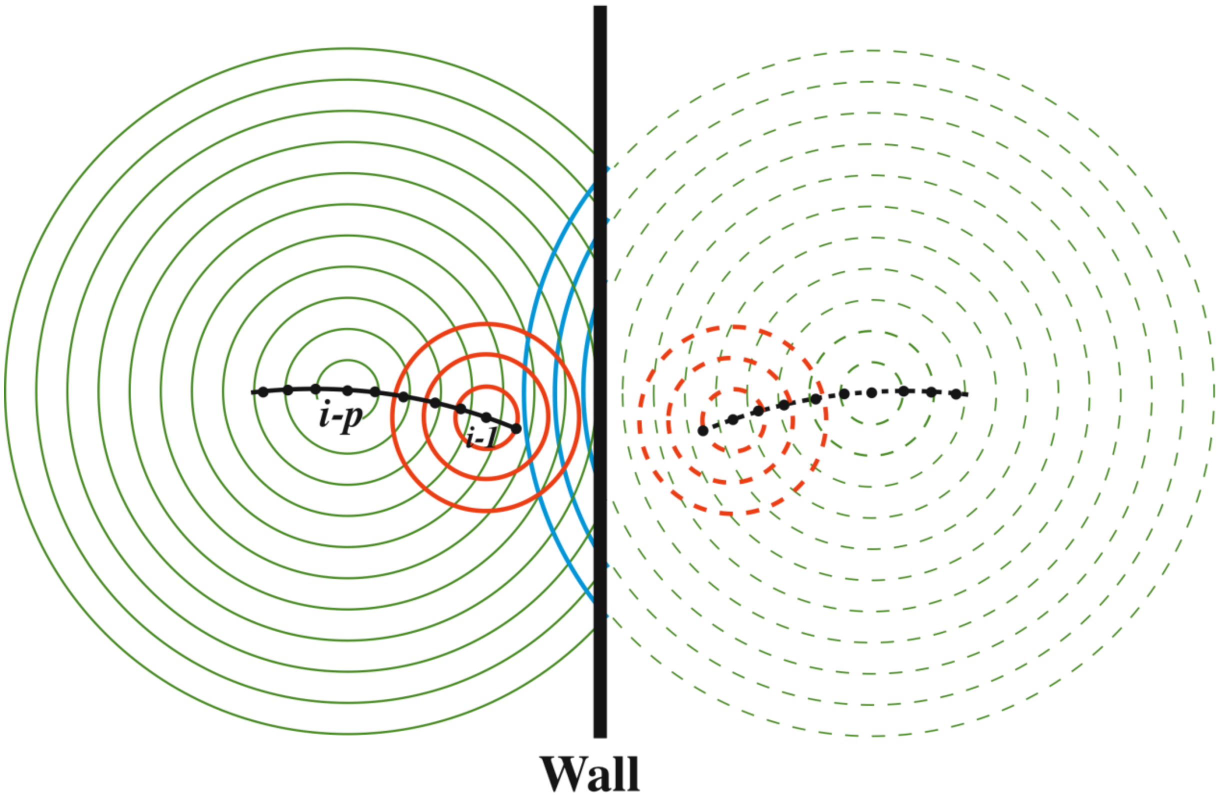


The local velocity of the walker along the main axis of the cavity

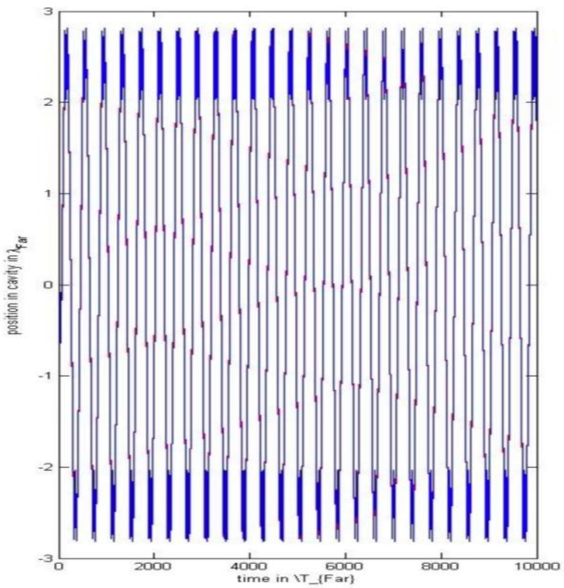


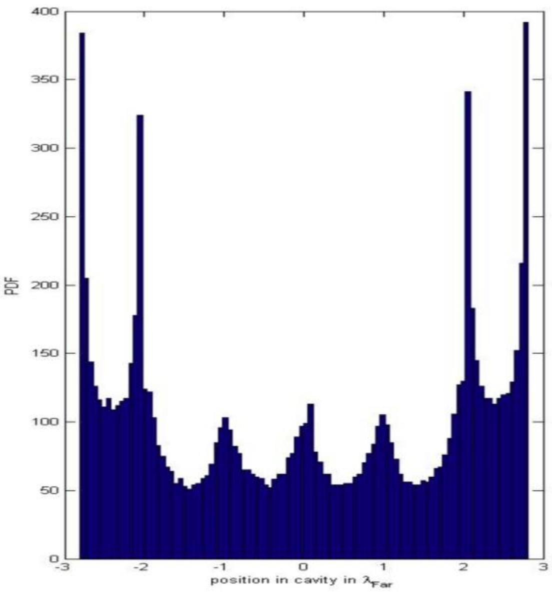
The numerical model of walkers in the vicinity of boundaries

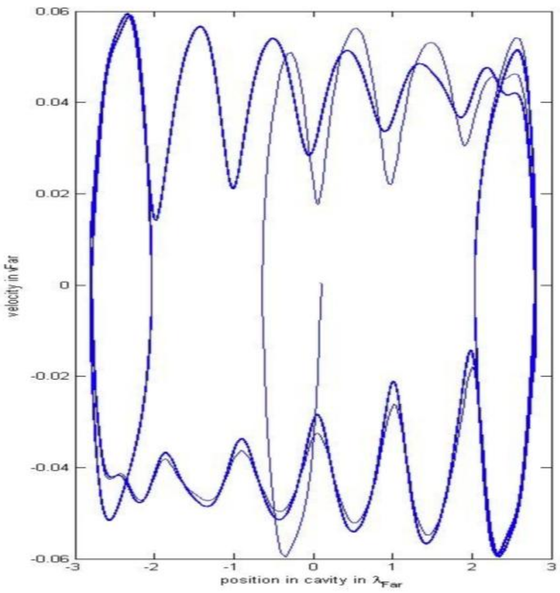
The echolocation of the walker: interaction with boundaries,
reflected waves are also taken into account

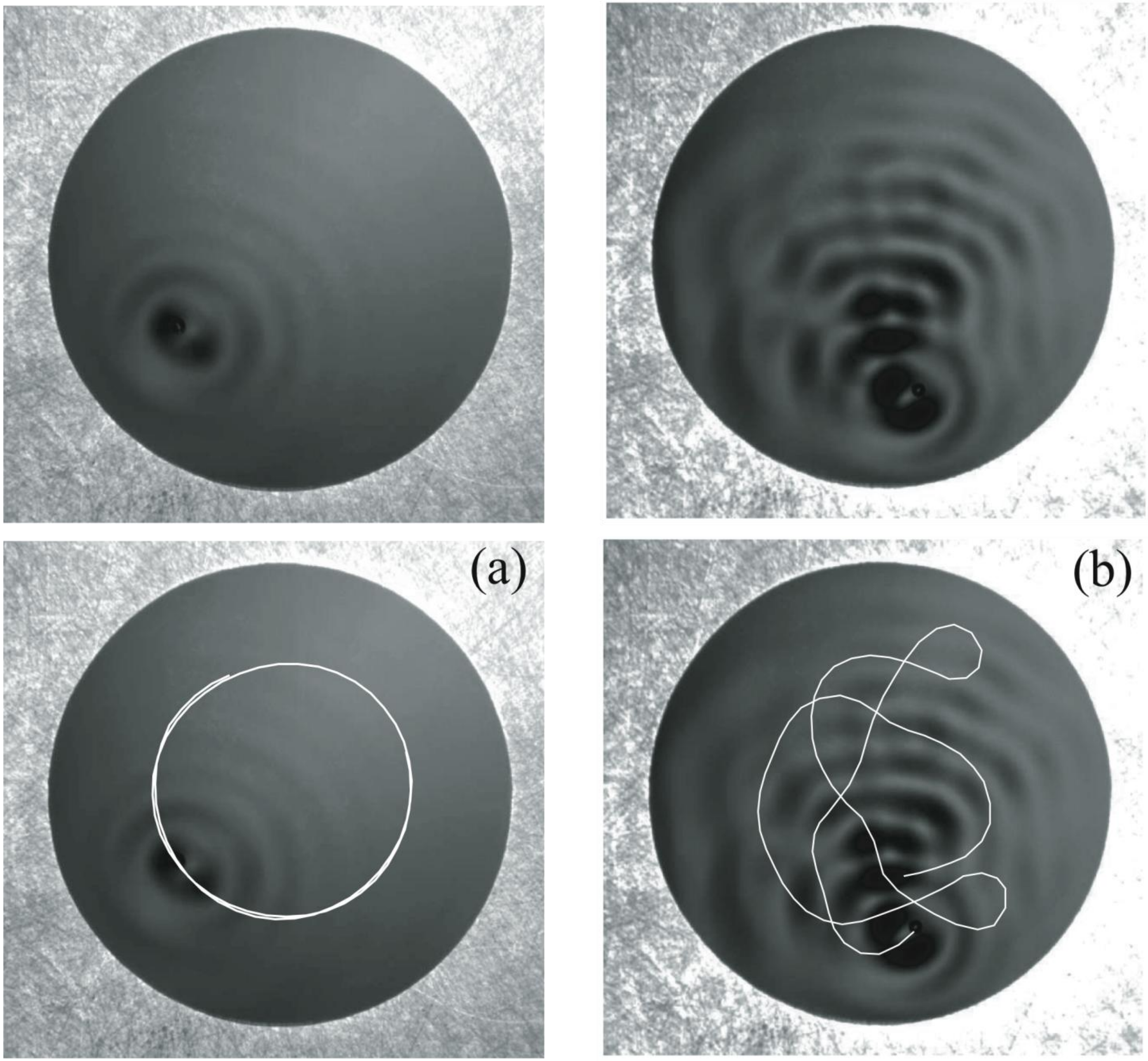


The 1D numerical simulation

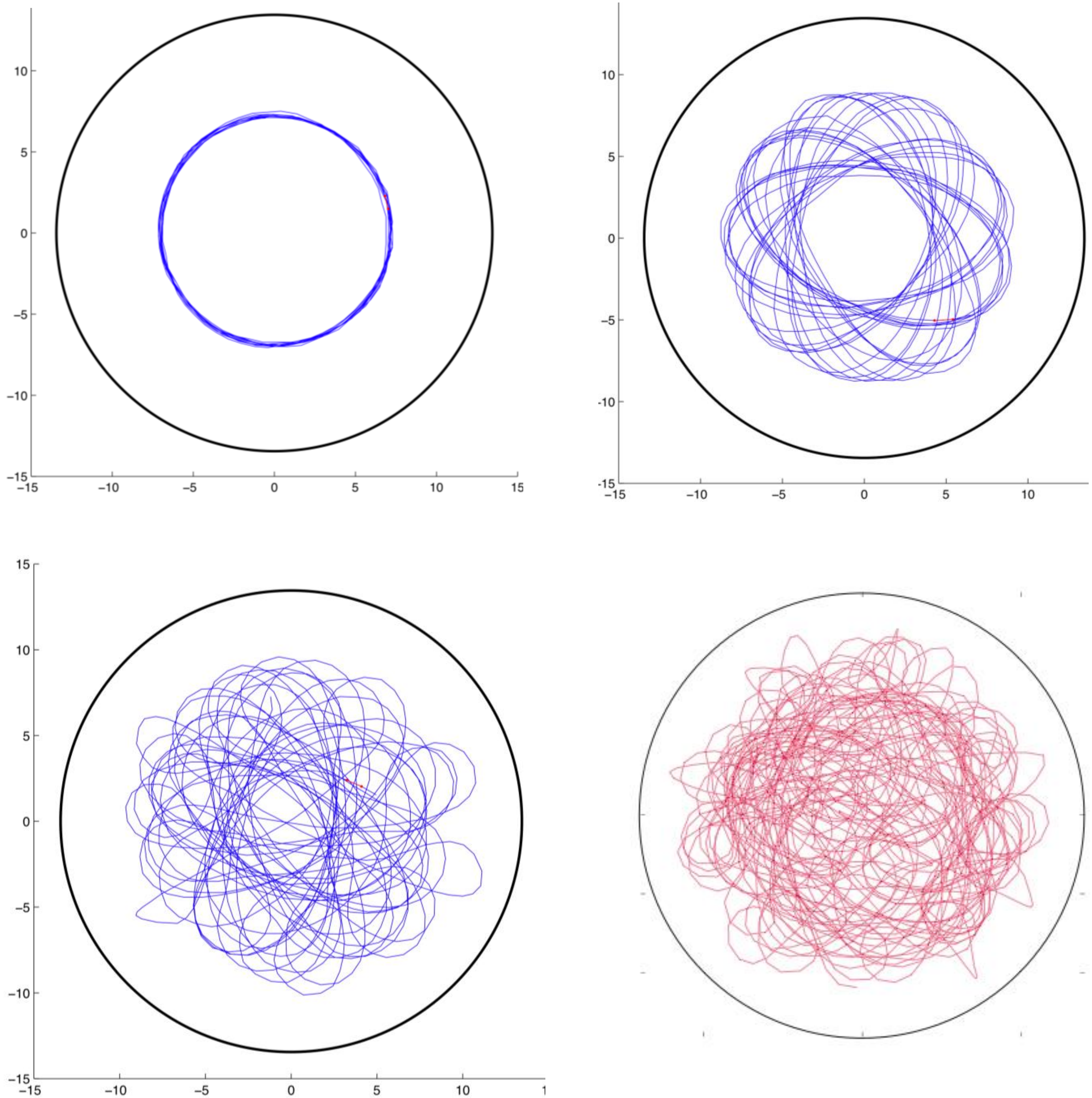




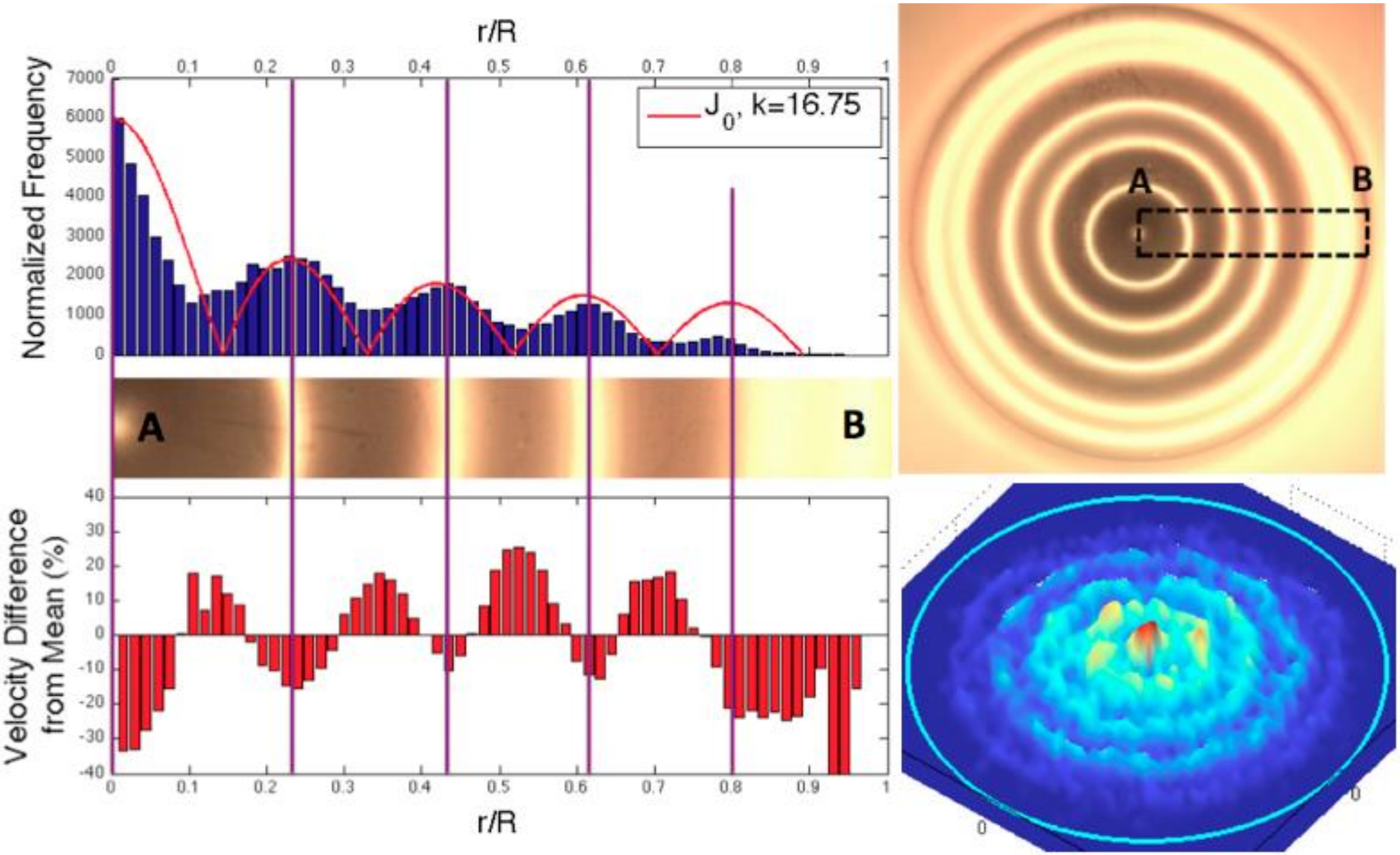



(a)
(b)

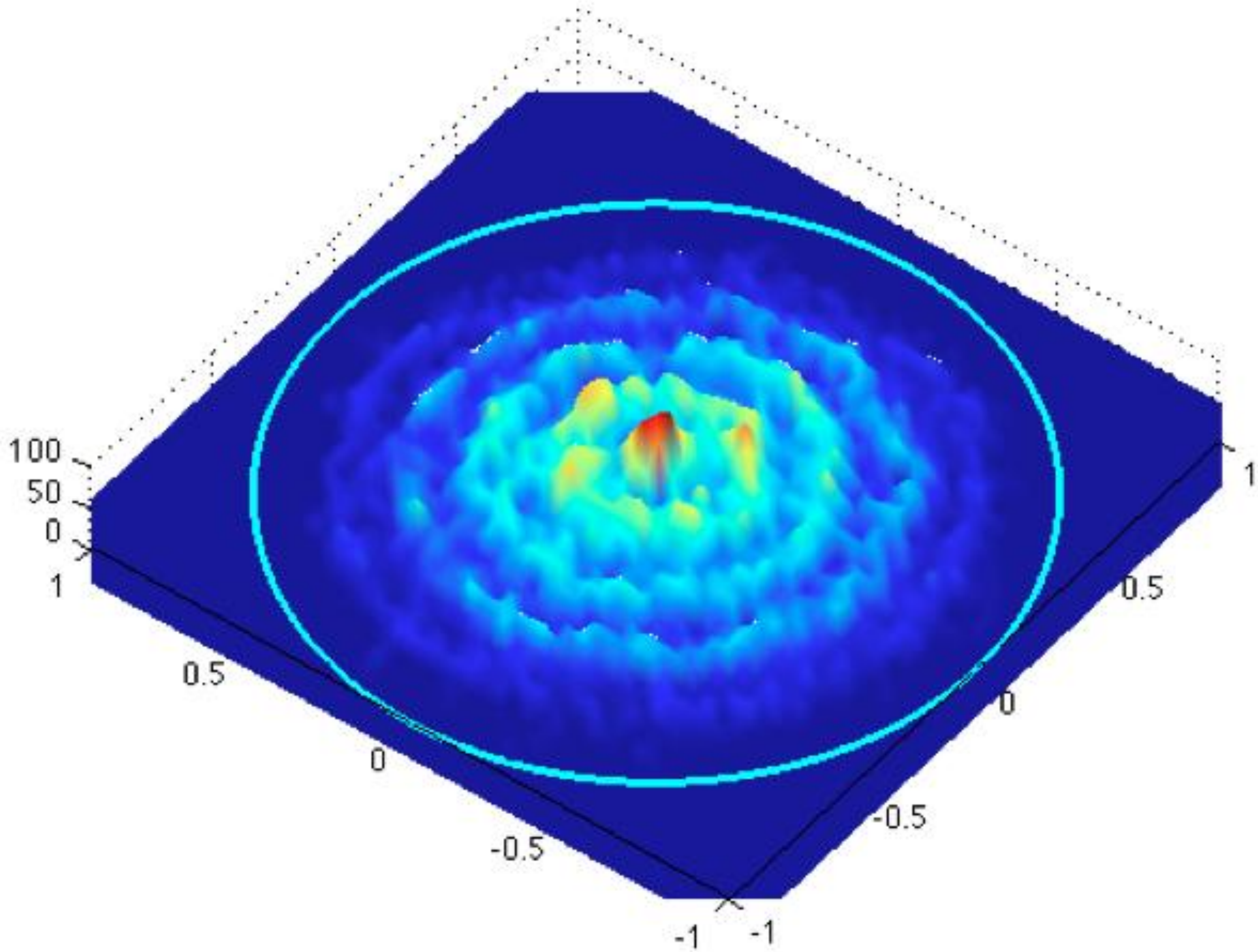
Boites circulaires



The same experiment by Dan Harris et John Bush (MIT)



The probability of visit (21966 position measurements)



Courtesy Dan Harris and John Bush

Part III (b)

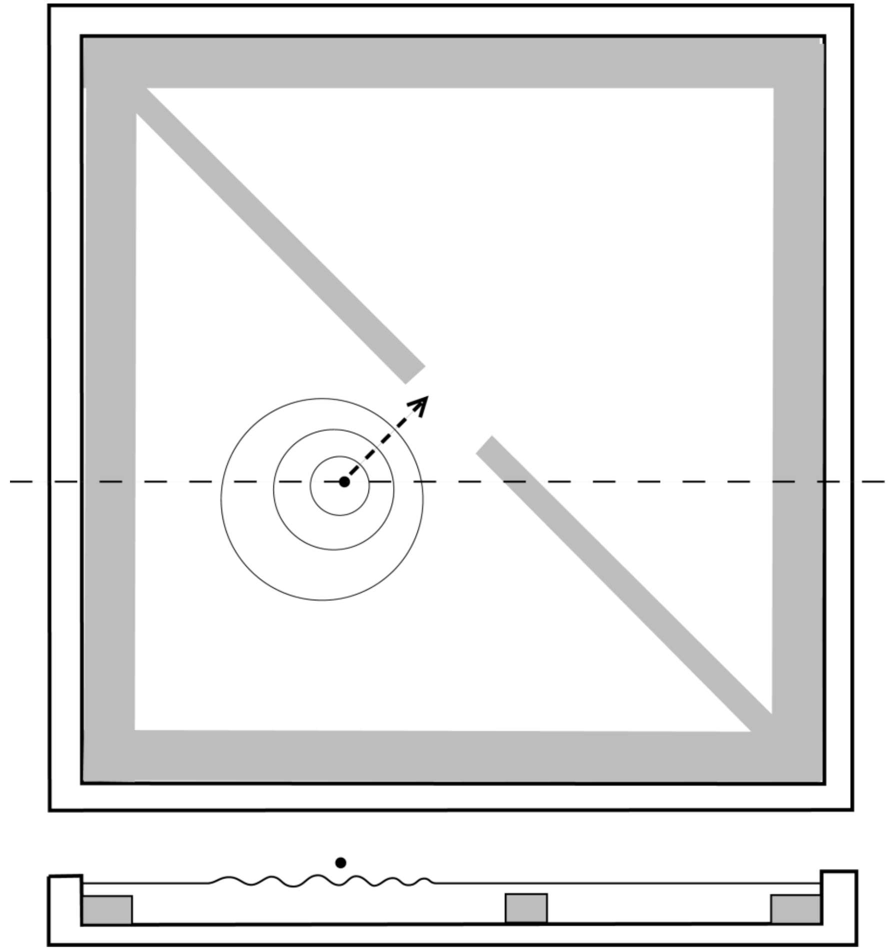
Single particles diffraction and interference

Couder Y. & Fort E. PRL,. 97, 15101, (2006)

The experimental set up for diffraction and interference experiments

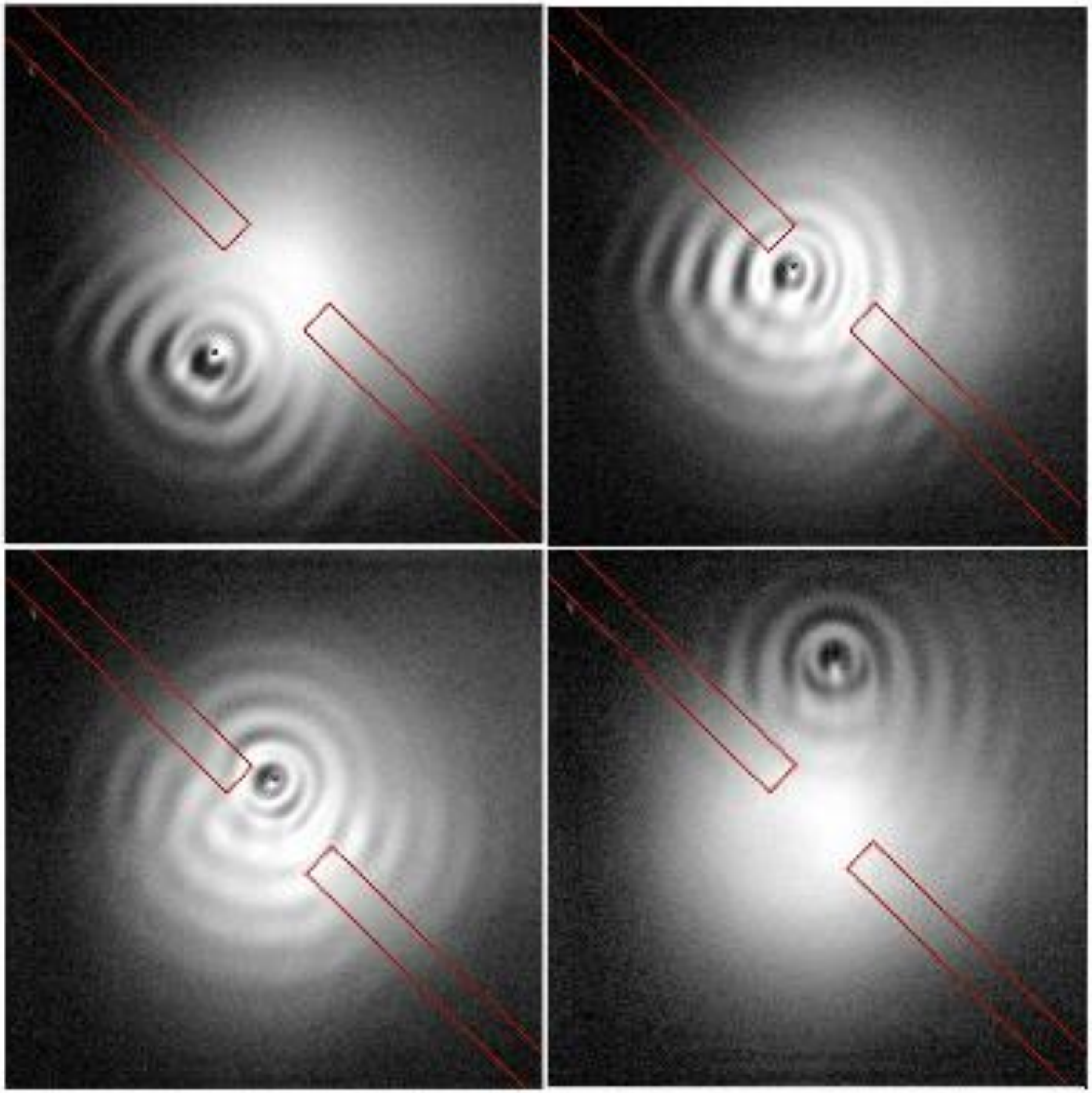
In the grey regions the fluid layer thickness is reduced to h_1=1mm (h_0=4mm elsewhere)

In these regions the Faraday threshold being shifted, the walkers do not propagate



Cross section

Four
photographs of
the
wave pattern
during the
diffraction
of a walker



Measurements on the droplet's trajectory

The relevant parameters:

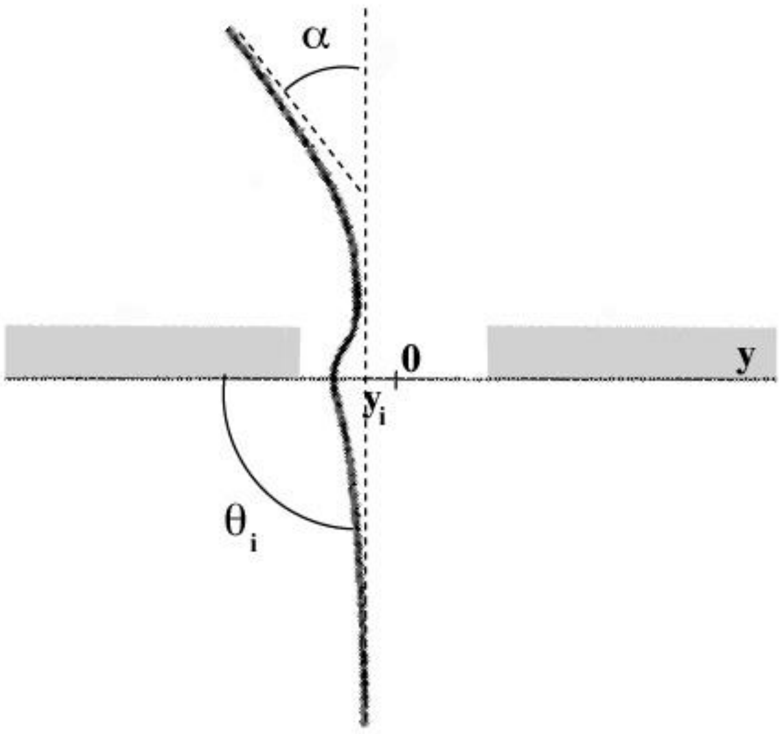
L : the width of the slit,

θ_i : angle of incidence (chosen $\theta_i = \pi/2$),

α : the angle of deviation

$Y_i = y_i/L$: the impact parameter

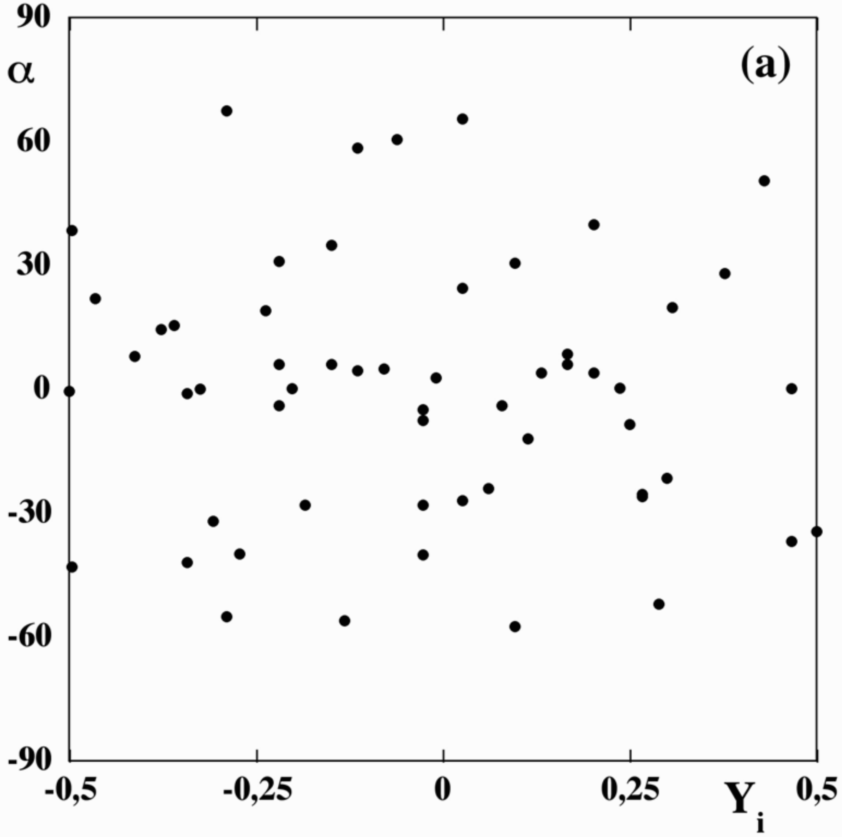
(With $-0.5 < Y_i < 0.5$)



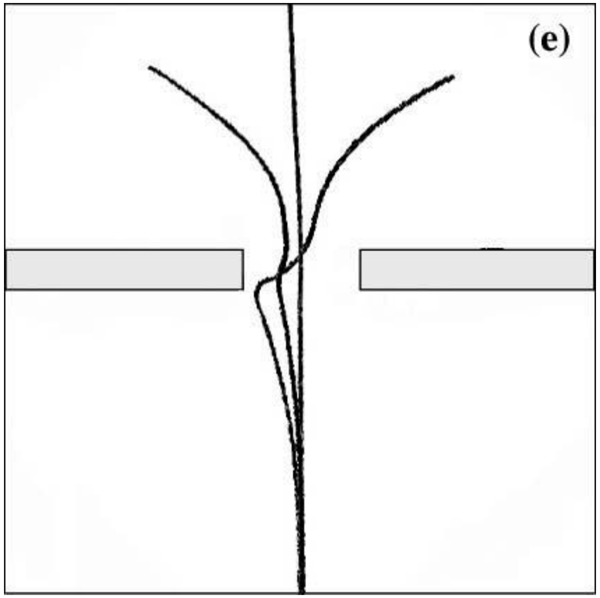
Is there a link between the deviation α and the impact parameter Y_i ?

The measured deviation in experiments performed with the same walker, the same angle of incidence, but various impact parameters

$L/\lambda_F=3.1$ ($L=14.7$mm and $\lambda_F= 4.75$ mm).

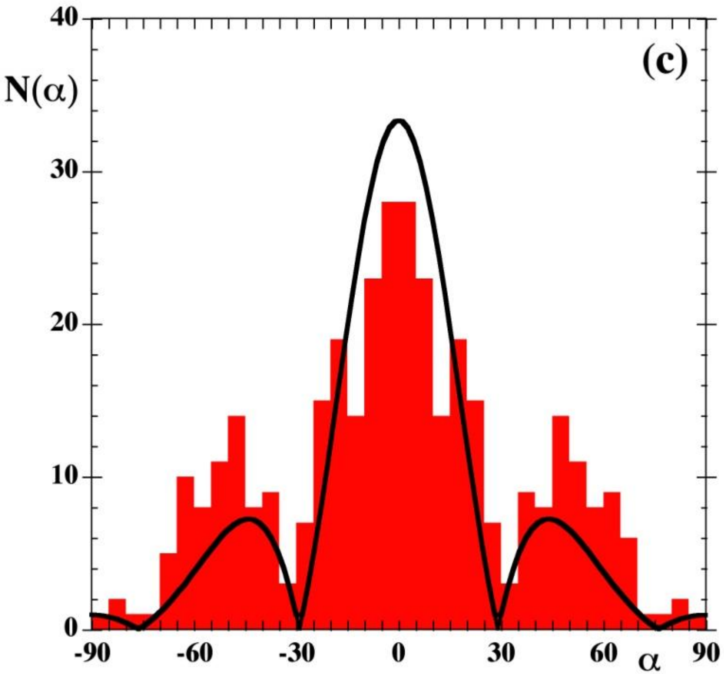


Three independent trajectories with the same initial conditions (within experimental accuracy)



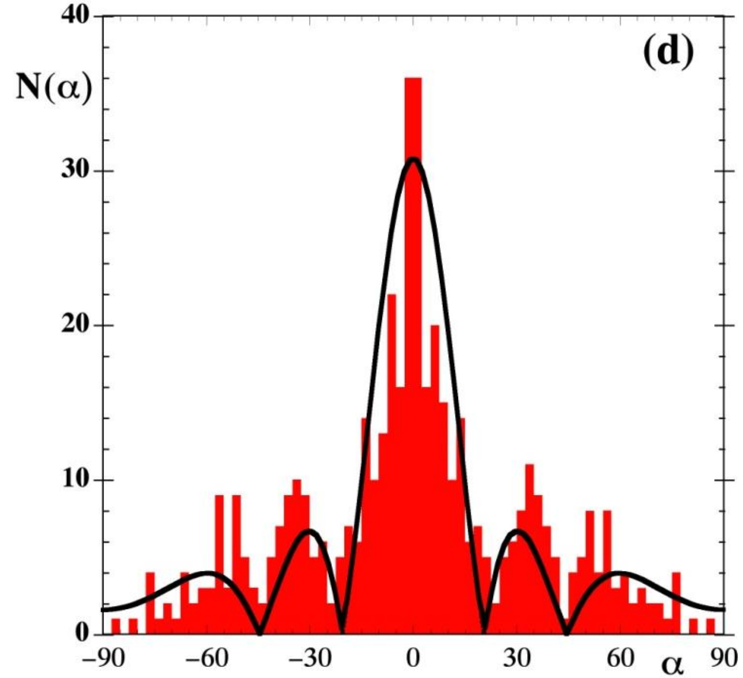
What about statistical properties?

Cumulative histograms of the observed deviations in N independent crossings



$L/\lambda_F = 2.1$

(L=14.7mm, λ_F= 6.9 mm)



L/λ_F=3.1

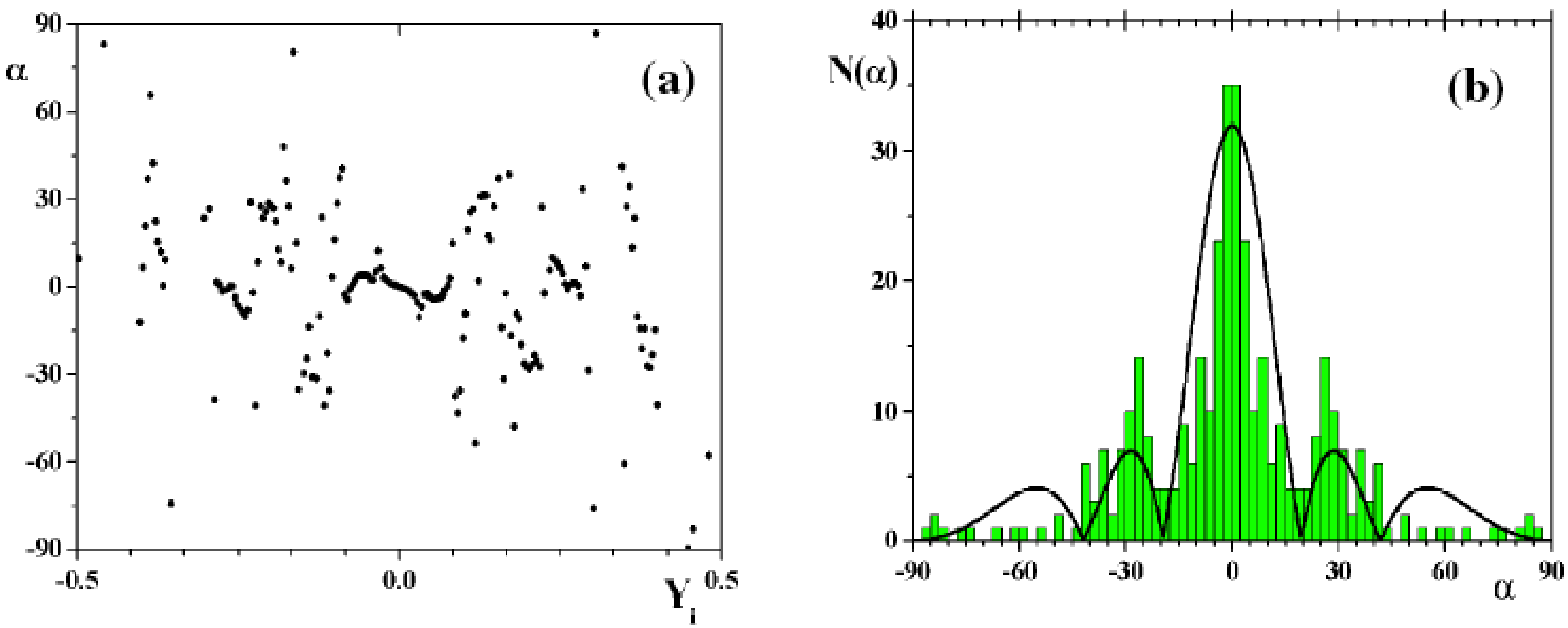
L=14.7mm, λ_F= 4.7 mm

The curve is the modulus of the amplitude of a plane wave of the same wavelength diffracted through a slit of the same width

$$f(\alpha) = A\left|\frac{\sin(\pi L \sin \alpha / \lambda_F)}{\pi L \sin \alpha / \lambda_F}\right|$$

The numerical simulation of the diffraction

In the presence of path memory the deviation becomes a complex function of the impact parameter
And the pdfs of deviations are similar to those observed experimentally



$$L/\lambda_F = 3$$

Diffraction of waves…

It is not a surprise that a wave passing through a slit is diffracted. This is the standard result from Fourier transform. The wave truncation results in its spreading in the transverse direction.

… or diffraction of particules?

Here, however, we do not measure the wave-field but the trajectories of successive particles. Their individual deviations are unpredictable, exhibiting an uncertainty linked with the spreading of the wave:

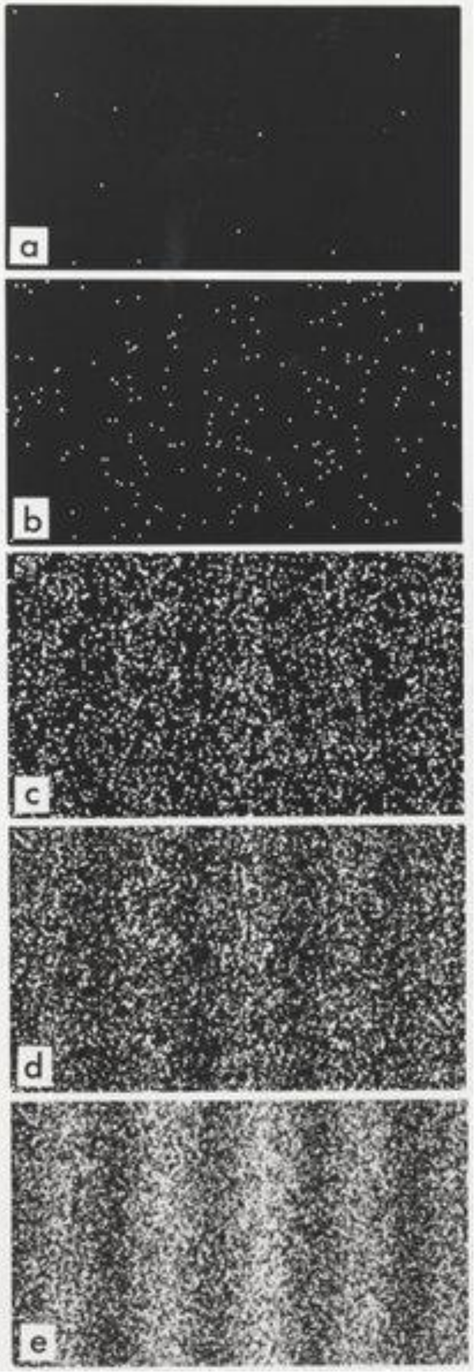
The Fourier spreading of the wave generates an uncertainty for the direction of the velocity of the particle and thus for its momentum.

The Young double-slit interference with single particles

A phenomenon which is assumed to have no equivalent in classical physics

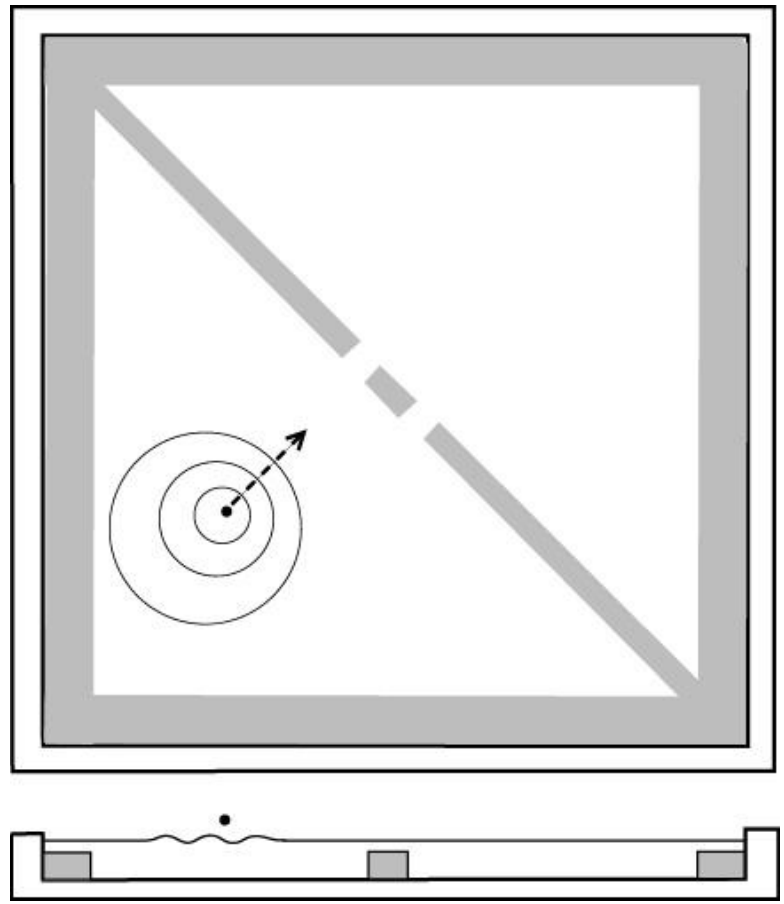
*R. Feynman's, Lectures on Physics, vol. 3, Quantum Mechanics, (*First chapter)

« … In this chapter we shall tackle immediately the basic element of the mysterious behavior in its most strange form. We choose to examin a phenomenon which is impossible, absolutely impossible, to explain in any classical way and which is at the heart of quantum mechanics. In reality it contains the only mystery. We cannot make the mystery go away by explaining how it works . We will just tell you how it works.… »

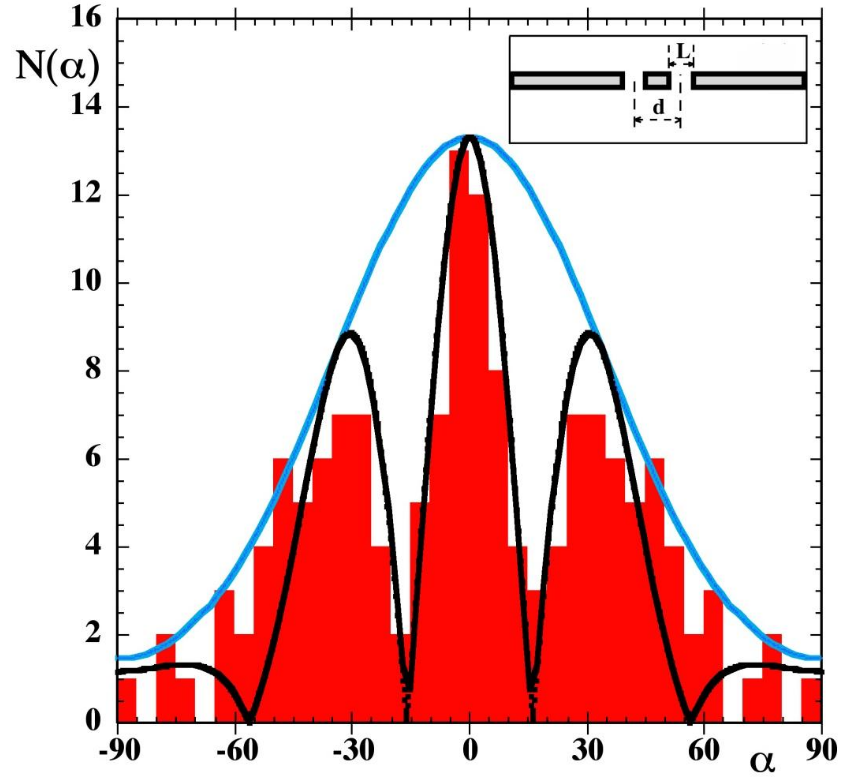


The build-up of the interference pattern (with electrons, after Tonomura)

Young's two slits experiment



Interference: histogram for 75 realizations



The curve is the modulus of the amplitude of the interference of a plane wave through two slits with L/λ_F= 0.9 and d/λ_F=1.7.

$$f(\alpha) = A\left|\frac{\sin(\pi L \sin\alpha/\lambda_F)}{\pi L \sin\alpha/\lambda_F}\cos(\pi d \sin\alpha/\lambda_F)\right|$$

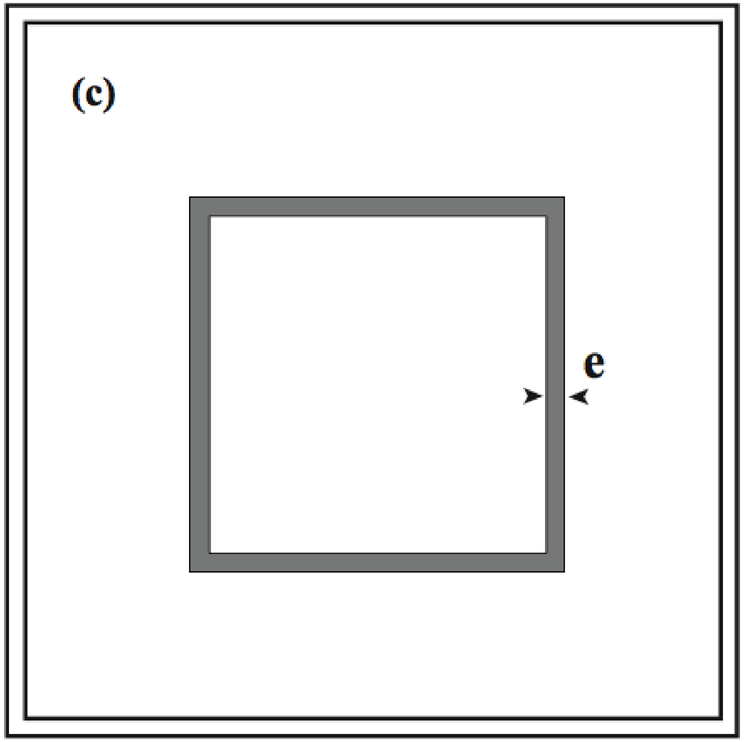
Part II (b)

Tunneling through a barrier

With Antonin Eddi

A.Eddi, F. Moisy, E. Fort & Y.Couder
" Unpredictable tunneling of a classical wave-particle association”
Phys. Rev. Lett. **102**, 240401, (2009)

First experimental set-up

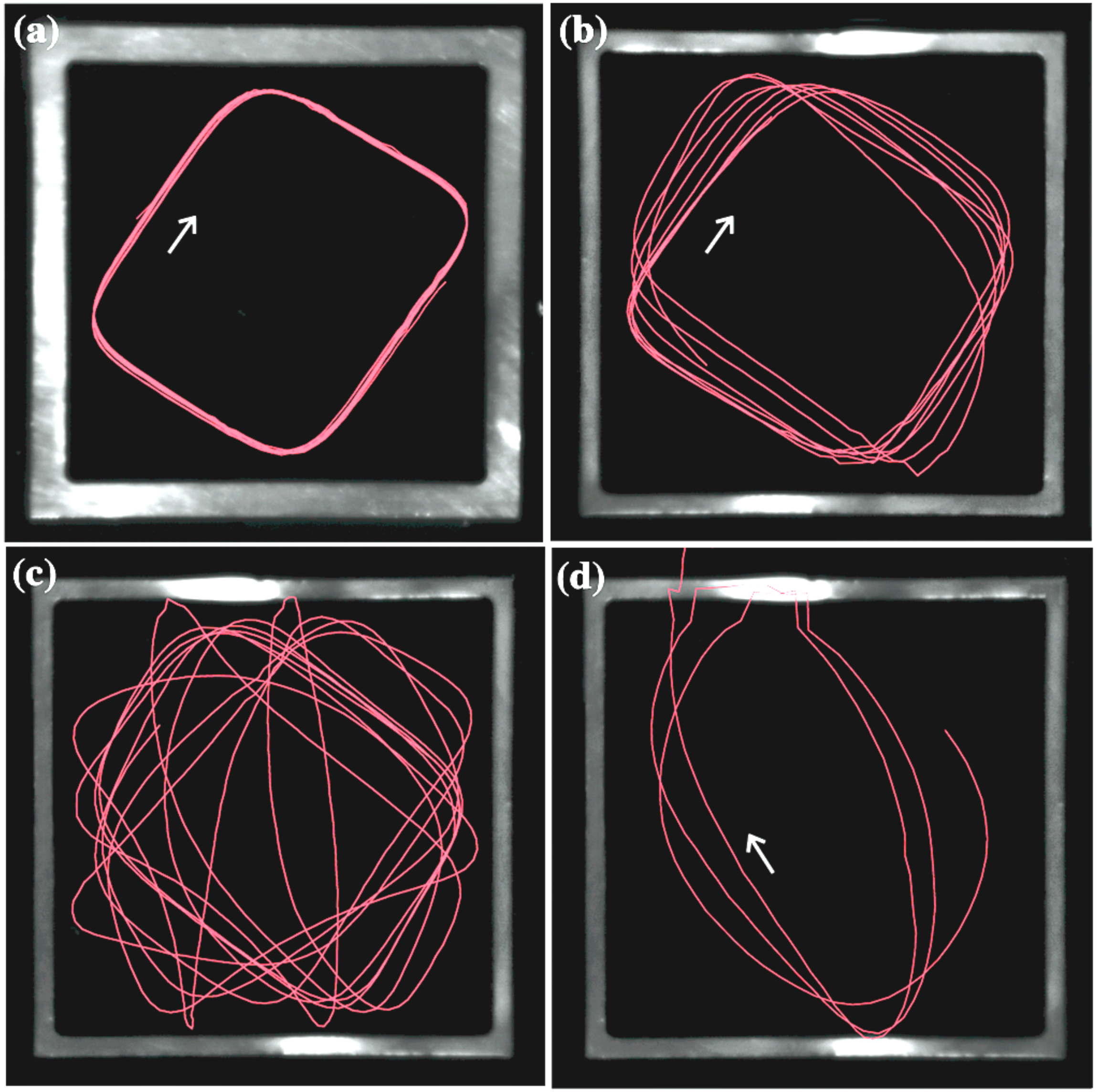


Escape out of a closed cavity

The origin of the probabilistic behaviour: the trajectories inside the frame

For thick walls the walker reaches a stable limit cycle

With thin walls the reflections are imperfect, leading to different types of collisions with the wall and to a probability of escape

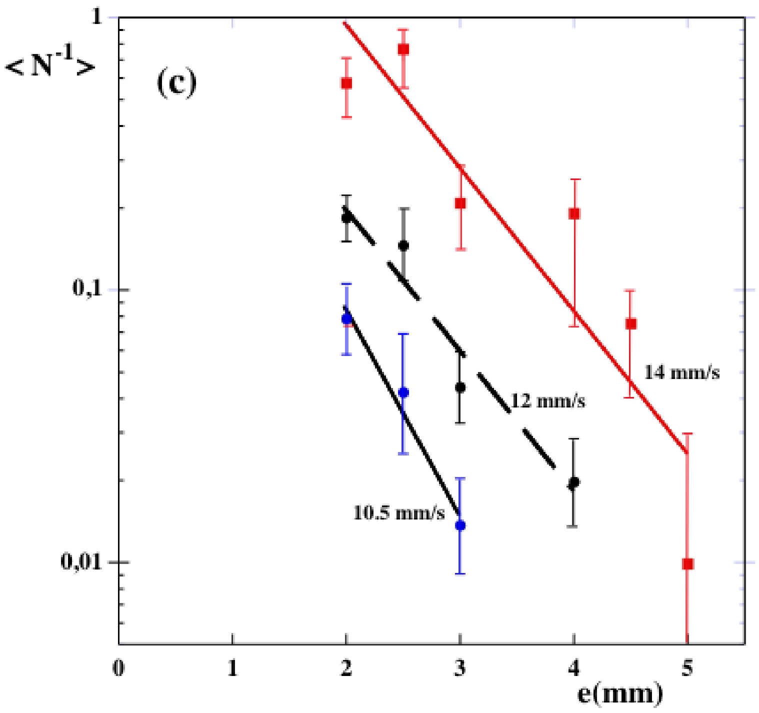


A probability of escape can be measured

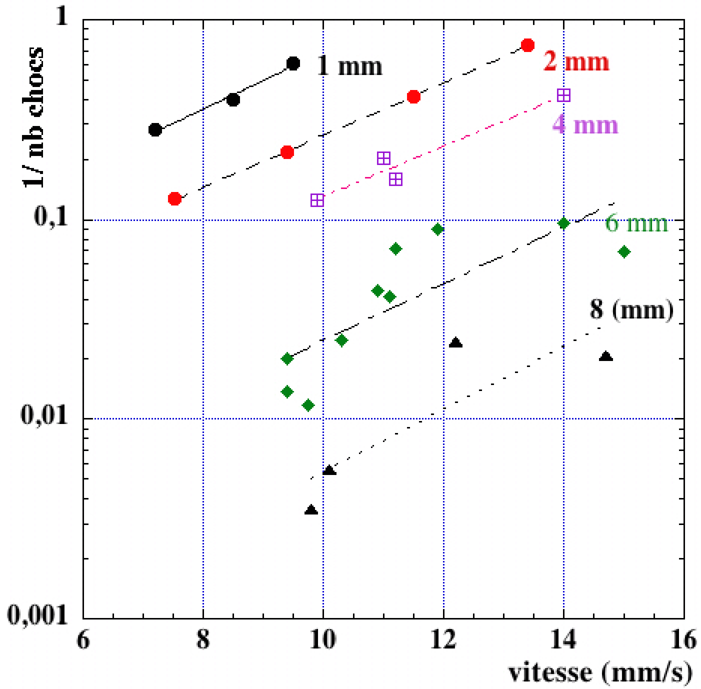
The walker has a billiard motion inside the frame and escapes out of it once in a while.

Repeating the experimen, a probability of escape can be defined as the ratio of the number of escapes over the total number of collisions with the wall

It turns out to depend on the barrier thickness and the velocity of the walker

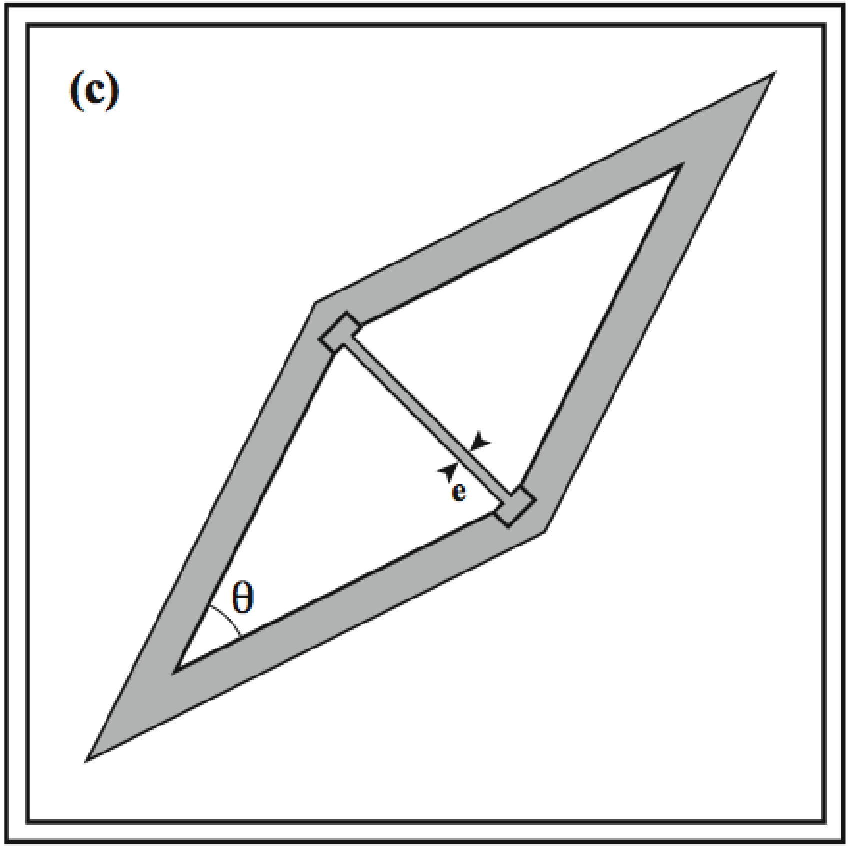


Semi-log plot of the probability of crossing as a function of the barrier's tickness



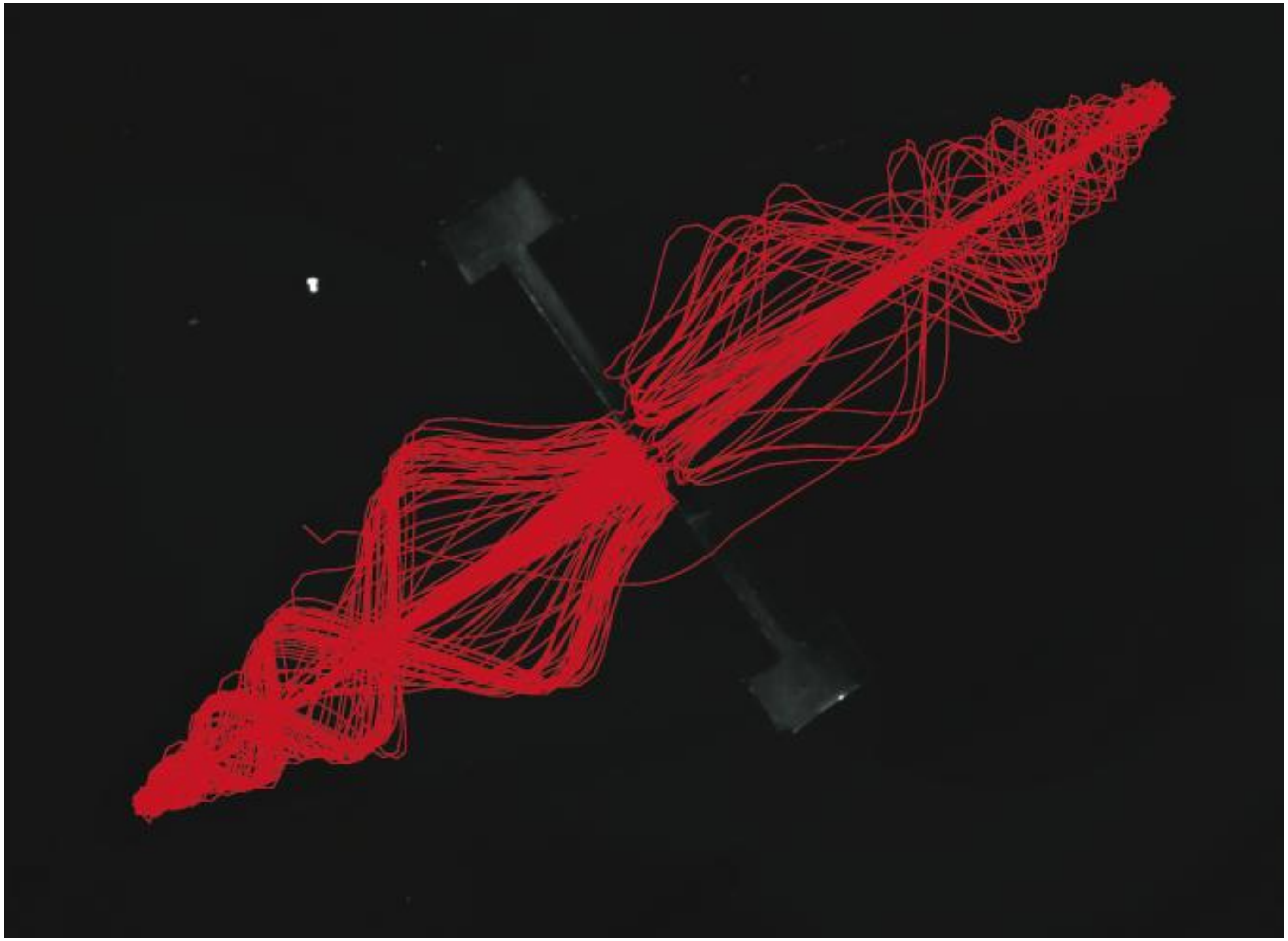
Semi-log plot of the probability of crossing as a function of the walker's velocity

Second experimental set-up

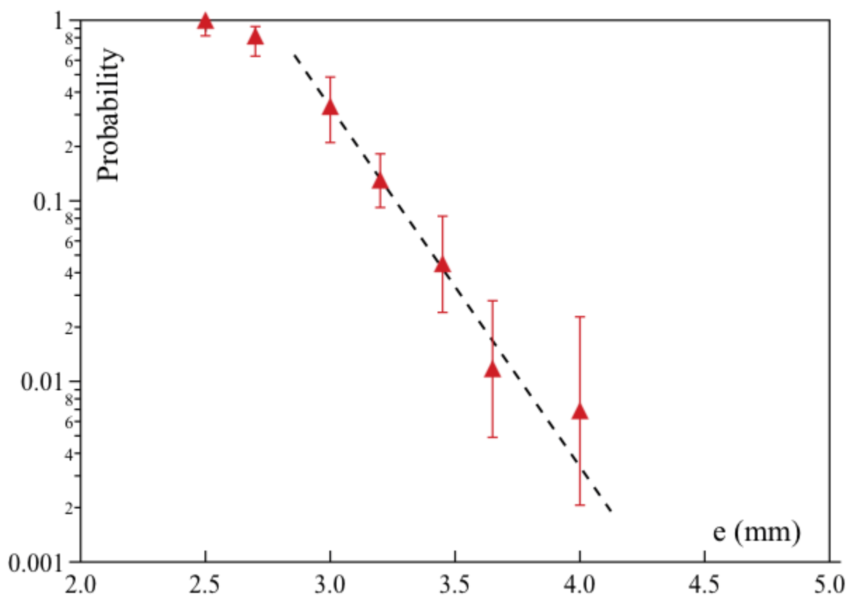


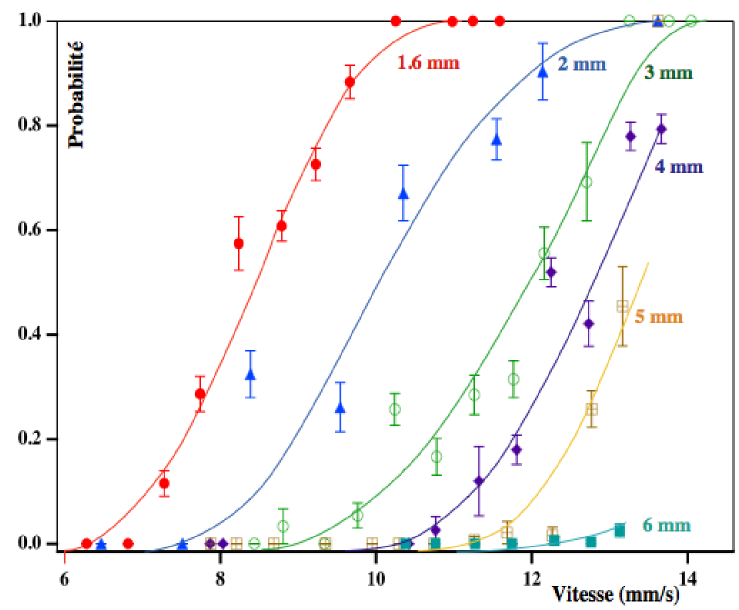
The particle is guided by the divergent walls so that it impiges perpendicularly on the barrier of thickness e.

Accumulation of events



Probability of crossing with normal incidence

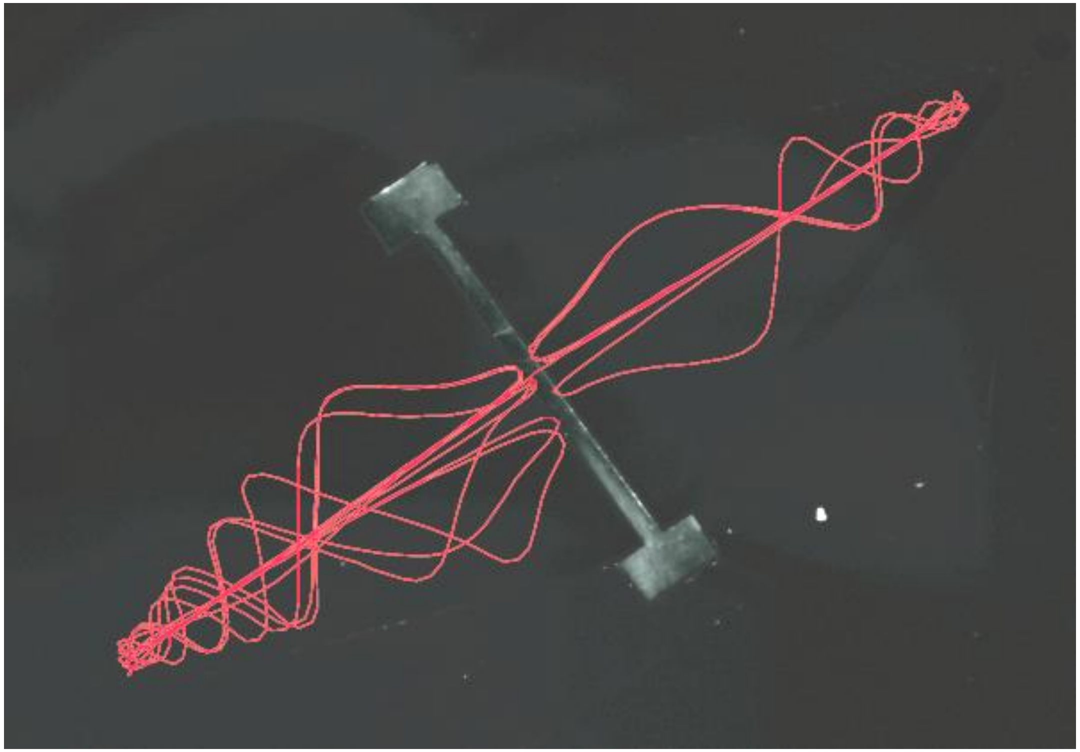




Semi-log plot of the probability of crossing as a function of the barrier's tickness

Linear plot of the probability of crossing as a function walker's velocity

The relation to the incident trajectory



Far from the barriers, all walkers have a normal incidence.

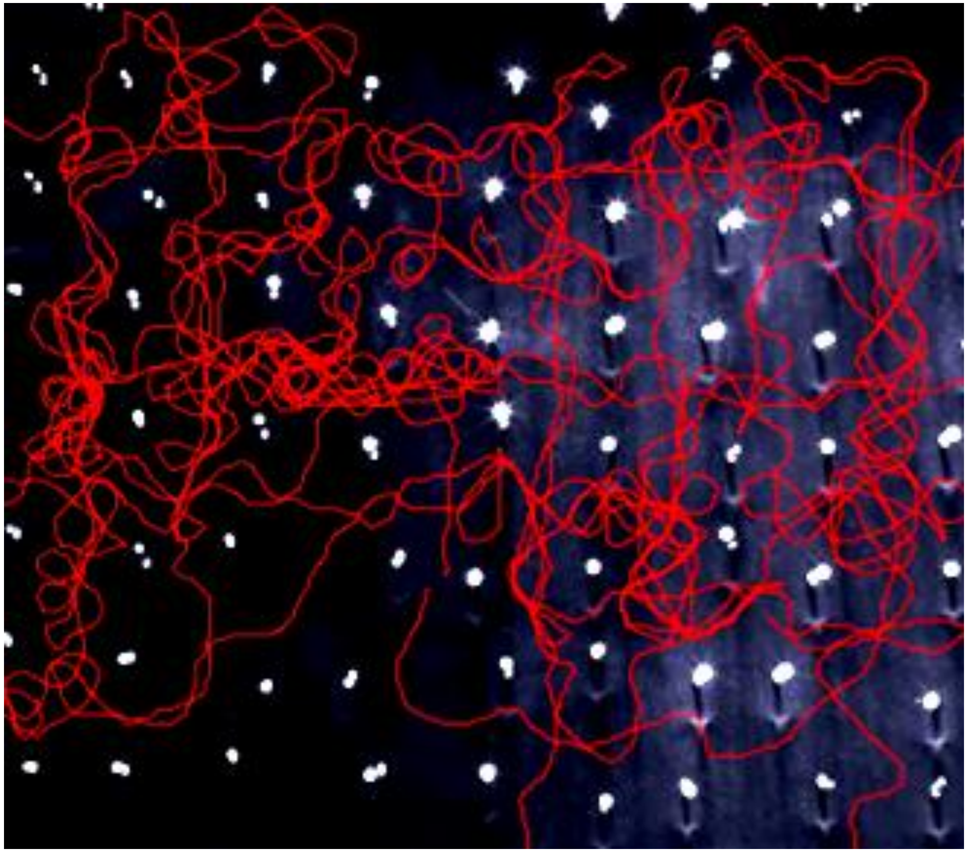
They deviate because of the reflected waves

Only those walkers which have had a weak deviation have a probability of crossing

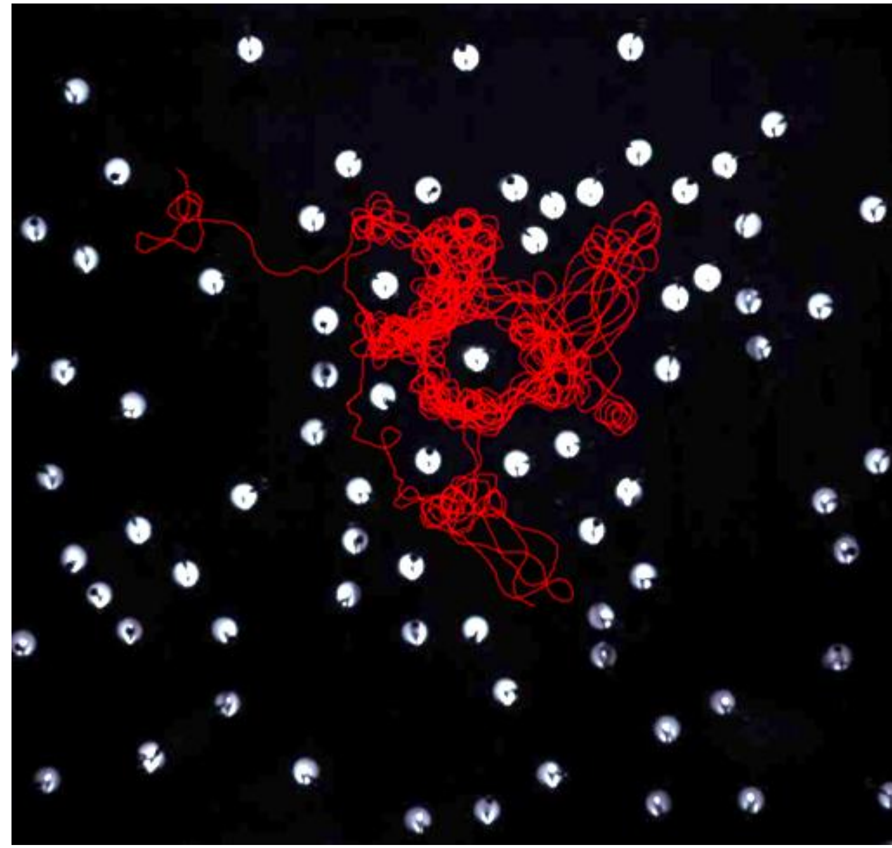
The walker deviates have a weaker probability of being deviated when the reflected waves are weaker (thin barriers), hence a larger probability of crossing

Propagation and localization in a random medium

Trajectoires d’un marcheur dans un milieu désordonné



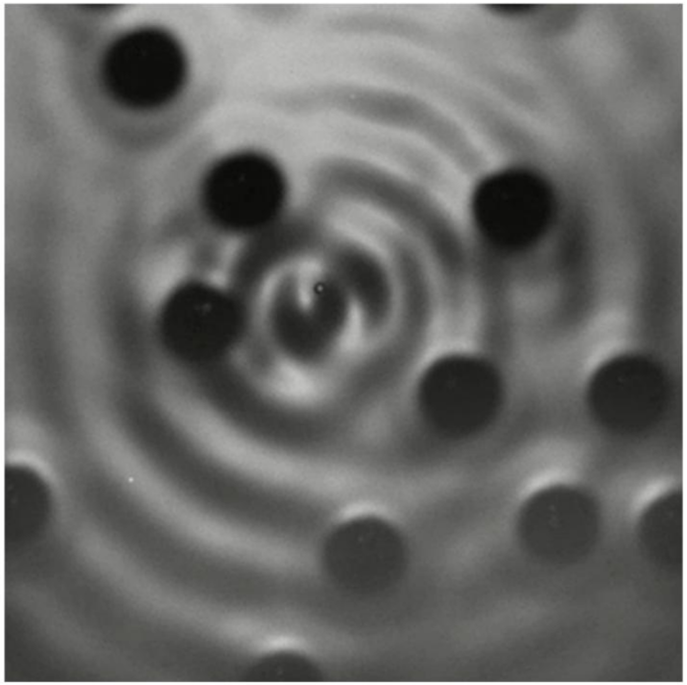
Mémoire à court terme
et faible désordre

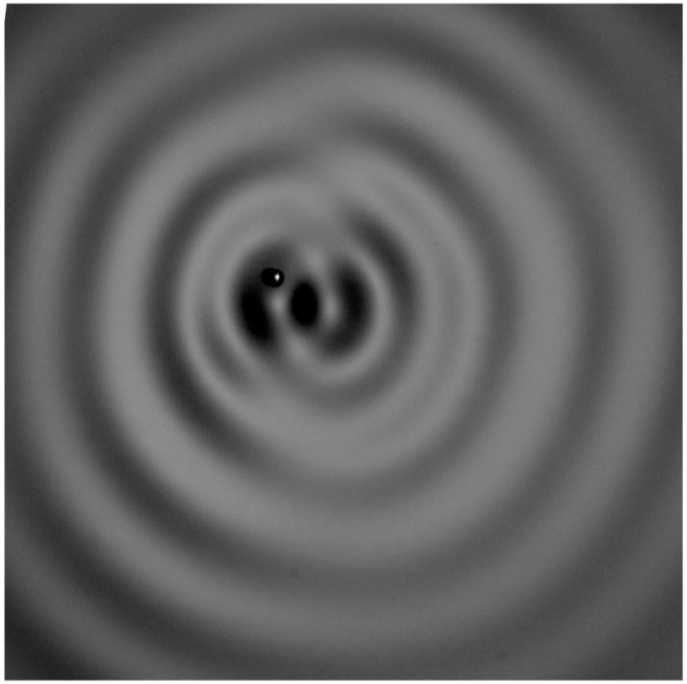


Mémoire à long terme
et fort désordre

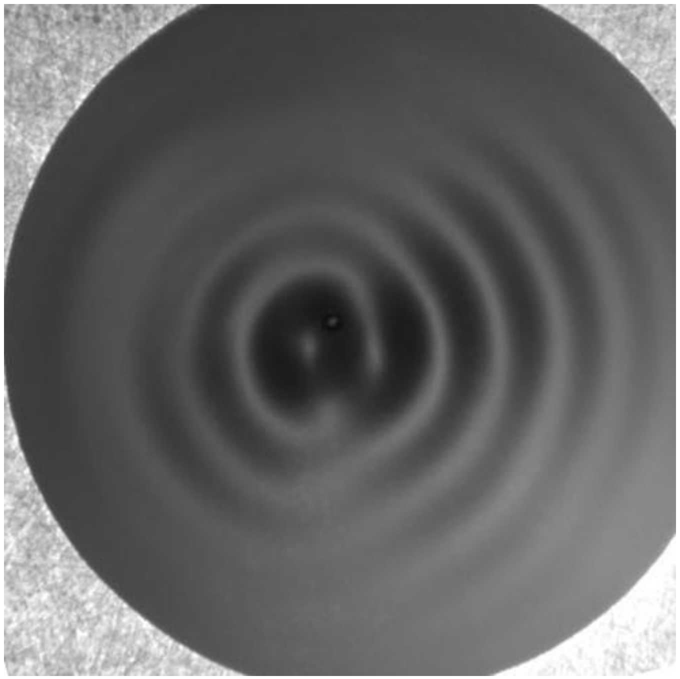
Localisation d’Anderson?

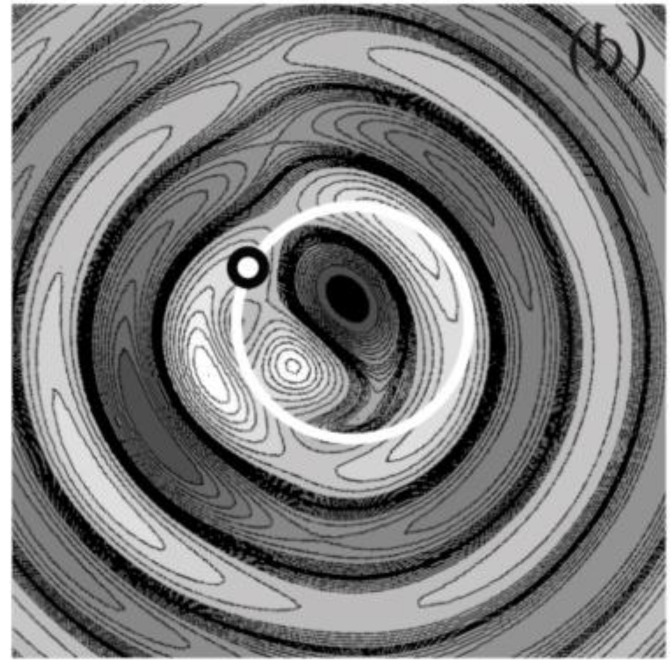
The localization appears dominated by "self orbiting" motions on orbits similar to the tightiest orbit observed in the rotation experiment





… also observed in cavities





Discussion:

is there a relation to quantum mechanics?

In our system we clearly have a particle driven by a wave it generates. It is therefore interesting to revisit the unorthodox "pilot wave" models of quantum mechanics.

The "pilot wave" models

The association of particles to waves was initially proposed by de Broglie

L. de Broglie, *Ondes et mouvements*, (Gautier Villars Paris) (1926).

In 1952 D. Bohm obtain trajectories by deriving an equation of motion out of the Schrödinger equation

D. Bohm, *Phys. Rev.* 85: 166-179, and 180-193, (1952),
***Phys. Rev.* 89: 458-466 (1953).**

These two approaches are often identified to each other and called the de Broglie-Bohm pilot-wave models.

They are in fact very different from each other and should be dissociated.

D. Bohm uses the Madelung transformation of the Schrödinger equation

revisited recently by John Bush

$$i\hbar\frac{\partial\Psi}{\partial t}=-\frac{\hbar^2}{2m}\nabla^2\Psi+V\Psi$$

$$\Psi=\sqrt{\rho}\;e^{imS/\hbar}$$

Continuity: $$\frac{\mathrm{D}\rho}{\mathrm{Dt}}+\rho\nabla.\mathbf{u}=0$$

Quantum Hamilton-Jacobi $$\frac{\partial S}{\partial t}+\frac{1}{2}\mathbf{u}^2-\frac{\hbar^2}{2m^2}\frac{1}{\sqrt{\rho}}\nabla^2\sqrt{\rho}+\frac{V}{m}=0$$

where:

$\rho=|\Psi|^2$ is the probability density

$\mathbf{u}=\nabla S$ is the quantum velocity of the probabil

$\mathbf{j}=\rho\mathbf{u}$ is the quantum probability flux

The quantum Hamilton-Jacobi can be written:

$$\frac{\partial S}{\partial t}+\frac{1}{2}\mathbf{u}^2=-\frac{Q}{m}-\frac{V}{m}$$

Where Q is the quantum potential

$$Q=-\frac{\hbar^2}{2m^2}\frac{1}{\sqrt{\rho}}\nabla^2\sqrt{\rho}$$

Taking the gradient

$$m\frac{Du}{Dt}=-\nabla Q-\nabla V$$

Equating the quantum velocity and the "particle" velocity

$$m\,\ddot{\mathbf{x}}_p=-\nabla Q-\nabla V$$

Bohmian mechanics consists in solving Schrödinger equation for Ψ, from which Q is then computed, before solving the trajectory equation.

The limit of Bohmian mechanics

The trajectory equation:

$$m\,\ddot{\mathbf{x}}_p = -\nabla Q - \nabla V$$

does not define the trajectory of the particle but the trajectory of the probability density.

For this reason de Broglie wrote in 1953 :

“A year and a half ago, David Bohm took up the pilot-wave theory again. His work is very interesting in many ways (…) But since Bohm’s theory regards the wave Ψ as a physical reality, it seems to me to be unacceptable in its present form”.

de Broglie original model

L. de Broglie, *Ondes et Mouvements*, (Gautier-Villars Paris) (1926).

COLLECTION DE PHYSIQUE MATHÉMATIQUE

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ONDES ET MOUVEMENTS

PAR

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Docteur ès Sciences

PARIS

GAUTHIER-VILLARS ET C^ie^, ÉDITEURS

LIBRAIRES DU BUREAU DES LONGITUDES, DE L'ÉCOLE POLYTECHNIQUE

Quai des Grands-Augustins, 55

1926

The pre-Shrödinger de Broglie model (1926)

de Broglie assumes that there are well defined particles that he considers as point sources.

This material point is considered as having an internal oscillation and emitting in the surrounding medium a wave of frequency :

$$\nu_0 = \frac{1}{h} m_0 c^2$$

The phase of the particle oscillation and that of the wave are locked to each other

The particle is surrounded by a stationary spherical wave, the superposition of a divergent and a convergent wave.

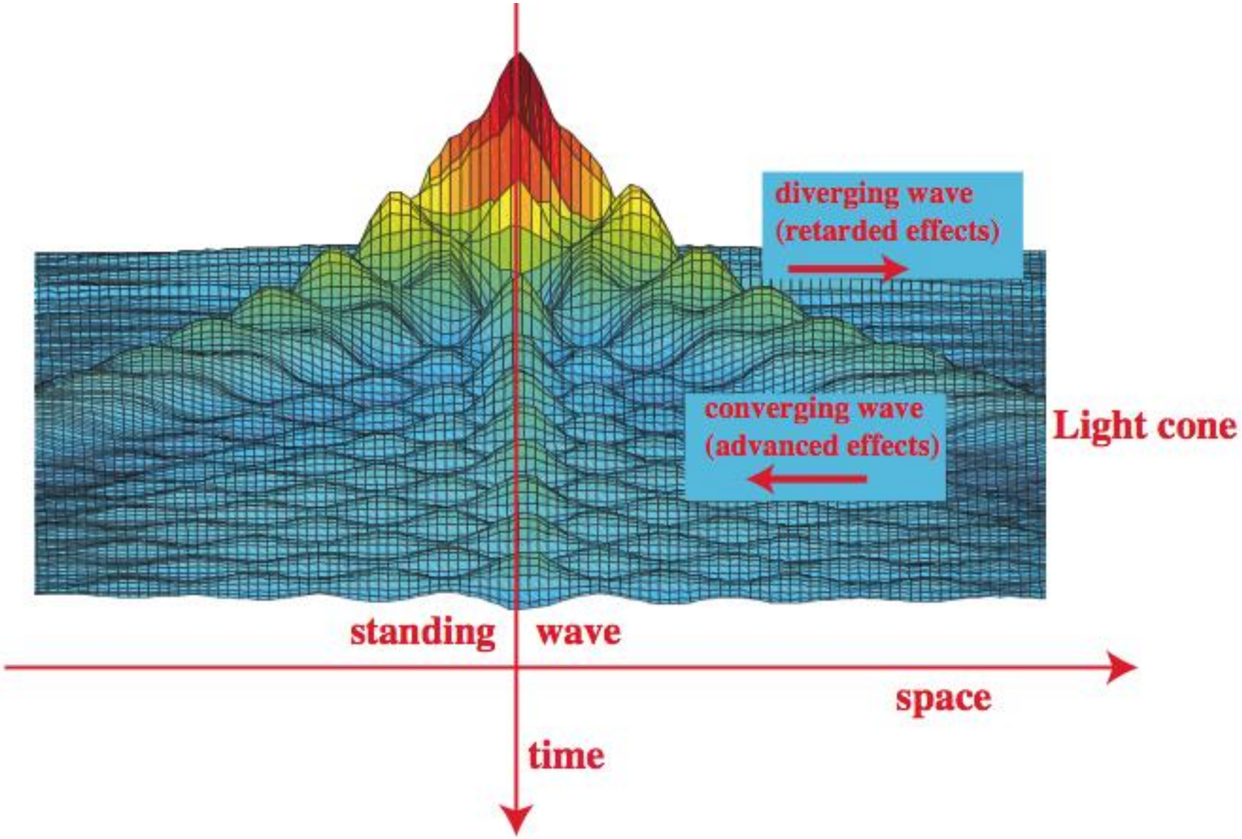
$$\varphi(r_0, t_0) = \frac{A}{2r_0}\left\{\cos\left[2\pi\nu_0\left(t_0 - \frac{r_0}{c}\right) + c_1\right] - \cos\left[2\pi\nu_0\left(t_0 + \frac{r_0}{c}\right)\right] + c_2\right\}$$

He writes :

« But there is also the convergent wave, the interpretation of which could raise interesting philosophical issues, but that appears necessary to insure the stability of the material point »

In our system a standing wave is also associated to the particle. How is it generated?

Measured spatio-temporal evolution of the radial profile of the wave emitted by one bounce



A travelling droplet. Because of the excitability of the medium, it leaves behind a Faraday standing wave

Each point of the wave front emits a wave moving backwards towards the source.

De Broglie proposed what he called a "double solution"

First solution
The particle has an oscillation at a frequency $\nu_0 = m_0 c^2/h$ and is surrounded by a standing wave with a singularity (or non linear region) at its core. This structure forms the individual particle and has well-defined trajectories in space-time.

The second solution
A linear and smooth wave, solution of the Schrödinger equation that corresponds to the averaged behaviours.

De Broglie "double solution"

The first solution can be written

$$u = f\, e^{\frac{2\pi i}{h}\varphi}$$

Where f has very large values in a singular region

The second solution can be written

$$\psi = a\, e^{\frac{2\pi i}{h}\varphi}$$

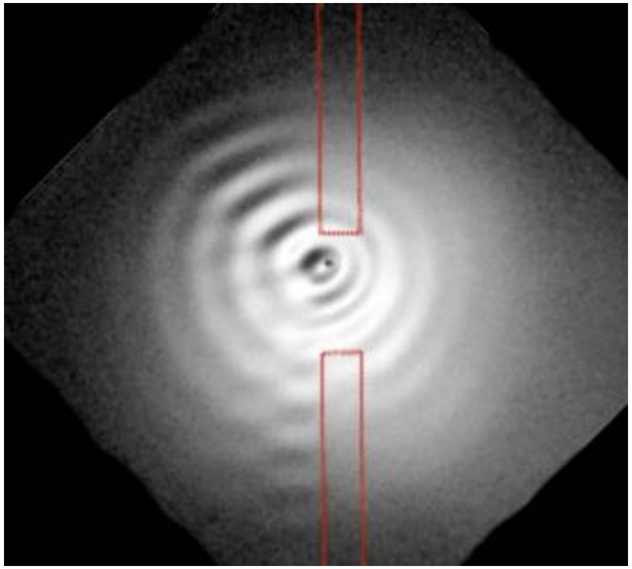
Where a and φ are continuous.

Ψ *is the solution of the Shrödinger equation*

The velocity of the particle is given by

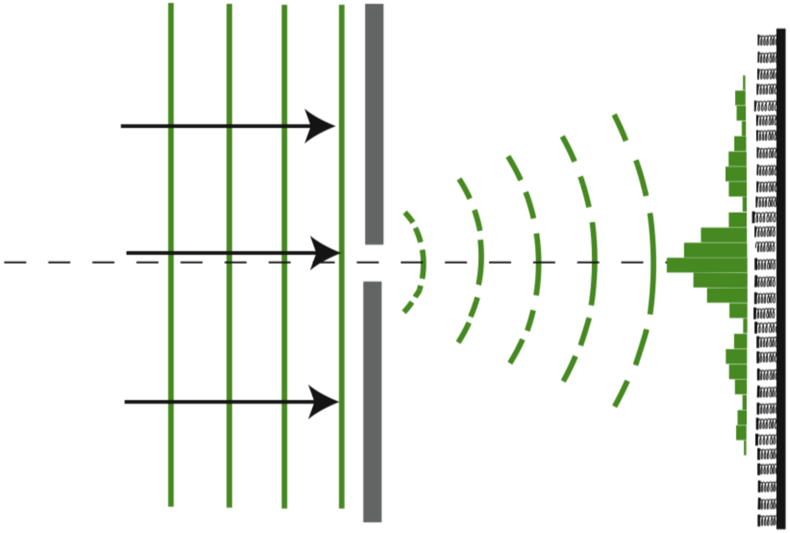
$$\vec{\mathbf{v}} = -\frac{1}{m}\vec{\nabla}\varphi$$

In our experiment we have a double solution situation of the type proposed by de Broglie.



There is a particle: the droplet. It is guided by a wave of wavelength λ_F but this wave is not a plane wave

Analogous to de Broglie's first solution



The probabilities of the various angles of deviation correspond to a diffracted plane wave of wavelength λ_F

Analogous to a Schrödinger wave, de Broglie's second solution

The double solution in cavities

First solution

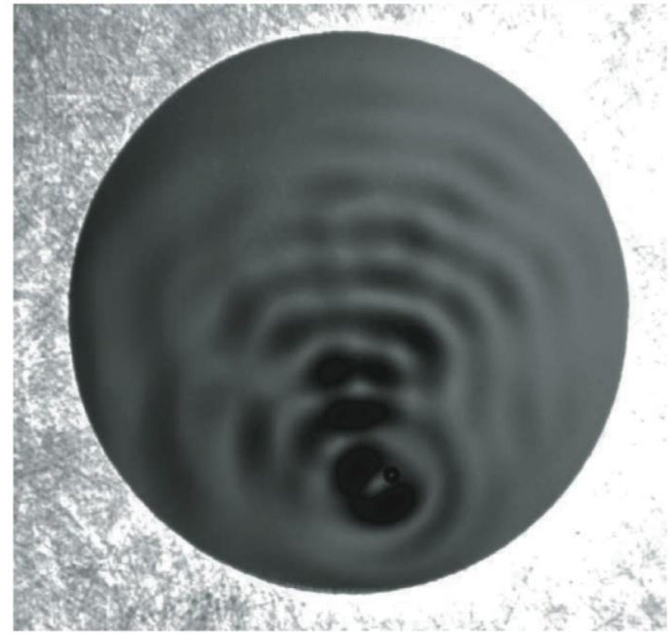
-One given walker is piloted by the wave it has generated and has an individual complex trajectory.

-The structure of the pilot wave is complex as it contains a memory of the past trajectory

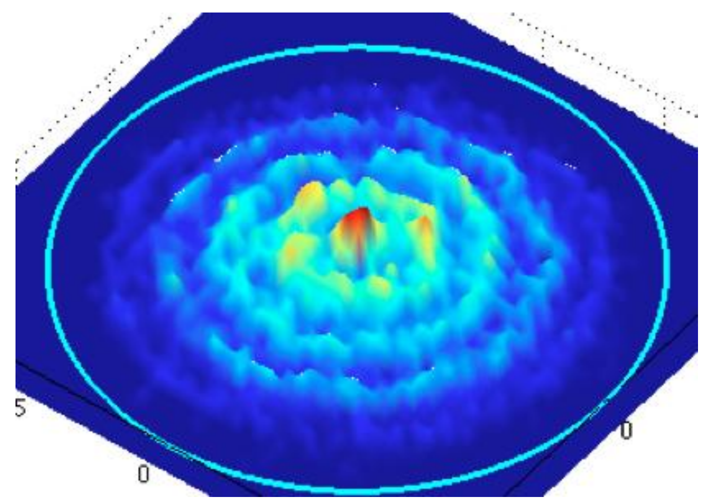
Second solution

- The probabilities are given by an underlying mean wave structure linked to the resonant modes of the cavity or more generally to the environment

First solution



Second solution



Courtesy Dan Harris

Returning to our experiment.
Its main drawback:
it is very far from quantum mechanics

- **Macroscopic scale : no relation with Planck constant.**
- **The system is two-dimensional.**
- **The system is dissipative and sustained by external forcing.**
- **This forcing imposes a fixed frequency: the “energy” is fixed**
- **The waves live on a material medium: there is an “ether”.**
-

Its main interest:
it is very far from quantum mechanics,

-At quantum scale the Planck limitation imposes itself to all phenomena. It is not possible to do a non-intrusive measurement.

-Intrusive measurements generate a projection onto states, so that only the probabilities of those states can be measured.

-***Here we can do either intrusive or non intrusive measurements.***

- If we try to know the position of a walker by confining it in a cavity or by having it pass through slits we find probabilistic behaviours.

- The observation with light is non intrusive so that the undisturbed trajectory of the particle and the wave can be observed directly.

- Non intrusive observations done during an intrusive measurement show that the latter generates chaotic trajectories that are responsible for the observed statistical properties.

All the observed quantum-like properties emerge from what we have called the “wave-mediated path-memory”.

This “path memory” generates a particular type of space and time non locality.

For this reason we believe the debate on hidden variables is not closed

Walkers

Couder Y., Protière S., Fort E. & Boudaoud A., ,
Nature 437, 208. (2005)

Protière S., Boudaoud A. & Couder Y.
J. Fluid Mech. 554, 85-108, (2006)

Diffraction

Couder Y. & Fort E.
Phys. Rev. Lett. 97, 15101, (2006)

Tunnel effect

Eddi A., Fort E., Moisy F., & Couder Y.
Phys. Rev. Lett. 102, 240401, (2009)

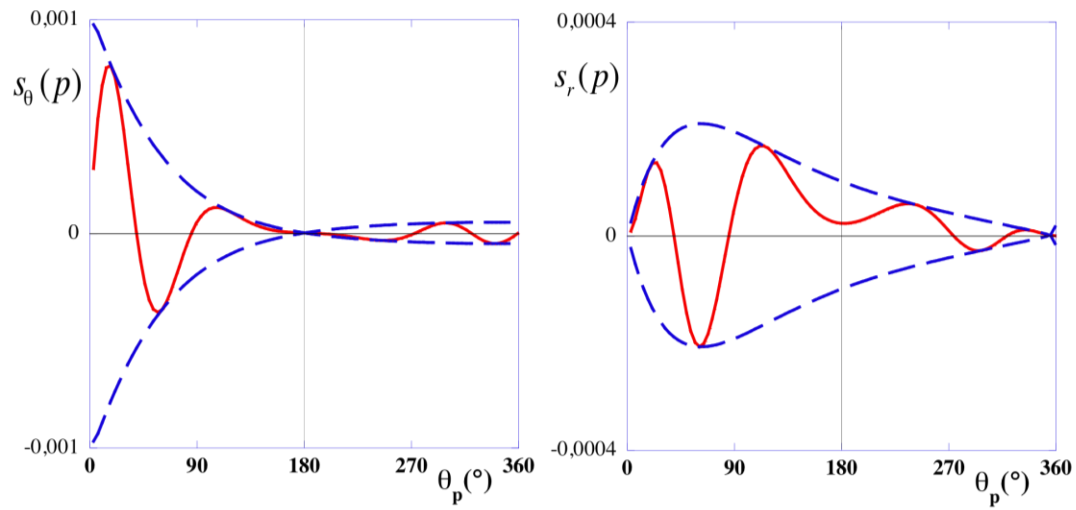
Quantized orbits

Fort E., Eddi A., Boudaoud A., Moukhtar J., and Couder Y.,
PNAS 107, 17515-17520, (2010).

Memory and wavefield structure

Eddi A., Sultan E., Moukhtar J., Fort E., Rossi M., and Couder Y.,
J. Fluid Mech., 674, 433- 464, (2011).

The individual contributions to the local slope under the droplet



Their summation