# The Design of Low-Noise Amplifiers

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Abstract-The essential theory and practical considerations for the design of low-noise amplifiers are gathered and organized to a uniform presentation. The relevant material is quite simple and straightforward, hopefully bringing within the reach of the interested circuit designer the "art" of low-noise-amplifier design.

# I. INTRODUCTION AND SOME SIGNIFICANT CONCLUSIONS

ESPITE numerous papers published on the subject of noise in electronic devices, circuits, and networks in general, the design of low-noise amplifiers is still often regarded by circuit designers as obscure and esoteric. The reason for this seems to be, at least in part, due to the fact that information on actual design of low-noise amplifiers is widely scattered. Most of the published materials are either of theoretical nature—which often tend to discourage the reader, or too superficial for a serious low-noise amplifier design. However, the fact is that circuit design of low-noise amplifiers requires no special knowledge in semiconductor physics, network theory, or probability theory.

Bearing in mind that electronics is a practical science this paper aims to provide a guide and reference source for designing low-noise amplifiers. A comparison is made between the junction transistor, field-effect transistor (FET), and monolithic amplifiers in terms of noise characteristics and their dependence on the bias point, device parameters, and frequency. Noise is treated in terms of the equivalent input noise sources rather than by noise figure, which is less efficient and often confusing. Similarly, the techniques of noise matching are regarded as modifying the input noise sources in such a way that the noise for a given source impedance is minimized.

Some conclusions of practical significance follow.

1) The noise performance of amplifiers, besides being dependent on the amplifier, is also a function of the signal source impedance and frequency range. These two factors determine the optimum input stage.

2) Impedance matching at the amplifier input and source matching techniques for best noise performance are entirely different.

3) For narrow-band reactive sources, the total noise can be reduced by the addition of a suitable reactance at the amplifier input.

4) The noise performance of FET's at low frequencies is related to  $g_m$ , and at high frequencies to  $f_T$ . Low-noise junction transistors should have a high current gain  $-\beta$ , a minimum base resistance  $-r'_b$ , and high cutoff frequency  $f_T$ .

5) Noise performance of monolithic amplifiers is usually inferior to that of discrete amplifiers. However, mainly at low frequencies, monolithic amplifiers may prove sufficient and should be considered first.

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6) For resistive signal sources and low-frequency inductive sensors, "ideal" amplifiers can be designed where the added noise is negligible in comparison to the inherent thermal noise generated in the source.

7) Input devices are now available with 1/f noise component reduced to an amount which is virtually insignificant for any practical purpose.

8) High precision in noise calculations serves little purpose, not only because of manufacturing spread of the parameters but also because noise sources are nearly always uncorrelated. As a result, secondary noise sources, such as second stage noise, should have only minor effect.

9) Common-base (or common-gate) input stage has the same input-noise sources as a common-emitter (or common-source) stage. The total performance is inferior, however, except for source impedance of the order of  $1/g_m$ , especially at high frequencies.

## II. Some Noise Characteristics

Noise can be considered as anything which, when added to the signal, reduces its information content. Here, we shall deal only with noise generated in amplifiers as a result of physical processes occurring in electronic components. This noise is random and, mostly, Gaussian.

Noise cannot be predicted as a function of time. It can, however, be characterized in terms of average values. The most widely used characteristic is the root mean square (rms). For a noise n(t) the rms is defined as follows:

$$\sqrt{n^2(t)} = \sqrt{\frac{1}{T} \int_0^T n^2(t) dt}$$
 (1)

where the bar designates an average value for a relatively long time T.

When two noise sources  $n_1(t)$  and  $n_2(t)$  are summed, the instantaneous output is the sum of the individual instantaneous values. The *average* output power would be

$$\overline{[n_1(t) + n_2(t)]^2} = \overline{n_1^2(t)} + 2\overline{n_1(t)n_2(t)} + \overline{n_2^2(t)}$$
$$= \overline{n_1^2(t)} + 2\gamma \sqrt{\overline{n_1^2(t)}} \sqrt{\overline{n_2^2(t)}} + \overline{n_2^2(t)}.$$

The second term, which is proportional to the average product of the two sources, is a measure of their correlation.  $\gamma$  is defined as the normalized correlation coefficient and its absolute value may vary from 1 to 0. On one extreme, the two sources are identical and differ only in amplitude. On the other extreme, the two sources are totally uncorrelated, and the second term is zero. For zero correlation, the rms of the sum  $n_1(t) + n_2(t)$  would be the square root of the sum of the squared individual rms values. In other words, uncorrelated noise adds as orthogonal vectors. For other values of correlation coefficient, the angle between the vectors differs from 90°. Thus, for example, the addition of a 1-mV noise source to a 2-mV uncorrelated source will increase the latter  $\sqrt{5}/2$  times or about 10 percent. Thus one noise source usually dominates and efforts to reduce secondary noise sources are wasteful.

An ideal Gaussian-distribution noise can assume any amplitude as a function of time. In practice, however, even disregarding dynamic range and bandwidth limitations, the percentage of the time during which the instantaneous value exceeds a given amplitude sharply diminishes as a function of this value. For example, the instantaneous noise will exceed an amplitude corresponding to 3.3 times the rms value only 0.1 percent of the time. Similarly, it will exceed an amplitude of 2.5 of the rms 1 percent of the time. Practically, rms values can be measured rather accurately by displaying the noise on an oscilloscope display. The observed noise "thickness" would, roughly, be five times the rms. This method can obviate special measurement equipment and can be refined by overlapping two traces. Accuracies on the order of 10 percent can thus be achieved [1]. Another advantage of measuring noise with an oscilloscope rather than with an rms meter or a calibrated average reading ac meter is that power supply ripple or externally induced interferences are not mistaken as "true" noise.

Random noise can also be characterized in the frequency domain. The most important characteristic is the spectral density function (SDF) which is defined as the Fourier transform of the temporal autocorrelation function [2]. Practically, it represents the time averaged noise power  $n^2(t)$  over a 1-Hz bandwidth as a function of frequency. If the SDF is independent of frequency in the range of interest, it would be referred to as "white noise." The SDF of a voltage  $e_n(t)$  is designated  $\overline{e_n^2(f)}$ , and of a current  $I_n(t)$  by  $\overline{I_n^2(f)}$ . For short, the designations  $\overline{I_n^2}$  and  $\overline{e_n^2}$  will be used. If  $\overline{e_n^2}$  and  $\overline{I_n^2}$  are correlated, a normalized correlation coefficient can be defined as for the temporal case. Now, however, it can attain a complex value as well. The meaning of an imaginary component of a correlation coefficient is that a component of one noise can be obtained from the other one by a suitable phase-shifting network, mostly by differentiation or integration. Fortunately, however, the correlation coefficients rarely have any significance in practical low-noise design.

Noise sources are often characterized in terms of spot value, defined as the square root of the power density designated as  $V(rms)/\sqrt{Hz}$  or  $A(rms)/\sqrt{Hz}$ . Practically convenient, but lacking intuitive physical meaning, the spot noise is numerically the rms value over a 1-Hz bandwidth. Due to the similarity to vector quantities as for the temporal definitions, we shall use vector notation for noise sources defined as  $\vec{e_n} = \sqrt{\vec{e_n}^2}$  and  $\vec{I_n} = \sqrt{\vec{I_n}^2}$ .

To calculate the rms value of white noise over a frequency bandwidth B, its spot value should be multiplied by  $\sqrt{B}$ . This is because the noise at different frequency bands is uncorrelated. For similar reasons, the rms value of white noise passing through a network with a transfer function  $H(j\omega)$  will be proportional to

$$\sqrt{\int_0^\infty |H(j\omega)|^2 d\omega}.$$

If, for example

$$H(j\omega) = \frac{1}{1 + j\omega/\omega_0}$$



Fig. 1. Noise bandwidth of a low-pass amplifier.

and the input noise has a uniform density of  $K(V(rms)/\sqrt{Hz})$ , the overall noise power at the output will be

$$\int_0^\infty |H(j\omega)|^2 K^2 \, d\omega = \int_0^\infty \frac{K^2}{1 + (\omega/\omega_0)^2} \cdot d\omega$$
$$= \frac{\pi}{2} \omega_0 K^2 (\mathbf{V}^2).$$

In other words, the total noise output is the same as to that of an ideal low-pass filter of bandwidth  $\pi/2 \omega_0$ . This bandwidth (see Fig. 1) is referred to as the noise bandwidth of the filter, and accounts for the noise passed beyond the 3-dB frequency, which is usually used to define cutoff. Thus it serves as a correction factor but will assume different values for other than white noise.

The noise bandwidth of an amplifier or network with a known gain can, in principle at least, be determined by supplying a calibrated white noise to the input and measuring the output rms noise. The simplest way to generate such noise is by means of a solid-state noise diode (specially processed Zener diodes) which may give a flat noise density spanning the range  $10-10^7$  Hz, a typical noise density of the CND6000-series noise diodes made by Standard-Reference Labs. Inc., is 0.05  $\mu V/\sqrt{Hz}$ .

The spectral density of a voltage developed on some complex impedance Z(s) at a particular frequency  $\omega$  due to the passage of a current noise  $\overline{I_n^2}$  would be  $|Z(j\omega)|^2 \overline{I_n^2}$ . Over a finite bandwidth the rms value would be

$$\sqrt{\int_{\omega_1}^{\omega_2} |z(j\omega)|^2 \overline{I_n^2} d\omega}.$$

#### **III.** Noise Sources in Electronic Devices

Noise in electronic devices can be attributed to two main processes: thermal noise and shot noise. As a result of thermal fluctuations of charge carriers a noise voltage can be measured in series with a resistor of a value R, whose power density is

$$\overline{e_n^2} = 4kTR(V^2/Hz)$$
 (Johnson formula) (2)

where T is the absolute temperature of the resistor and  $k = 1.38 \times 10^{-23}$  J/K is Boltzmann constant. This density is constant to frequencies up in the infrared, where it begins to drop due to quantum-mechanical effects. It follows, an actual resistor can be represented by a noiseless resistor in series with a voltage noise source. Or, equivalently, by a parallel current noise source  $I_n^2 = 4kTG(A^2/Hz)$ , where G = 1/R as shown in Fig. 2.

For room temperature (300 K), substitution of the constants yields convenient approximate expressions. So, if R is the resistance in kilohms, the corresponding noise voltage is ap-



proximately

$$4\sqrt{R} (nV/\sqrt{Hz})$$
 (2-a)

whereas, the equivalent current source is  $4/\sqrt{R}$  (pA/ $\sqrt{Hz}$ ). For example, the noise voltage associated with a 9-k resistor at room temperature would be  $\vec{e_n} = 4\sqrt{9} = 12 \text{ nV}/\sqrt{Hz}$  and the noise current  $\vec{I_n} = 4/\sqrt{9} \sim 1.3 \text{ pA}/\sqrt{Hz}$ . Across a 10-kHz bandwidth the noise voltage will be  $12\sqrt{10^4} = 1.2 \mu \text{V}$  (rms) and the noise current  $1.3\sqrt{10^4} = 130 \text{ pA}$  (rms).

The generation of thermal noise is ideally not affected by the flow of current through the resistor. However, carbon resistors, in particular, have an additional current dependent noise which makes them unsuitable for critical applications. In contrast to resistors, ideal capacitors and inductors do not generate noise. For complex impedance  $Z(\omega) = R(\omega) + jX(\omega)$ the spot value of the thermal noise will be due to resistor  $R(\omega)$ , and its density will, thus, be frequency dependent.

Since electric current is composed of discrete charge carriers, fluctuations are present in the current crossing a barrier where the charge carriers pass independently of one another. Examples are: the p-n junction diode where the passage takes place by diffusion; a vacuum-tube cathode where electron emission occurs as a result of thermal motion, and photodiodes where the absorption of photons is involved. This effect does not exist, for example, in metallic conductors because of longrange correlation between charge carriers. The fluctuations manifest themselves as a noise component named shot noise, which can be represented by an appropriate current source in parallel with the dynamic impedance of the barrier across which it is generated. The spectral density of this source is given by the expression

$$\overline{I_n^2} = 2qI_0 \text{ (A}^2/\text{Hz)} \text{ (Schottky formula)} (3)$$

where  $q = 1.6 \times 10^{-19}$  C is the electron charge, and  $I_0$  the dc current. The above expression applies up to frequencies close to  $1/\tau$ , where  $\tau$  is the transit time through the barrier and thus throughout the useful frequency range of any device.

Photoconductive detectors produce generation-recombination (GR) noise in response to a steady irradiance. Hole-electron pairs are generated randomly and recombine randomly by a statistically unrelated process. Thus full GR noise neglecting that which is thermally generated, corresponds to twice shot noise on the absorbed background photon rate for intrinsic photoconductors. As the dc bias is increased, a voltage is reached at which the minority carrier (hole) transit time is less than the lifetime. At this bias, the carriers are swept out of the device before they recombine and the GR noise approaches shot noise on the photocurrent.

For practical calculations it is convenient to substitute for the constants of formula (3). The current spot noise result-



Fig. 3. Flicker noise representation.

ing from  $I_0$  expressed in microampere is:

$$0.57\sqrt{I_0} (pA/\sqrt{Hz}).$$
 (3-a)

For example, the current spot noise associated with an average current of 100  $\mu$ A, would be 0.57 $\sqrt{100}$  = 5.7 pA/ $\sqrt{\text{Hz}}$ ; across a 10-kHz bandwidth it will amount to 5.7 $\sqrt{10^4}$  = 0.57 nA (rms).

As noted above, thermal noise and shot noise have a constant spectral density. Semiconductor devices as well as vacuumtubes show, an additional noise component that is inversely proportional to frequency. Hence, it has the name 1/f noise (also referred to as excess, flicker, or pink noise) (see Fig. 3). This noise in semiconductors is associated, mainly, with crystal surface conditions. It occurs, however, in nonelectrical phenomena, as well [3]. When associated with current noise, it can be described by the spectral density

$$\overline{I_n^2} = \overline{I_{n0}^2} \, (1 + f_L / f^n) \tag{4}$$

 $I_{n0}^2$  represents the white shot noise component, in the excess noise component  $f_L$  is the empirical value of the break frequency where the two noise components are equal and is usually subject to process spreads. The actual value of *n* is not necessarily fixed with frequency. However, in junction transistors it is in the vicinity of 1.1. In some operational amplifiers this value was verified down to  $10^{-7}$  Hz-a one year period! [4]. Apparently, this may lead to very high noise amplitude. However, for the type of spectral density in (4) the noise power in each frequency *decade* is approximately constant. Thus, for example, if  $f_L$  is 1 kHz the noise power in the  $10^{-7}$  to  $10^3$ frequency band is approximately equal to the noise power in the 1- to 30-kHz band. In practice, flicker noise is often regarded as a dc instability.

# IV. THE CHARACTERIZATION OF NOISE IN AMPLIFIERS BY EQUIVALENT INPUT SOURCES

As a result of the mechanisms described above, the output of any real amplifier is accompanied with a noise which depends on the measurement bandwidth, the overall gain and the noise properties of the various stages. The sensitivity of the amplifier is best characterized by the minimum signal at the input, still detectable at the output-rather than by the actual



Fig. 4. Noise sources in a three-terminal amplifying device.

noise measured at the output. This input signal may conveniently be defined as equal to a virtual noise source located at the input, which is obtained by dividing the actual output noise over a given bandwidth by the overall gain. As shown later, the total equivalent input noise of a real amplifier also depends on the impedance of the signal source besides thermal noise that may accompany this impedance.

The device shown in Fig. 4 may represent a junction transistor, FET, or vacuum-tube gain stage in a somewhat simplified manner, and can be considered to have the following noise sources:

- 1) shot noise which accompanies the bias current  $I_B$  of the control electrode (base, gate, or grid) and is given by  $2qI_B A^2/Hz$ ;
- shot noise of the quiescent current I<sub>C</sub>, given by λqI<sub>C</sub> A<sup>2</sup>/ Hz; λ is dependent upon the particular device, in the junction transistor; it is equal to 2, that is, a "full" shot noise;
- 3) thermal current noise of the load resistor  $R_L$ , given by  $4kT/R_L$  (A<sup>2</sup>/Hz).

In relating these noise sources to the input of the stage, the first source, being already located at the input, can be represented by a current source shunting the input. The second source can be represented by a noise voltage source  $e_n^2$  in series with the input and given by

$$\overline{e_n^2} = \frac{\lambda q I_C}{g_m^2} \left( \frac{\mathbf{V}^2}{\mathbf{Hz}} \right).$$

In these devices, the mutual conductance  $g_m$  increases with the quiescent current  $I_C$  [5], although not necessarily linearly and the actual noise is also found to be proportional to absolute temperature T. If we assume  $g_m \sim I_C$  then

$$\overline{e_n^2} \sim \frac{q T}{g_m} \left( \frac{\mathbf{V}^2}{\mathbf{Hz}} \right).$$

Thus the noise may alternatively be attributed to a thermal origin in  $1/g_m$ .

The power density of this voltage noise is thus inversely proportional to the mutual conductance of the device. Assuming that the dc voltage on the resistor  $R_L$  is  $V_L$ , then the shot noise density of  $I_C$  would at most be  $2q V_L/R_L$  compared to  $4kT/R_L$  of the resistor thermal noise. A simple calculation shows that if  $V_L > 50$  mV, the contribution of the shot noise exceeds that of the thermal noise and the thermal noise of  $R_L$ can practically be ignored. A resistive load is usually preferable to an active current source biasing since the latter would add its own shot noise which is comparable to that generated in the active device. However, "quiet" current sources can be obtained, for example, by means of a bipolar transistor with an emitter degenerating resistor. It is apparent that any biasing resistor in parallel with the input will add its thermal noise to the input equivalent current source. Similarly the noise of a resistor  $R_e$  in series with the emitting electrode can be accounted for by adding  $4KT/R_e g_m^2$  to  $e_n^2$ .

The above simplified model is applicable over a considerable frequency range, and indicates that the noise sources are determined mainly by the input bias current and transconductance. These sources seem to have a constant spectral density and be statistically independent. However, as shown later, at high frequencies the density of these sources increases due to decreasing gain, inversely to the device cutoff frequency. At low frequencies the spectral density increases as a result of excess noise effects.

To calculate the noise contributed by the second stage, we observe that its bias current shot noise  $2qI_{B2}$  is adding directly to the shot noise  $2qI_{C1}$ . Due to  $I_B$  being small compared to  $I_C$ , it is negligible. Similarly  $e_{n2}$  of the second stage should be compared with  $e_{n1}$  after dividing by the voltage gain  $g_{m1}R_L$ . From Section I, a voltage gain of only 2 in the first stage may still be sufficient to render its contribution negligible. Thus first stage dominates the amplifier noise performance, unless for some reason the second stage has an unusually high noise.

The two equivalent input noise sources model applies to any amplifier regardless of the nature of its components [6]. In fact, different combinations of external equivalent noise sources along with their correlation coefficient can be used to represent the actual noise sources within any amplifier. However, the above representation is the most convenient for practical purposes.

### V. INFLUENCE OF SIGNAL-SOURCE IMPEDANCE ON THE TOTAL NOISE

Fig. 5 shows schematically an amplifier with input impedance  $Z_{in}$  and white input noise sources  $\overrightarrow{e_n}$  and  $\overrightarrow{I_n}$  which are statistically uncorrelated. If a voltage signal source  $e_s$  with internal impedance  $R_s$  is applied to the amplifier input, the signal voltage at the input would be

$$e_s \frac{Z_{in}}{Z_{in} + R_s}$$

and the total noise at the input would be

$$(\sqrt{4kTR_s} + \vec{e_n} + \vec{I_n}R_s) \frac{Z_{\text{in}}}{Z_{\text{in}} + R_s}.$$
 (5)

This does not include any noise, other than thermal, which may be present in the source which for our treatment would be regarded as a signal.

The signal-to-noise ratio (S/N) at the virtual amplifier input (and the actual S/N at the output) will be  $e_s/(\sqrt{4kTR_s} + \vec{e_n} + \vec{I_n}R_s)$  and is at maximum for low source resistance. The input impedance  $Z_{in}$  apparently does not affect the S/N, since by definition, any noise generated at  $Z_{in}$  is implicit in  $\vec{I_n}$ .

In general, a signal-source can be represented by either a voltage or current source. In practice, however, signal sources



Fig. 5. Noise at the input of an amplifier.

often have one "natural" representation even disregarding their internal impedance. Thus in a current-signal source the short circuit signal current is essentially independent of its internal impedance, as opposed to a voltage-signal source. An analysis similar to the above would show that for best noise performance a current-signal source should have a minimum shunt admittance, even neglecting possible thermal noise and, again, independent of the amplifier input impedance. For a given amplifier, however, an added impedance at the input will adversely affect its performance. It is easy to show, for example, that a shunt parasitic capacitance, such as that associated with a coaxial cable, which may be noiseless by itself, tends to deteriorate the noise performance, especially for current-signal sources at relatively high frequencies.

The above classification does not include parametric sensors, i.e., sensors in which the signal is related to the source impedance, such as infrared resistive detectors. These sensors should be supplied with either current or voltage biasing and the corresponding signal would be accordingly read as a voltage or current. The signal-to-noise ratio is, theoretically at least, proportional to the bias and independent on the actual biasing method, as long as any noise that may be added by the biasing network is negligible.

The total input voltage noise of a given amplifier or device can be determined by measuring the output noise and dividing it by the voltage gain. When the input is shorted, the contribution of output noise is due to  $\overrightarrow{e_n}$  only. However, when the input is biased with a large enough impedance, the contribution will come mostly from  $\overrightarrow{I_n}$ . If an individual selection of devices for low noise is desired—usually in the low-frequency range, the input noise sources can be measured regardless of the actual gain of the device by a fixture in which the gain is held constant by a feedback network [7]. In this way, individual selection of premium devices can be facilitated.

#### VI. NOISE MATCHING

The total input noise current of an amplifier with input equivalent sources  $\vec{e_n}$  and  $\vec{I_n}$ , and correlation coefficient  $\gamma$  fed by a signal source with internal complex impedance  $Z_s$  is given by

$$\sqrt{\overline{I^2}} = \sqrt{4kT/R_e |Z_s| + \overline{e_n^2}/Z_s^2 + \overline{I_n^2} + 2\gamma \cdot \overline{e_n} \cdot \overline{I_n}/Z_s^2}.$$
 (6)

This usually determines the minimum signal that can be handled. Nevertheless, this threshold can often be lowered by noise matching. Noise matching is based on the fact that a coupling transformer with turns ratio 1:n in series with the amplifier input (Fig. 6(a)) yields an equivalent amplifier with input noise sources modified to  $nI_n^2$  and  $e_n^2/n$ , the correlation coefficient being unchanged [8].

The input noise now becomes a function of the turns ratio, and reaches a minimum when

$$n^{2} = n_{opt}^{2} = \frac{1}{Z_{s}} \cdot \frac{\overline{e_{n}}}{\overline{I_{n}}}$$
(7)

the corresponding expression for the minimum noise voltage is

$$Z_{s}[4kT+2(1+\gamma)\overrightarrow{e_{n}}\cdot\overrightarrow{I_{n}}].$$
(8)

Thus a step-up transformer is needed, when the voltage noise predominates, and vice versa. If the input noise sources are frequency dependent  $n_{opt}$  will be obtained by differentiating the overall noise over the bandwidth with n as a parameter.  $n_{opt}$  will be a function of the frequency range but still independent of the correlation coefficient.

From expression (8), the contribution of the amplifier to the input noise is  $2Z_s(1+\gamma)\overrightarrow{e_n}\cdot\overrightarrow{I_n}$  thus neglecting  $\gamma$  the magnitude  $\overrightarrow{e_n}\cdot\overrightarrow{I_n}$  characterizes the inherent noise performance of an amplifier or an input device, provided noise matching is feasible. In practice, the selection must bring other factors into consideration since coupling by means of a transformer is often incompatible with solid-state circuits, and may involve bandwidth limitations, intrinsic resistance noise, bulkiness, and sensitivity to external magnetic interferences.

In certain situations there is a latitude in the selection of the transducer impedance. For example, in a flux measuring magnetic transducer, such as a reproducing tape head, the desired signal is proportional to the flux in the magnetic core. The short-circuit current, assuming purely inductive impedance, can be shown to be directly proportional to the flux and inversely proportional to the number of turns. However, for any input device the total input current noise at a given bandwidth is also a function of source inductance. Thus there is an optimum number of turns as a function of the magnetic core and the input equivalent noise sources of the amplifier, similar to selecting an optimum turns ratio of a matching transformer. In a sense, the transducer serves as its own matching transformer. In practice, the windings resistance must also be considered and, for the analogy to remain valid, this implies that the wire cross section should be inversely proportional to the number of turns. The total cross section of the winding which is assumed constant should be the maximum possible to minimize source thermal noise.

The reduction of noise with a step-up transformer may in a sense be regarded as due to noiseless voltage amplification prior to the actual amplifier, equivalently reducing the input voltage noise. This may also be effected for a narrow-band resistive source by means of a series resonant circuit, as shown in Fig. 7. At the resonant frequency the overall S/N is identical for the two configurations and is given by

$$\frac{e_s^2}{4KTR_s + \overline{I_n^2}R_s^2\left(1 + L/R_s^2C\right) + \overline{e_n^2}R_s^2C/L}$$

as compared to

$$\frac{e_s^2}{4KTR_s + \overline{I_n^2}R_s^2 + \overline{e_n^2}}$$

without the resonant circuit. The two input equivalent noise sources are thus effectively modified in a nearly reciprocal manner, and by properly selecting L and C the narrow-band noise can be minimized. However, as shown below, for a low resistance source the amplifier noise can in most cases be made negligible by merely selecting a proper junction transistor as an input stage.

In cases when  $\overrightarrow{e_n}$  predominates, noise matching can also be effected by connecting several input devices in parallel. This technique is based on the fact that *n* identical devices in paral-





Fig. 7. Input noise sources modification in a resonating resistive signal source.



Fig. 8. Input noise tuning for a reactive source.

lel are equivalent, as far as noise sources are concerned, to a single device preceded by an input transformer having the turns ratio  $1:\sqrt{n}$ —i.e., the source  $\vec{e_n}$  is decreased and the source  $\vec{I_n}$  proportionally increases (8). Here, too, the correlation coefficient remains unchanged.

In determining the transformer turns ratio, the number of devices in parallel—or the quiescent current in a bipolar transistor for minimum noise (see below), the expression for the total noise is of form x + 1/x + c where x is the turns ratio, the number of devices in parallel or the emitter current. This function has a shallow minimum and as a result, noise matching is not critical. For example, if x deviates from its optimal value by a factor of 2 the total noise increases by no more than 25 percent.

#### VII. NOISE REDUCTION FOR REACTIVE SOURCES BY INPUT TUNING

The resonant matching method mentioned above can effectively be applied to inductive narrow-band sources by merely adding proper resonating capacitance. For reactive sources in general, the type of reactance to be applied at the amplifier input depends on the nature of the signal source and the dominant noise source. For a capacitive current signal source  $I_s$ , an inductance L in series or in parallel to the amplifier input will have different effects on the S/N as shown in Fig. 8.

The overall current signal at the input of the amplifier may be obtained by shorting B to ground. For a series inductance the signal component in the short-circuit current will be

$$I_s \cdot \frac{1}{1 + S^2 LC}$$

whereas the noise component is

$$\frac{\overrightarrow{I_n}S^2LC_s + \overrightarrow{I_n} + \overrightarrow{e_n}SC_s}{1 + S^2LC}$$

The signal component is now frequency distorted. After equalization the S/N would be

$$\frac{S}{N} = \frac{I_s}{\overrightarrow{e_n} \cdot SC_s + \overrightarrow{I_n} (1 + S^2 LC)}$$

whereas, the S/N without the inductance would have been

$$\frac{S}{N} = \frac{I_s}{\overrightarrow{e_n}SC_s + \overrightarrow{I_n}}.$$
(9)

The inductance in combination with the source capacitance cancels the source reactance at the resonant frequency and modifies a formerly white current noise  $\vec{I_n}$  to one with a spectrum proportional to  $|1 - \omega^2 LC|$ , disappearing totally at the resonant frequency. However, at higher frequencies the S/N deteriorates compared to L = 0. In a similar manner an inductance in parallel to the input yields a S/N

$$\frac{S}{N} = \frac{I_s}{\overrightarrow{I_n} + \overrightarrow{e_n} (SC_s + 1/SL)}.$$

Now  $\vec{e_n}$  vanishes at  $\omega = 1/\sqrt{LC}$ , however, the S/N deteriorates at low frequencies compared to L = 0.

The techniques discussed, basically different from bandpass filtering, can be combined with transformer action to take care of the remaining noise source, subject to practical limitations. The improvement in S/N is inversely proportional to the band width; however, the technique can still be applied to broadband sources. Thus an inductance in series with a broad-band capacitive current source and paralleling several FET's, may decrease the total noise by a factor of 2 [9].

## VIII. NOISE FIGURE CHARACTERIZATION OF AMPLIFIERS

An older characterization of amplifier's noise performance is by means of the noise figure (NF). It is defined as the S/Nat the amplifier output divided by the corresponding ratio at the input, expressed in decibels

NF = 10 log<sub>10</sub> 
$$\left[ \frac{(S/N)_{out}}{(S/N)_{in}} \right]$$
. (10)

Equivalently, NF is the log ratio of the total output noise to its portion originating as thermal noise in the signal source resistance. Thus an NF of 3 dB means that half the output noise is due to the amplifier. For many amplifiers and resistive source combinations much lower NF are obtainable. In terms of the input equivalent noise sources NF can be expressed as follows:

NF = 10 log<sub>10</sub> 
$$\frac{4kTR_s + \overline{e_n^2} + 2\gamma \overline{e_n^*} \overline{I_n^*} R_s + \overline{I_n^2} R_s^2}{4kTR_s}$$

 $\gamma$  is the correlation coefficient, if any, between the two input noise sources.

Differentiation of the latter expression with respect to  $R_s$ yields the so-called optimal source resistance  $R_{s \text{ opt}} = \vec{e_n}/\vec{I_n}$ where NF attains a minimum. In Fig. 9, NF is plotted as a function of  $R_s$  for three combinations of  $\vec{e_n}$  and  $\vec{I_n}$ , but with fixed  $R_{s \text{ opt}}$  showing the effect of  $\vec{e_n} \cdot \vec{I_n}$  on the function. For given source resistance  $R_s$ , NF can be improved by modifying the source resistance to its optimal value, apparently by the addition of a series or shunt resistance. Actually, however, this would lower the S/N at the output because of the added thermal noise prior to amplification. Furthermore, for  $R_s = 0$ , NF reaches infinity although the actual output noise is less than that corresponding to any other source resistance, including  $R_{s \text{ opt}}$ . The "paradox" lies in the fact that the reduction of NF proportionally improves the S/N ratio at the output



Fig. 9. NF as a function of source resistance for various amplifier figure of merit.

only if the S/N at the source remains unchanged. This can be satisfied by coupling the signal source by means of a transformer. Provided the turns ratio is  $\sqrt{R_{s \text{ opt}}/R_s}$  NF is brought to its minimum, and the S/N to a maximum. This, however, is another interpretation of noise matching as already described.

In general, NF by itself cannot fully characterize the noise performance of an amplifier, nor does it provide a basis for prediction of noise with an arbitrary source impedance. Furthermore, NF does not apply to current signal source or reactive signal sources which ideally have no thermal noise. On the other hand, in RF communications and devices, noise figure is commonly used because of the convenience of optimum matching by a transformer or other source coupling reactive networks.

## IX. NOISE IN JUNCTION TRANSISTORS

The equivalent input noise sources of the junction transistor at midfrequencies are given by

$$\overline{I_n^2} = 2qI_B = 2q\frac{I_e}{\beta}$$

$$\overline{e_n^2} = 4KT\left(r_{b'} + \frac{r_e}{2}\right), \quad r_e = \frac{kT}{qI_e} = \frac{1}{g_m}$$
(12)

where  $I_B$  and  $I_e$  are the base and emitter currents and  $r_{b'}$  and  $r_e$  are, respectively, the spreading base resistance and emitter small-signal resistance. The current noise is obviously the shotnoise of the base current, whereas the voltage-noise source corresponds to the thermal noise of the base resistance in series with one-half the small signal resistance of the base-emitter junction. At relatively low emitter currents, when  $r_e > r_b \cdot e_n^2$  is inversely proportional to  $I_e$ , whereas  $I_n^2$  is directly proportional to it as shown in Fig. 10, this means that the operating current can be matched to the signal source impedance in order to minimize the overall noise in much the same way as with a transformer.

The noise sources are independent of the collector voltage as long as the leakage current is negligible and they are essentially the same for various connections [10], however, the commonemitter is usually preferred due to the higher gain. From (12), a transistor with a relatively high  $\beta$  and a low  $r_{b'}$  is potentially best for minimum noise. When the source resistance is  $R_s$ , the



Fig. 10. Graphic representation of junction transistor input noise sources as a function of  $I_{\rho}$ .

optimum emitter current obtained by differentiation is

$$H_{\text{opt}} = \frac{kT}{q} \frac{\sqrt{\beta}}{R_s + r_{b'}}$$
$$= \frac{25\sqrt{\beta}}{R_s + r_{b'}} \quad (\text{mA})$$

For signal sources other than purely resistive, the optimum current will depend on the bandwidth as well. For small source impedance of the order of  $r_{b'}$  the input noise sources are no longer reciprocal and the noise performance, and NF, deteriorate. The value of  $r_{h'}$  is usually not supplied in data sheets and is subject to manufacturing spread. Typical values vary from several hundred ohms for super- $\beta$  transistors, to several tens of ohms in certain types [10, pp. 11, 68]. Evidently,  $r_{b'}$  can be reduced by a parallel connection of several transistors. This technique is employed in the LM194 matched pair which-not being optimized for low noise, have  $r_{b'} = 30 \Omega$ . However, some special geometry transistors have  $r_{b'}$  reduced to a few ohms (types 2SD786 and 2SB737,<sup>1</sup> n-p-n and p-n-p transistors with a typical  $\beta = 400$  and the specified value of  $r_{b'}$  of 4 and 2  $\Omega$ , respectively). A junction transistor first stage may thus contribute insignificant noise even with very low resistance sources.

At frequencies where  $\beta$  falls off the input current noise source increases with frequency and is given by

$$\overline{I_n^2} = 2qI_B\left(1 + \beta \frac{f^2}{f_T^2}\right).$$
(12-a)

The corner frequency is thus  $f_T/\sqrt{\beta}$  where  $f_T$  is the cutoff frequency. Similarly,  $e_n^2$  starts to increase near the upper use-

ful frequency range of the transistor, along with some correlation to  $\overline{I_n^2}$  [12]. Considering the fact that  $f_T$  increases with  $I_e$ , the optimal current for a given source impedance will increase at high frequencies reflecting the fact that the effective  $\beta$  is decreasing.

The effect of operating the junction transistor at low temperatures is, apparently, to reduce  $\vec{e_n}$  proportionally to T. In silicon planar transistors  $\vec{e_n}$  was found to reach a minimum of around 150 K-[13]; however at still lower temperatures, it increases again, accompanied by a sharp drop in  $\beta$  and the cutoff frequency, deteriorating  $\vec{I_n}$  as well. A similar trend is found also in germanium transistors.

#### X. NOISE IN FIELD-EFFECT TRANSISTORS

The noise sources of FET's above the excess noise region are given by the following expressions [14], [15]:

$$\overline{e_n^2} = 0.7 \cdot 4kT/g_m$$
  
$$\overline{I_n^2} = 2qI_g + 0.7 \cdot 4kT/g_m \cdot \omega^2 C_{gs'^2}$$
(13)

 $g_m$  is the mutual conductance of the FET,  $I_g$  is the gate leakage current, and  $C_{gg'}$  is the internal input capacitance of the FET (roughly  $\frac{2}{3}$  of the total capacitance  $C_{gs}$ ). As expected, the noise sources have a form similar to that in the junction transistor. However, the leakage current  $I_{g}$  is usually much smaller than a typical base current. It is also not proportional to the operating current, it may, however, increase substantially at drain voltage above several volts, thus there is no interdependence between the two noise sources. The noise sources of the MOSFET are much the same as those of the junction FET but  $I_g$  is negligible. In junction FET's  $e_n^2$  is found in practice to be somewhat larger than calculated from the measured  $g_m$ . This is due to the thermal noise of the bulk resistance of the source terminal, the actual measured  $\overrightarrow{e_n}$  is usually not less than 2 nV/  $\sqrt{\text{Hz}}$ . Special geometry junction FET's such as the 2N6550 achieve  $\overrightarrow{e_n} = 0.8 \text{ nV}/\sqrt{\text{Hz}}$  by increasing  $g_m$  along with  $C_{gs}$ .

A special meshed-gate geometry developed by TOSHIBA yields extremely low noise FET's for audio frequencies. In this family the p-channel FET type 2SJ72 and the n-channel type 2SK147 (with its dual version 2SK146) have  $\overline{e_n} = 0.75$  nV/ $\sqrt{\text{Hz}}$  with  $C_{gs} = 130$  pF and 50 pF, respectively. This noise level is equivalent to that of a 35- $\Omega$  resistor and is achieved at a drain current of 2 mA. For still higher frequencies, the 2SK117 is available with  $\overline{e_n} = 1 \text{ nV}/\sqrt{\text{Hz}}$  and  $C_{gs} = 10$  pF at a drain current of only 0.5 mA.

To reduce  $\overrightarrow{e_n}$  in FET,  $g_m$  apparently, must be increased as much as possible by maximizing drain current  $I_D$ . However, because  $g_m$  is proportional to  $I_D^{1/2}$ ,  $\vec{e_n}$  is proportional to  $I_D^{-1/4}$ , and with such a mild dependence, there is no advantage in increasing  $I_D$  beyond a value dictated by other considerations. In addition, excessive heat dissipation reduces the effective  $g_m$ and increases the leakage current  $I_g$ . More effectively, noise can be reduced, when necessary, by connecting several FET's in parallel or selecting a large geometry FET's mentioned above, however, due to the corresponding increase of  $C_{gs}$ , high frequency noise performance is impaired and there is an optimal number of FET's. For capacitive source, the optimum occurs when the total capacitance at the input becomes roughly equal to that of the source [9], and in general, the noise quality of the FET is dependent on the ratio  $g_m/C_{gs}$ , which is also the high-frequency figure of merit. An example of a high-quality FET is U309, with  $g_m = 15$  mmho and  $C_{gs} = 4.3$  pF. Another

<sup>&</sup>lt;sup>1</sup> Made by TOYO Electronics Ind. Corp., Central Kyoto, Japan (represented by R-Ohm Corp., P.O. Box 4455, Irvine, CA 92761).



Fig. 11. Noise sources of a differential input stage.

|                            | en(nV/√Hz) | e Low-Frequency<br>Corner (Hz) | Î <sub>n</sub> (pA//Hz) | I Low-Frequency<br>Corner | Gain Bandwidth<br>Product (MHz) | Notes                |
|----------------------------|------------|--------------------------------|-------------------------|---------------------------|---------------------------------|----------------------|
| HA-909                     | 7          | 100                            | 0.2                     | 2.0 kHz                   | 7                               |                      |
| HA-4602                    | 7          | 300                            | 0.15                    | 1.5 kHz                   | 8                               | Quad                 |
| NE-5534A                   | 4          | 100                            | 0.4                     | 200 Hz                    | 10                              |                      |
| OP-27/37                   | 3          | 3                              | 0.4                     | 140 Hz                    | 8/63                            |                      |
| OP-07                      | 10         | 10                             | 0.1                     | 50 Hz                     | 0.6                             |                      |
| OPA 101/102 BM, Burr-Brown | 8          | 100                            | 0.002                   |                           | 20/40                           | Spec Guaranteed)     |
| PM 156/157 A               | 12         | 50                             | 0.01                    | <100 Hz                   | 4.5/20                          | Bife                 |
| MA-334                     | 8          | 60                             | 0.005                   | 60 Hz                     | 15                              | ,                    |
| MA-322 Analog-             | 3.5        | 100                            | 0.5                     | 400 Hz                    | 50                              |                      |
| MA-106 Systems             | 0.6        | 1000                           | 2                       |                           | 15 (3 dB))                      |                      |
| ZN-459 Perranti            | 0.8        | 50                             | 1                       |                           | 15 (3 dB)                       | Video-<br>Amplifiers |
| SL1205C Plessey            | 0.8        | <100                           |                         |                           | 6.5 (3 dB)                      |                      |

Fig. 12. Currently available low noise monolithic amplifiers.

high-performance FET is TOSHIBA type 2SK61, this device has  $g_m = 10 \text{ mmho}$ ,  $C_{gs} = 4 \text{ pF}$ , and a very small reverse capacitance  $C_{ed} = 0.1 \text{ pF}$ .

The ratio  $g_m/C_{gs}$  is still higher in FET's of the D-MOS type, such as Signetics type SD203, with  $g_m = 15$  mmho and  $C_{gs} =$ 2.4 pF. A disadvantage of MOSFET's, in general, is the high level of flicker (1/f) noise in  $\vec{e_n}$  which precludes its use at the audio range. Compared to silicon FET's gallium-arsenide FET's have higher  $g_m/C_{gs}$  ratios and, thus, are potentially lower in noise. However, presently they suffer from very high 1/f noise and high gate leakage current. Consequently, they are useful only for very high frequencies.

In silicon junction FET's,  $g_m$  usually increases by decreasing temperature, in addition to the implicit effect of T in the expressions for the two noise sources and the leakage gate current. A FET front-end may thus be cooled to advantage in case the signal source is already cooled. It is found, however, that the improvement in  $\overrightarrow{e_n}$  reaches a maximum around 100 K, depending on the specific device [16]. MOSFET's, on the other hand, can be operated at still lower temperatures [17].

#### XI. NOISE IN MONOLITHIC AMPLIFIERS

The noise sources in monolithic amplifiers are essentially those expected from a discrete equivalent, with the first stage usually being of a differential junction or FET pair. Monolithic operational amplifiers, as well as discrete junction transistors, may suffer from additional type of noise, beside those expected from (12). This is called "burst" or "popcorn" noise and is associated with the base current noise (see Section XII). "Burst" noise is not a Gaussian noise but more like random jumps between two levels with characteristic times spanning a large range. However, it is virtually absent in most modern devices due to improved manufacturing technologies. The differential input amplifier can be represented in terms of four noise sources, two at each input (Fig. 11) similar to dc drift representation. The two voltage noise sources can be vectorially combined and represented by a single equivalent source connected in series with one of the inputs. Then the differential input stage is represented by two current noise sources and one voltage noise source.

Monolithic operational amplifiers have usually been optimized for input dc characteristics such as low bias current high dc gain, and low power consumption rather than for low noise. Moreover, some of the circuit techniques utilized such as use of Darlington input stage, active loads in the first stage or the use of resistive input protection network tend to degrade the noise performance. In addition, high  $\beta$  is accompanied with large  $r_{b'}$ . In some operational amplifiers the input bias current is internally supplied, with the result that the input current noise is much more than expected due to the specified bias current. The above drawbacks combined with high-frequency limitations and the inability to modify the first stage current for matching various sources have tended to limit the use of operational amplifiers and other monolithic amplifiers in various critical applications. However, operational amplifiers and other monolithic amplifiers in particular, have been made available in recent years specifically for low noise (Fig. 12), and can fit many low-noise applications with the benefits of small space and low price. For comparison, a  $\mu$ A741 amplifier has-depending on the manufacturer-a typical  $\vec{e_n} = 25$  nV/ $\sqrt{\text{Hz}}$  and  $\vec{I_n} = 0.6$  pA/ $\sqrt{\text{Hz}}$ . This means that for any source impedance the amplifier noise will exceed the source thermal noise, this is not true for the above amplifiers each within its typical range of source impedance.

In cases where lowest noise is desirable for a given range of source impedance, a discrete differential input stage can be



Fig. 13. Noise sources in a noninverting-connection operational amplifier.





Fig. 14. (a) A discrete input stage differential amplifier. (b) An improved discrete input stage differential amplifier.

combined with a monolithic operational amplifier. The input stage being optimized for noise while most of the loop gain is supplied by the operational amplifier. However, the addition of the input stage often makes the amplifier unstable in closed loop without further compensation.

The general expression for the total input noise in a noninverting operational amplifier (Fig. 13) with a resistive source is

$$\overline{e_n^2} + \overline{I_n^2} (R_1 / / R_2)^2 + 4kT(R_1 / / R_2) + 4kTR_s + \overline{I_n^2} R_s^2.$$
(14)

The value of the feedback network resistors must be low enough to ensure minimum added noise. In comparison the inverting-type configuration is not suitable for low noise due to the series input resistor. For a similar reason the conventional four resistors configuration of a differential amplifier is not suitable for low noise applications. When low noise differential input is needed, the differential input pair can be left outside the feedback loop as in Fig. 14(a) where the low frequency gain is  $g_m R$ . A more elaborate configuration is shown simplified in Fig. 14(b). In this scheme the input pair is within the feedback loop, yet the signal source can directly coupled to the bases because feedback is applied to the emitters. The differential gain of the amplifier is  $2R_1/R_E + 1$ , and the common mode as well as power supply rejection ratio can be made very high by selecting matched components. It may be necessary though, to bypass the positive and negative supplies near the input stage to eliminate possible high-frequency supply noise from coupling through unbalanced stray capacitances at the collectors. This is true also for operational amplifier preamplifiers due to the finite power supply rejection ratio.

In general, even though a single-ended signal source would enjoy less added noise when coupled to an amplifier with a single-ended input stage, say a common emitter followed by an operational amplifier, the power supply sensitivity, components count, and size of coupling and bypass capacitors may still make the differential input amplifier a better choice.

In some cases an amplifier must have both low input noise, possibly with wide bandwidth, and good dc properties (offset voltage and input bias current) as well. These requirements tend to conflict; however they can be met using a composite amplifier [18]. Similarly to the classical chopper-stabilized configuration, the amplifier is separated into two channels, one which has a differential input stage and is dc coupled to the signal source, and another channel which is a low-noise ac amplifier. The two output signals are then combined to a single channel. If the dc amplifier output is low passed, its input voltage noise source will not contribute to the output noise above the cutoff frequency. The total input current noise however originates from the two inputs: By design, however, the input bias current of the dc amplifier is small and has a low shot noise. At high frequencies the current noise which flows through the source impedance tends to increase, but its effect can be suppressed by a decoupling choke in series with the input of the dc channel.

# XII. INPUT DEVICE SELECTION FOR VERY-LOW FREQUENCIES

Expressions (12) and (13) give the noise sources in the FET and junction transistors but do not take into account excess noise effects at low frequencies, which are less predictable.

As a general rule, in junction transistors low-frequency excess noise is associated with the current noise source  $I_n$  [19], [20]. Another noise associated with the base current but which is usually of no concern in discrete devices is the *burst* noise mentioned earlier [21], [22]. The opposite occurs in the FET and especially MOSFET's where the voltage noise source  $e_n$  is affected.

In junction transistors  $\overrightarrow{I_n}$  is given by

$$\overline{I_n^2} = 2qI_B + \frac{KI_B^m}{f^n} \tag{15}$$

whereas in FET's  $\overrightarrow{e_n}$  is approximated by

$$\overline{e_n^2} = 4kT \frac{0.7}{g_m} \left( 1 + \frac{f_L}{f^n} \right).$$
(16)

In the above expressions,  $n \sim 1$ , K, and  $f_L$  are subject to spread whereas 1 < m < 2 [23]. The low-temperature dependence of low-frequency noise is pronounced and is such that n is not fixed [17], [24].

In FET's, not specified for low noise at low frequencies, the corner frequency  $f_L$  may be of the order of many kilohertz; on the other hand, in some N-channel type,  $f_L$  can be as low as 1 kHz. These FET's are usually characterized by  $\overrightarrow{e_n}$  at 10 Hz. Devices such as 2N6483, 2N5592, 2N4867A, 2N6550, and NF 101, may have  $\overrightarrow{e_n}$  as low as 6 nV/ $\sqrt{\text{Hz}}$  which can of course, be further lowered by paralleling. By far, however, the best low-noise junction FET's are TOSHIBA family of meshedgate devices, mentioned earlier. The corner frequency may be as low as 15-20 Hz and at operating current  $I_d = 2$  mA the

voltage noise at 10 Hz is 2 nV/ $\sqrt{\text{Hz}}$  for the 2SJ72 and 1.3 nV/ $\sqrt{\text{Hz}}$  for the 2SK147.

Before the advent of low flicker noise FET's, the only means for obtaining low  $\overrightarrow{e_n}$  and a negligible  $\overrightarrow{I_n}$ , at low frequencies had been the varactor-bridge amplifier (also referred to as a lowfrequency parametric amplifier). This amplifier is based on signal dependent imbalancing of a voltage-controlled capacitor (varactor) bridge, which is driven by a reference high-frequency carrier. The output of the bridge is a carrier modulated by the signal and after being further amplified is synchronously demodulated. The initial gain is achieved through the voltage dependent capacitance of the varactors, rather than by an active device. The source  $\overrightarrow{e_n}$  being essentially free of 1/f noise is determined by passive components (25) and may be superior to J-FET's at subaudio frequencies. However, for practically all applications this technique is now regarded obsolete.

In junction transistors, the low-frequency noise corner is current dependent due to m and the corner frequency is usually in the order of several hundred hertz. From expression (15), the excess noise in junction transistors is equivalent to a decrease in  $\beta$ . Consequently, the optimal current for a given source resistance would be lower at the flicker noise region than it would be at medium frequencies. It has also been found that 1/f noise can significantly increase as a result of avalanching the base-emitter junction [26]. This may happen during supply turn-on or input overloading and may be prevented by parallel protection diodes. Excess noise in junction transistors is observed also in  $\vec{e_n}$  and is due to base-current shot-noise passing through  $r_{b'}$ . However, the corner frequency of  $\vec{e_n}$  may be very low and is usually of no concern.

## XIII. IMPEDANCE, NOISE, AND NEGATIVE FEEDBACK

A preamplifier for a specific transducer should in general meet two main requirements: 1) the output voltage be proportional to the desired signal over the bandwidth of interest, and 2) the equivalent input noise should not exceed a certain minimum related to the signal expected amplitude. The first requirement is usually easier to comply with since one can always design a network which will restore a frequency distorted signal. To minimize the complexity of such a network and, perhaps, eliminate it altogether one should distinguish between two main types of transducers, as in Section V, i.e.: 1) those in which the desired signal is proportional to the source short-circuit current, such as reverse biased photodiodes or magnetic inductive transducers, and 2) those in which the signal is proportional to the open-circuit voltage, such as piezoelectric detectors. The first-order equivalent circuits of these two types of sources would be a current source with a shunt admittance, and a voltage source with a series impedance, respectively. For the current signal source, in order to eliminate the effect of the shunt admittance the ideal preamplifier would have zero input impedance, with an output voltage proportional to the input current. Similarly, for a voltage-type signal source, a high input impedance voltage amplifier is in order. Evidently, there are sources for which the equivalent circuit is more complex, necessitating a signal restoring network. One example is a reactive source with a noise tuning reactance (see Section VII). Another example is a flux measuring inductive sensor with internal resistance as discussed below.

In establishing an appropriate input impedance there is no need to compromise noise performance since noise is not



Fig. 15. (a) Parallel feedback amplifier with a current signal source at the input. (b) Total noise and signal at the input of the amplifier.



Fig. 16. Extending low-frequency response for inductive source with a negative input impedance amplifier.

necessarily associated with it. The reason is that using a negative feedback, one can modify the input impedance without significantly affecting the input equivalent noise sources. In contrast to this flexibility, the noise performance depends heavily on the input stage.

Fig. 15(a) shows a current signal source  $I_s$ , with internal admittance  $Y_s$ , terminated with a voltage amplifier with input admittance  $Y_{in}$ . In Fig. 15(b) the input network has been replaced by its Norton equivalent where the total input current noise is given by

$$\overline{I_n^2} + \overline{e_n^2} \cdot |Y_s|^2 + 4kTR_e[Y_s].$$
(17)

Now, if we add a parallel feedback admittance  $Y_f$ , it will add to the source admittance since, for this purpose, the output side of the feedback admittance is at ground. Consequently, the total current noise at the input would be

$$\overline{I_n^2} + \overline{e_n^2} |Y_s + Y_f|^2 + 4kTR_e [Y_s + Y_f].$$
(18)

The input network can be regarded as an equivalent current source comprised of a signal component and a noise component. When feedback is applied it obviously exerts the same effect on the signal source as on the total noise source so that their ratio remains unchanged at any frequency. However, this may well cause a change of the transfer function from input current to output voltage. So that if the spectral contents of the signal and noise are different, the integrated S/N over the bandwidth of interest may change. However, for both cases, as long as the transfer function is equalized, there would be no difference compared to the open loop. Matters become different, if for the sake of fair comparison we assume  $Y_F = 0$  in the nonfeedback amplifier, which then theoretically appears to be superior. Practically, this means that depending on the feedback type, series or parallel, the impedance level of the feedback elements should be sufficiently small or sufficiently large to minimize the added noise.

The transimpedance amplifier in which  $1/Y_F = R_F$  [27] provides an example where negative feedback serves for obtaining wide bandwidth by reducing the input impedance. It is advantageous for capacitive current sources since ideally no signal integration would take place at the input. Input impedance can certainly be made arbitrarily low with a parallel input resistor R; however, the thermal current noise 4kT/R would adversely affect the S/N. The transimpedance amplifier, on the other hand, has an input impedance which is equal to  $R_F/A$ , but the thermal noise added is that of the resistor  $R_F$  only. Thus a low noise and a large bandwidth can be achieved simultaneously if  $R_F$  and A are sufficiently large. Similarly a charge amplifier is obtained when  $Y_F = \omega C$ , i.e., the feedback



Fig. 17. Two configurations of the cascode connection.

is by means of capacitor (in parallel with a large bias resistor) and is ideal for sources represented by a voltage source in series with a capacitance.

Magnetic flux-measuring inductive signal sources such as magnetic reproducing heads and current transformers are another example where the signal is proportional to the short-circuit current. In practice, however, there is always certain winding resistance which would, in combination with the inductance, limit the low-frequency response even if the amplifier has zero input impedance. Neglecting parasitic capacitance, the source can be represented as in Fig. 16 where it is coupled to an amplifier with a combined positive and negative feedback. A negative input resistance is thus realized which can nearly cancel the winding resistance effect and appreciably extend the low-frequency cutoff. The thermal noise of the source resistance cannot, obviously, be eliminated. The feedback resistor  $R_F$  should be large enough to add a negligible thermal noise, while the positive feedback resistors  $R_1$ ,  $R_2$  should similarly be small enough. In the ideal case where the input resistance exactly cancels the source resistance the amplifier is in fact operated open loop. To ensure dc stability, the positive feedback is ac coupled. This circuit is simple and performs better than a high-input impedance amplifier and equalization network [28].

#### XIV. THE JUNCTION TRANSISTOR AND THE FET AS A FIRST STAGE

As shown in Section VI, the low-noise figure of merit of any amplifier or input device is the product of its two equivalent input noise sources at the frequency range of interest; theoretically at least, one should select a device in which  $e_n^2 \cdot \overline{I_n^2}$  is at minimum, and the source impedance would then only determine the turns ratio of the noise matching transformer or the number of input devices to be connected in parallel.

On the basis of formulas (12), (12a), (13), and assuming that  $r_{b'}$  is negligible, then neglecting the dependence of  $f_T$  and  $\beta$  on  $I_e$ , the figure of merit for the junction transistor would be

$$(2KT)^2 \left[ \frac{1}{\sqrt{\beta}} + \frac{f^2}{f_T^2} \right]$$

The figure of merit for the FET, substituting  $g_m/C_{gs} = f_T$  is found to be quite similar

$$(2KT)^2 \left[ \frac{1.5 \, qI_g}{kTg_m} + \frac{f^2}{f_T^2} \right]$$

Thus, at low frequencies, high  $\beta$  and low  $I_g$  characterize lownoise devices, whereas at high frequencies, high cutoff is the figure of merit. It should be kept in mind that the FET would usually consume more power due to the dependence of  $g_m$  on  $I_D$  and that noise matching is the basic condition for the above comparison. Thus applying the above result to a relatively low impedance source may necessitate the use of a transformer with an impractical turns ratio or, alternatively, a large number of FET's in parallel. However, as already shown, exact noise matching is not critical.

In general, the FET is a better choice for high-impedance wide-band signal sources. Since, at low frequencies, high  $\beta$ junction transistors operated at sufficiently small collector currents may achieve base currents comparable to and at high temperatures even smaller than typical gate leakage currents. In FET's, on the other hand, the low noise performance spans a wider range of signal sources and frequencies. For very low impedance sources the junction transistor is superior due to its potentially higher  $g_m$  when accompanied with low  $r_{b'}$ . As far as noise is concerned the actual selection should eventually be made by a quantitative comparison of the integrated noise over the signal bandwidth.

As already mentioned, for both devices, the input noise sources are essentially independent on the configuration; however, the voltage gain, current gain, and input impedance are different for the common-base and the common-emitter. For example, due to its unity current gain, the common-base is a bad choice for a current-signal source. With voltage-signal sources, on the other hand, sufficient voltage gain may be obtained as long as  $R_s$  is of the order of  $1/g_m$ . As a result the common-base can be preferable when its low input impedance is an advantage, mainly for source impedance matching in communications applications and in general, where its wide bandwidth and low reverse-capacitance are important as in infrared low resistance with bandwidth HgCdTe detectors.

In most cases, the common-emitter or common-source are preferable as a first stage owing to the high gain and input impedance. However, the voltage gain A of such stage tends to increase the input capacitance due to Miller-effect by  $A C_r$ where  $C_r$  is the reverse capacitance of the device. ( $C_r$  tends to be very small in D-type MOSFET's [26] and some junction FET's-see Section X). In other than low-frequency amplifiers this is undesirable, and a classical input configuration, the cascode, is often used to minimize this effect (see Fig. 17). In this configuration the second stage which serves as a load for the first stage is a common-base operated at sufficient current so as to decrease the voltage gain of the input stage, typically not much greater than unity. The common base is a unity current amplifier and the total voltage gain is  $g_m R_L$ . Thus the cascode is a combination of unity voltage and unity current gain stages and has high speed capabilities, besides low input capacitance. The noise of the second stage can be accounted for by vectorially adding its base-current noise, divided by  $g_{m1}$ , to the first stage noise-voltage source. As already mentioned this base noise tends to increase at low as well as at high frequencies. Thus a junction FET second stage may sometimes be preferable.

Some care is usually necessary in the selection of passive components in low noise design [10, ch. 9]. As already mentioned, carbon composition resistors in particular, as well as various potentiometers develop extra noise, which is proportional to the dc current. Usually, metal film resistors are the best for an input stage. In addition, electrolytic capacitors may contribute noise due to leakage current and should be avoided if possible.

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