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The Action of a Direct Radiator Loudspeaker with a Non-Linear Cone Suspension System

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During the past few years a number of mathematical investigators have directed their efforts toward the solution of differential equations with variable coefficients. These analyses are useful in explaining some of the phenomena which occur in electroacoustic vibrating systems with non-linear elements. In particular, this mathematics may be used to explain the various phenomena exhibited by a direct radiator loudspeaker with a non-linear cone suspension system. One of the effects is a jump phenomenon in the response frequency characteristic. Another effect is the production of harmonics and subharmonics.

INTRODUCTION

DURING the past few years a number of investigators[1] have directed their efforts toward the solution of differential equations with variable coefficients. These analyses are useful in explaining some of the phenomena which occur in electroacoustic vibrating systems with non-linear elements. In particular, this mathematics may be used to explain the various phenomena exhibited by a direct radiator loudspeaker with a non-linear cone suspension system.

The general trend in all types of radio receivers and phonographs is more output without a corresponding increase in the size of the loudspeaker. As a result, the maximum amplitude of the loudspeaker is also increased. Many apparently peculiar activities are manifested by the loudspeaker at the lower frequencies when the amplitude or excursion of the cone is large. Most of the unusual phenomena exhibited by the direct radiator loudspeaker at the lower frequencies are due to the non-linear characteristics of the suspension system. One of the effects of a non-linear

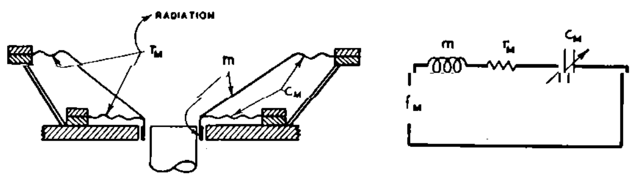

FIG. 1. Cross-sectional view of a dynamic type direct radiator loudspeaker and the mechanical circuit of the vibrating system. In the mechanical circuit: m, the mass of the cone and voice coil; C_M, the compliance of the suspension system; r_M, the mechanical resistance due to radiation and losses in the suspension system; f_M, the driving force.

[1] K. O. Friedrichs and J. J. Stoker, Quart. App. Math. 1, 97 (1943). This paper includes an extensive bibliography.

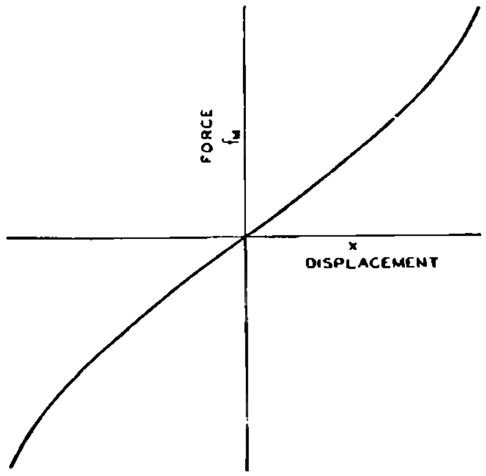

FIG. 2. The force displacement characteristic of the suspension system of the cone of a direct radiator loudspeaker.

cone suspension system is a jump phenomenon in the response characteristic. Another effect is the production of harmonics and subharmonics due to the non-linear cone suspension system. It is the purpose of this paper to demonstrate theoretically and experimentally the effect of a non-linear cone suspension system.

DIRECT RADIATOR LOUDSPEAKER

The simple direct radiator loudspeaker consists of a cone driven by voice coil located in a magnetic field. The essential mechanical elements and the mechanical circuit[2] of this loudspeaker for the low frequency range are shown in Fig. 1. The compliance or spring element in this system is the outside cone suspension and the voice coil centering suspension. In general, the compliance of the outside cone suspension is small compared to the compliance of the voice coil suspension and the cone suspension is therefore the controlling compliance. In any event, at the low frequencies the two compliances may be lumped as a single compliance as shown in the mechanical circuit of Fig. 1.

EQUATIONS DEPICTING THE ACTION OF A DIRECT RADIATOR LOUDSPEAKER WITH A NON-LINEAR SUSPENSION SYSTEM

The force displacement characteristic of a typical direct radiator loudspeaker cone suspension system is shown in Fig. 2. It will be seen that for small amplitudes the suspension system is linear. However, for large amplitudes the suspension system is non-linear.

The force deflection characteristic of the loudspeaker cone suspension system of Fig. 2 may be approximately represented by the expression

$$f_M = f(x) = \alpha x + \beta x^3, \tag{1}$$

where $\alpha = \text{constant} > 0$, $\beta = \text{constant} > 0$, and f_M = applied force which produces the displacement x.

The compliance of the suspension system of Fig. 2 may be obtained from Eq. (1) as follows:

$$C_M = \frac{x}{f_M} = \frac{1}{\alpha + \beta x^2}. \tag{2}$$

The differential equation of the vibrating system in Fig. 1 is

$$m\ddot{x} + r_M\dot{x} + \frac{x}{C_M} = F \cos \omega t, \tag{3}$$

where x = displacement, $\dot{x}$ = velocity, $\ddot{x}$ = acceleration, m = mass of the cone and coil, r_M = mechanical resistance due to dissipation in the air load, C_M = compliance of the suspension system, $F = Bli$, B = magnetic flux density in the air gap, l = length of the voice coil conductor, i = amplitude of the current in the voice coil, $\omega = 2\pi f$, f = frequency, and t = time.

Substituting the expression for C_M of Eq. (2) in Eq. (3), the differential equation becomes

$$m\ddot{x} + r_M\dot{x} + \alpha x + \beta x^3 = F \cos \omega t. \tag{4}$$

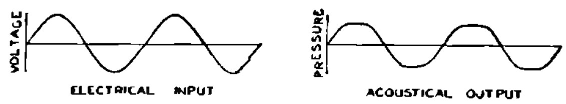

FIG. 3. The wave shapes of the electrical input and the acoustical output illustrate the effect of a non-linear element upon the acoustical output.

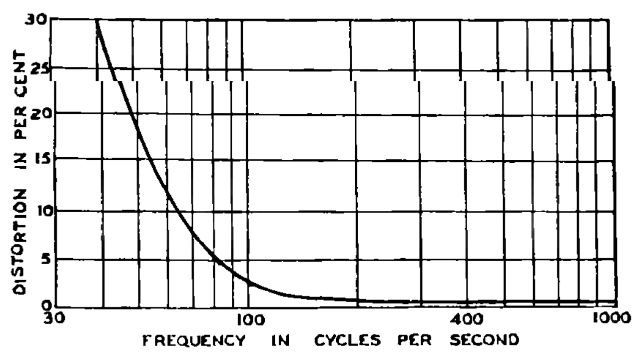

FIG. 4. Non-linear distortion frequency characteristic of an 8-inch dynamic type direct radiator loudspeaker mounted in a five-foot square baffle for an input of 5 watts.

[2] H. F. Olson, *Elements of Acoustical Engineering* (D. Van Nostrand Company, Inc., New York, 1940), Chapter VII.

Since the mechanical resistance r_M is quite small compared to the mechanical reactance, save over a very narrow frequency range near the resonance frequency, we can write Eq. (4) as follows:

$$m\ddot{x}+\alpha x+\beta x^3=F\cos\omega t. \qquad (5)$$

A number of investigators have obtained an approximate solution of this differential equation.

If β is considered to be small, the relation

$$\omega^2=\frac{\alpha}{m}+\frac{\frac{3}{4}\beta A^2}{m}-\frac{F}{Am} \qquad (6)$$

between the arbitrary *amplitude A* and ω may be obtained.

An approximate solution of the differential equation, for unit mass, is

$$x=A\cos\omega t+\frac{1}{32}\frac{\beta A^3}{\alpha+\frac{3}{4}\beta A^2-(F/A)}\cos 3\omega t. \qquad (7)$$

The sections which follow will show that these equations predict the performance of a loudspeaker with a non-linear cone suspension system.

NON-LINEAR DISTORTION CHARACTERISTICS

The well-known experimental result of a non-linear cone suspension system is the production of odd order harmonics when a sinusoidal input is applied to the loudspeaker. The wave shape under these conditions is shown in Fig. 3. The

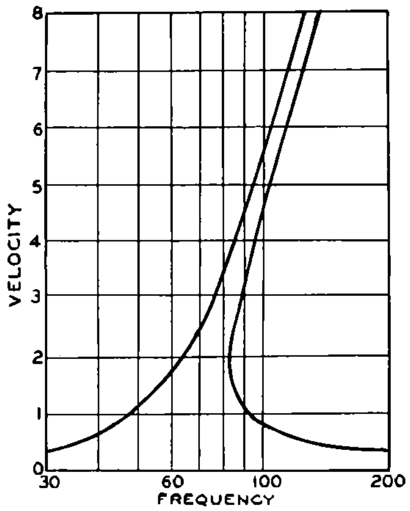

FIG. 5. Theoretical undamped response frequency characteristic of a direct radiator loudspeaker with a non-linear suspension system having a force displacement characteristic depicted in Fig. 2.

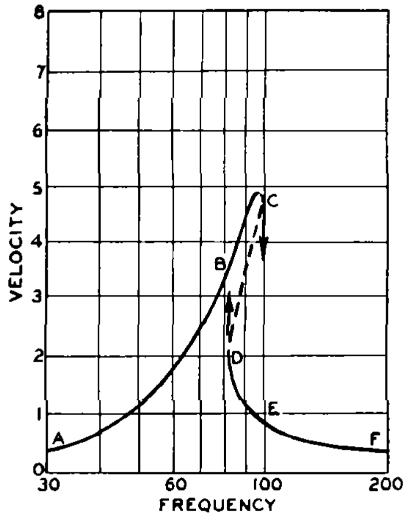

FIG. 6. Theoretical damped response frequency characteristic of a direct radiator loudspeaker with a non-linear suspension system. The dotted path represents a region of instability.

third harmonic is the preponderant distortion component. Equation (7) shows that a third harmonic term is introduced due to the suspension system. In the case of a direct radiator loudspeaker, the amplitude is inversely proportional to the square of the frequency for constant sound power output in the frequency region below the frequency of ultimate resistance. Consequently, the greatest distortion will occur at the low frequency end of the frequency range as shown by a typical experimental non-linear distortion frequency characteristic of Fig. 4. The manifestation and effect of this type of distortion upon the reproduction of sound are well known. It occurs in all amplifiers as well as loudspeakers. As a matter of fact, it is more troublesome in amplifiers because the distortion occurs over the entire audiofrequency range whereas the distortion is confined to the low frequency range in loudspeakers.

In the above considerations the distortion produced by the non-linear element comprises harmonics of the fundamental. Distortion components with frequencies of 1/2, 1/3, 1/4 . . . $1/n$ of the frequency of the applied force also occur in non-linear systems. Those familiar with the performance of loudspeakers have noticed the production of subharmonics. In general, these are very pronounced in the mid-frequency range. In the mid-frequency range the subharmonics are due to the non-linear properties of the cone. Particular solutions of Eq. (3) have been ob-

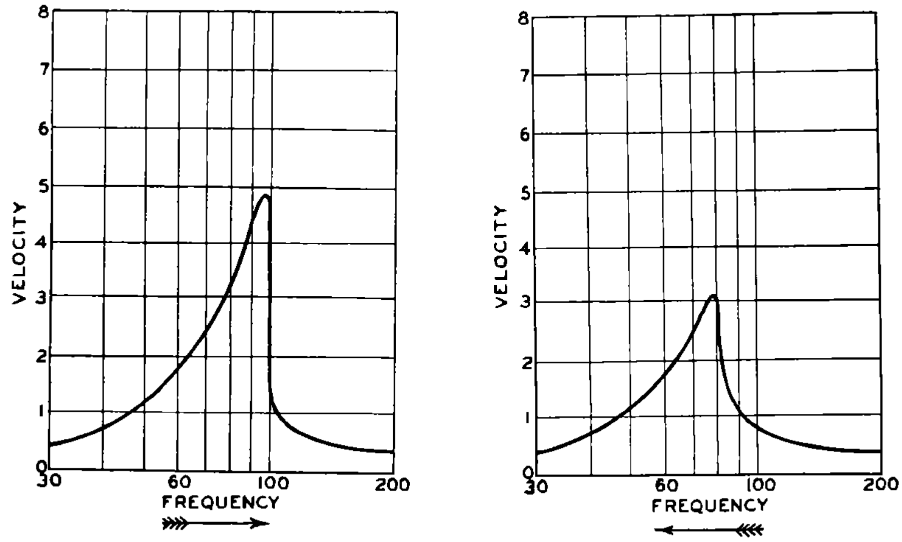

FIG. 7. Experimental response frequency characteristics of a direct radiator loudspeaker with a non-linear suspension system having a force displacement characteristic depicted in Fig. 2. A. (*left*) The response for an applied alternating voltage which continuously increases in frequency. B. (*right*) The response for an applied alternating voltage which continuously decreases in frequency.

tained which show that subharmonics are possible in a loudspeaker due to non-linear cone suspension system. As pointed out above, the amplitude of the cone of a direct radiator loudspeaker is inversely proportional to the square of the frequency for constant sound output. The large amplitudes are confined to the low frequency range. Therefore, these subharmonics will be of a very low frequency and difficult to detect. Careful experimental investigations have shown the existence of subharmonics due to a non-linear cone suspension system as predicted theoretically.

RESPONSE FREQUENCY CHARACTERISTICS

The velocity frequency characteristic of a loudspeaker with a non-linear suspension system may be obtained from the Eqs. (3) and (4). A typical theoretical velocity characteristic is shown in Fig. 5. In this analysis the mechanical resistance has been assumed to be zero. By analogy with the response curve in the linear case with damping one would expect the curve to be rounded off at the peak as shown in Fig. 6.

Suppose that we apply constant current to the voice coil of the loudspeaker and start at a low frequency A, Fig. 6. Then as we increase the frequency, the velocity increases steadily to the point C. At this point the velocity drops suddenly in a jump to point E. From point E on the velocity steadily decreases. Suppose that we now start at F and decrease the frequency. The velocity steadily increases to the point D. At point D the velocity suddenly jumps to the point B. From point B on the velocity steadily decreases.

Typical experimental velocity frequency characteristics are shown in Fig. 7. The velocity frequency characteristic for an increase in frequency is shown in Fig. 7A. The velocity frequency characteristic for a decrease in frequency is shown in Fig. 7B. These characteristics are quite similar to the theoretical characteristic of Fig. 6.

SUBJECTIVE ASPECTS OF SOUND REPRODUCTION BY A LOUDSPEAKER WITH A NON-LINEAR CONE SUSPENSION SYSTEM

The principal source of distortion due to a non-linear suspension system occurs in small table model and personal receivers, because in these receivers a relatively small cone is driven through relatively large amplitudes. Furthermore, to prevent damage to the cone and voice coil assembly the suspension is designed so that the amplitude of the cone is limited. When these systems reproduce speech and music at high levels non-linear distortion of the ordinary overload type is produced. There is in addition a peculiar type of distortion due to instability of the vibrating system. When a signal of varying frequency which includes the unstable frequency region is applied to the loudspeaker, extraneous frequencies are produced. The extraneous frequencies are manifested as grunting sounds. The loudspeaker jumps from one stable characteristic to the other with production of the extraneous sound or distortion during the transition period.