

A Linear-Phase IIR Filter for Audio Signal Interpolator

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Abstract—In the paper infinite impulse response (IIR) digital filters with linear-phase (LF IIR) for audio signal interpolator are discussed. This article presents the causal realization of LF IIR and especially the realization of LF IIR filter with overlap is considered. As the LF IIR filters are an alternative for ordinary finite impulse response (FIR) filters, a comparison of a particular design of FIR and LF IIR filter is made. There are also compared realizations of signal interpolators using FIR filter and LF IIR filter.

I. INTRODUCTION

In many applications it is desirable to use a linear phase shift filter, a condition which is satisfied by the typical FIR filters. However, the FIR filters require very large orders (hundreds), which results in a very large computational load. Therefore it is desirable to use IIR filters with linear-phase shift. The aim of a linear-phase IIR filter (LF IIR) is to obtain higher computational efficiency than that offered by FIR filters at similar performance levels.

In recent years there has been a significant interest in real-time implementation of IIR filters with a linear phase. Powell and Chau [6] have invented an efficient method for the design and realization of real-time LF IIR filters using suitable modification of a well-known time reversing technique. This realization has been modified by Willson and Orchard [10]. Kurosu et. al reduce LF IIR filter delay [4]. Also, Azizi has patented a signal interpolator using a zero-phase filter [2], [1]. An interesting solution of LF IIR without overlap is presented by Mouffak and Belbachir [5].

The aim of this paper is to present a new audio signal interpolator using LF IIR filter. The interpolator is designed for Class-D audio power amplifier.

II. LINEAR-PHASE IIR FILTER

A method which allows for a simple implementation of the IIR filter with linear-phase shift is a method with time reversal [7], [3] for realization of a non-causal IIR filter $H(z^{-1})$. Fig. 1 shows the filter with linear-phase shift in cascade connection. In this filter input signal is passed through a filter $H(z)$, then the order of the samples is reversed and again passed through the filter $H(z)$, and then the order of samples is once again reversed. In the block diagram of a

filter running by this method (Fig. 1), the symbol TR denotes time reversal. In the TR circuit the order of the signal samples is reversed. Time reversal can be described by

$$x(nT) \longrightarrow x(-nT) \quad , \quad (1)$$

and for Z transformation

$$X(z) \longrightarrow X(z^{-1}) \quad . \quad (2)$$

The whole filtration process realized by circuit from Fig. 1 can be described by equations:

$$\text{causal filter} \quad Y_1(z) = X(z)H(z) \quad , \quad (3)$$

non-causal filter

$$\begin{aligned} Y_2(z) &= Y_1(z^{-1}) = X(z^{-1})H(z^{-1}) \quad , \\ Y_3(z) &= Y_2(z)H(z) = X(z^{-1})H(z^{-1})H(z) \quad , \\ Y(z) &= Y_3(z^{-1}) = X(z)H(z)H(z^{-1}) = \\ &= X(z)|H(e^{j\omega T})|^2 \quad . \end{aligned} \quad (4)$$

The resultant amplitude characteristic of the filter is squared $|H(e^{j\omega T})|^2$ and should be taken in account in the filter design process.

Of course, methods with time reversal cannot be performed in real time, because it is impossible to reverse the flow of time. The implementation of such a system is only possible if the signal is divided into blocks of samples. However, there is a problem with connecting blocks of signal samples in the output signal. The filter $H(z)$ transient effects generate amplitude distortion at the end of signal samples blocks. To avoid this distortion, an overlap technique with additional N_{ov} samples can be used [9]. In Fig. 2 a block diagram of such solution is presented. If transient effects are bothersome

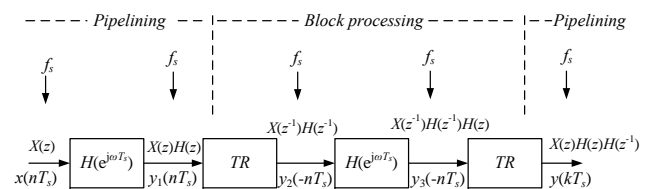


Fig. 1. The LF IIR filter

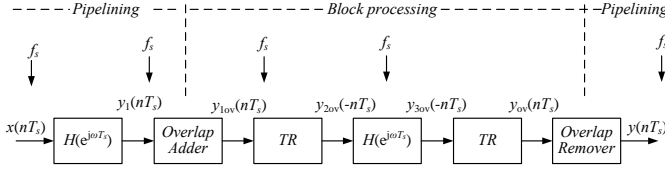


Fig. 2. The LF IIR filter with overlap

in a given application, discarding N_{ov} samples from the end of each signal block should be considered. The first $H(z)$ filter works continuously and the second filter uses a block of samples and it is reset before every block. Fig. 3 shows a realization diagram of such a LF IIR filter.

The LF IIR filter delay (N_d) in samples is equal to

$$N_d = 2(N + N_{ov}) \quad (5)$$

By selecting length of a block of signal samples N and length of overlap N_{ov} it is possible to choose quality of LF IIR filter. A good indicator of the filter quality is the signal-to-noise and distortion ratio ($SINAD$). In the next section is shown an example of a LF IIR filter. In Table I a comparison of proposed L -order LF IIR filter with L -order Powell and Chau LF IIR filter [6] is presented. Proposed filter has less delay and less multiplications and additions (MAC) per input sample.

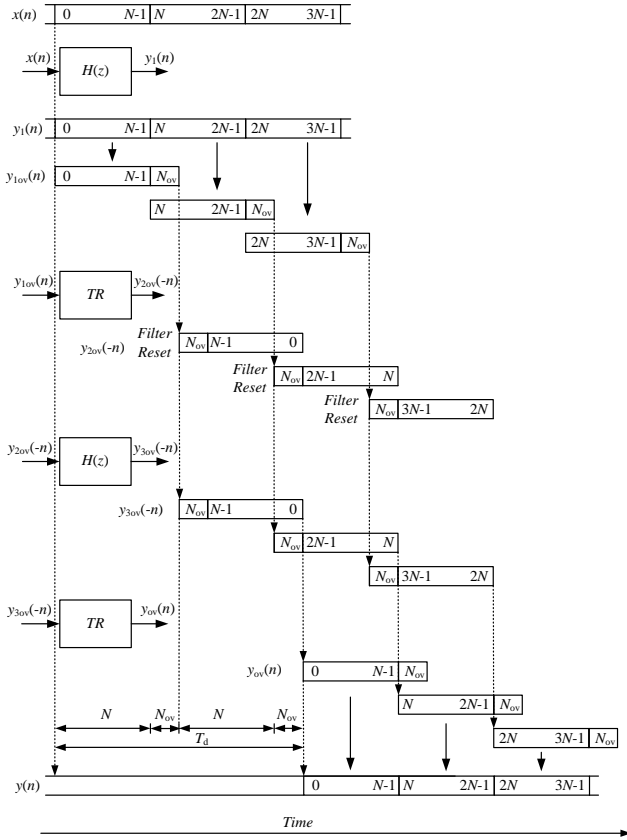


Fig. 3. Realization of LF IIR filter

TABLE I
COMPARISON OF LF IIR FILTERS REALIZATIONS

Filter type	Filter delay in sample N_d	Quantity of MAC operations per sample N_{MAC}
Powell and Chau	$3N$	$6L$
Proposed	$2(N + N_{ov})$	$2L(2 + N_{ov}/N)$

III. EXAMPLE OF A LINEAR-PHASE FILTER

As an example the author implemented an 8th-order IIR elliptic filter with crossing frequency $f_{cr} = 20$ kHz and sampling frequency $f_s = 352.8$ kHz. Input sinusoidal signal with frequency $f_{syg} = 19.98$ kHz is divided into blocks of length $N = 2048$ samples. That makes delay of output signal equal to $T_d = 2N/f_s = 11.69$ ms. Below is the author's Matlab listing for the realization of a LF IIR filter:

```
clear all;
fs=352.8e3; % sampling frequency
fb=20e3; % end of band of interest
fsyg=19e3; % signal frequency
ib=6; % number of blocks
N=2048; % length of block
Ns=N*ib; % length of input signal
t=(0:Ns-1)/fs; % time vector
y=zeros(1,Ns); %space for y
% coherent frequency of input signal
fsyg_k=round(fsyg/(fs/N))*fs/N;
x=sin(2*pi*fsyg_k*t); % input signal
% ----- Filter design -----
Fg=2*fb/fs;
[b a]=ellip(8,0.1,63,Fg,'low');
% ----- Causal filtering -----
y1=filter(b,a,x);
% Non-causal filtering, time reversing
N_ov=1024; % number of samples in overlap
y3=zeros(1,N+N_ov); %space for y3
for nb=1:ib-1
% time reversing and filtering
y3=filter(b,a,y1(nb*N+N_ov:-1:(nb-1)*N+1));
% time reversing
y3=y3(N+N_ov:-1:N_ov+1);
% output signal synthesis
y(1,(nb-1)*N+1:nb*N)=y3;
end
```

Fig. 4b shows results of connecting blocks of signal samples for LF IIR filter without overlap. The graph shows visible signal amplitude distortion associated with transient effects of the filter when connecting the blocks of signal samples. Using overlap as in Fig. 3, it is possible to reduce this distortion. In this particular case the overlap length used is equal to $N_{ov} = 1024$ samples. Fig. 4a shows results of connecting blocks of signal samples for LF IIR filter with overlap. In this case the output signal delay is longer

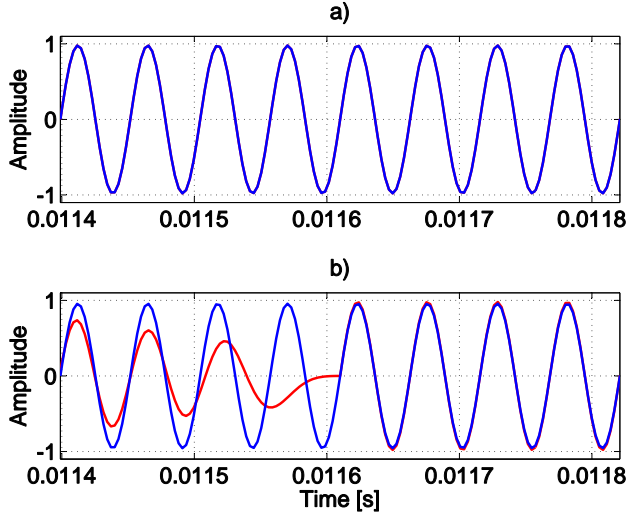


Fig. 4. Waveforms of linear-phase IIR filter output signal: a) with overlap $N_{ov}=1024$, b) without overlap

$T_d = 2(N + N_{ov})/f_s = 17.41$ ms. Additionally in Fig. 5 the difference between a reference signal and filter output signal for a filter with overlap and without overlap is shown.

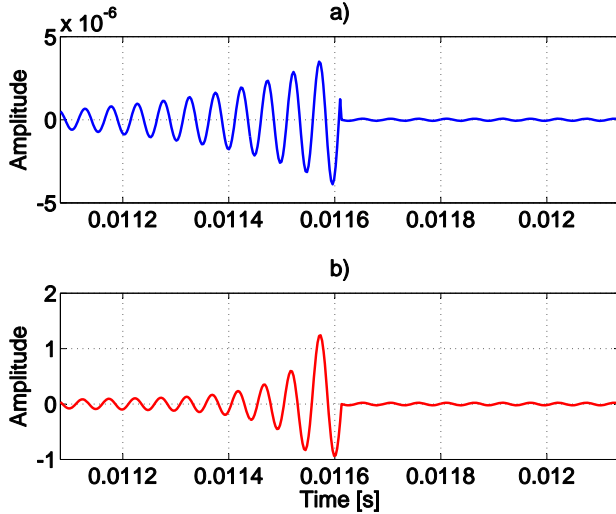


Fig. 5. Difference between reference signal and LF IIR filter output signal: a) with overlap $N_{ov}=1024$, b) without overlap

The illustration of the dynamic range of the signal is presented in Fig. 6, showing spectra of output signals and characteristics of the filter for both versions: with and without overlap. The input sinusoidal signal has unity amplitude and frequency $f_{syg} = 19.98$ kHz. The filter characteristic is a result of response of two IIR filters in cascade connection.

IV. COMPARISON OF FIR AND LF IIR

In order to make a sensible comparison, Parks-McClellan FIR filter with parameters similar to the LF IIR filter is

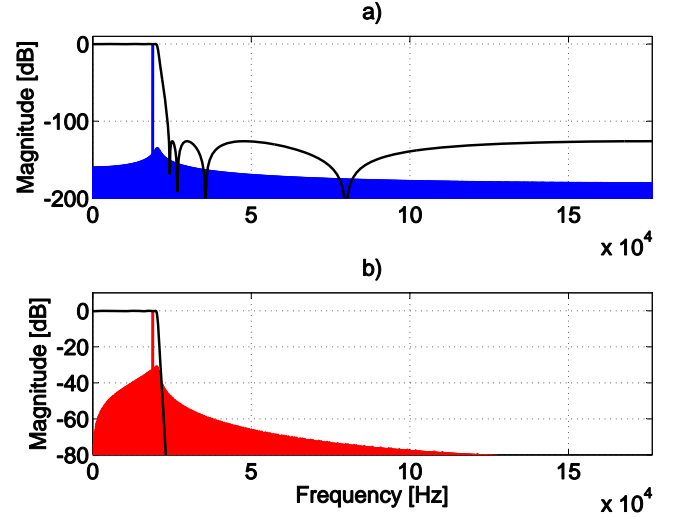


Fig. 6. Spectra of LF IIR filter output signal and filter characteristic: a) with overlap $N_{ov}=1024$, b) without overlap

designed. Design parameters of the two filters are shown in Table II.

TABLE II
FILTERS DESIGN PARAMETERS

Sampling rate	Pass-band frequency	Stop-band frequency	Pass-band ripple	Stop-band attenuation
f_s	f_p	f_z	A_p	A_z
352.8 kHz	20 kHz	24 kHz	0.1 dB	63 dB (IIR) 140 dB (FIR)

The frequency characteristics of designed filters are shown in Fig. 7. Filters were designed to achieve similar slope in transition band of amplitude characteristic.

Results of FIR filter and LF IIR filter design are shown in Table III. For the particular case FIR filter has 455th order and requires 456 multiplication and accumulation (MAC) operations per one input sample. Delay (N_d) for such a filter is equal to 228 samples. Solution with LF IIR filter requires two 8th order elliptic filters and 32+16 MAC operations per one input sample. The delay (N_d) is equal to 6144 samples.

It should be noted, however, that in the case of using an IIR filter LF, zero phase shift of the output signal is obtained.

V. SIGNAL INTERPOLATOR WITH LF IIR FILTER

In a typical signal interpolator a low-pass filter is used to suppress the aliasing components. The filter introduces a signal delay and when using IIR filter it has a nonlinear-phase response. Therefore, in order to obtain a linear-phase response, the FIR filter should be used. An alternative may be application of LF IIR filter. An example of such a solution is presented by Azizi [2], [1]. Also, the author has designed an interpolator using LF IIR filter. The interpolator is designed for class D

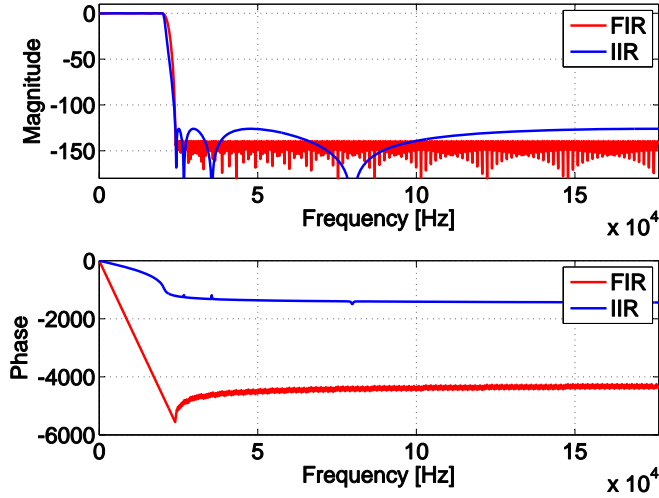


Fig. 7. FIR and IIR filters characteristics: a) amplitude response, b) phase response

TABLE III
COMPARISON OF FILTERS PARAMETERS

Filter type	Filter order	Filter delay in samples	MAC operations per sample
	L	N_d	N_{MAC}
Parks-McClellan FIR	455	228	456
Elliptic LF IIR $N=2048, N_{ov}=1024$	$2*8$	$4096+2048$	$32+16$

audio power amplifier using digital click modulator [8], [9]. The block diagram of a signal interpolator with the LF IIR filter proposed by the author is depicted in Fig. 8. The first filter works in pipeline mode and the second in block mode, similar as shown in Fig. 3. Number of MAC operations per

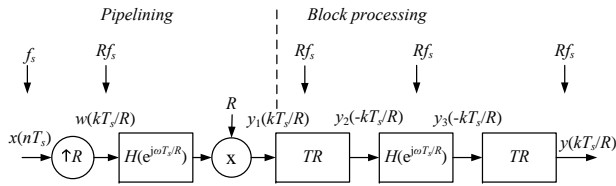


Fig. 8. Signal interpolator using LF IIR filter

one sample for signal intergrator using L -order LF IIR filter

$$N_{MAC} = 2RL(2 + N_{ov}/N) \quad (6)$$

where: R - signal oversampling ratio.

VI. COMPARISON OF INTERPOLATOR WITH FIR AND LF IIR FILTER

In order to compare the signal interpolators, was performed investigation using the FIR filter and IIR filter LF. The

oversampling ratio is equal $R = 8$, sampling ratio is typical for audio signal $f_s = 44.1$ kHz, end of signal band is equal to $f_b = 20$ kHz. Parameters of filters are as shown in Table II and III. The FIR filter order is $L = 455$, So interpolator requires $R(L + 1) = 3648$ multiplications and additions per one input sample, which is a huge amount of MAC operations.

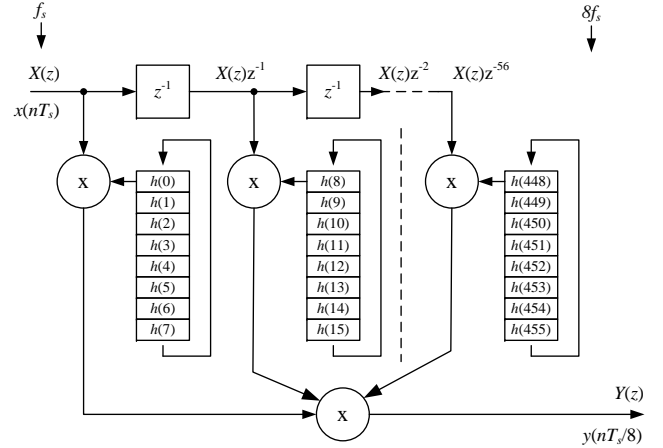


Fig. 9. Block diagram of FIR based signal interpolator for $R = 8$ with periodically switched coefficients and filter order $L = 455$

It is possible to decrease the quantity of arithmetic calculation by elimination of the multiplication and addition operations for zero value samples. In the case of applying a FIR filter it is very efficient to implement the system using polyphase circuit. The block diagram of such solution is shown in Fig. 9. This is a FIR based signal interpolator for $R = 8$ with periodically switched coefficients and filter order $L = 455$. In this case the interpolator requires $L + 1$ multiplication and addition operations per one input signal sample. This kind of filter structure is easily and efficiently realized by the DSP.

This FIR filter based interpolator is compared with interpolator based on LF IIR filter as shown in Fig. 8. Spectra of interpolators signals for $f_{sig} = 19.98$ kHz sinusoidal signal are shown in Fig. 10. Figure 10a shows spectrum of input signal after upsampler, Fig. 10b shows spectrum of FIR interpolator output signal and Fig. 10c shows spectrum of LF IIR interpolator output signal. Spectra of output signal of both interpolators are very similar.

In Tab. IV are compared parameters of both interpolators. Interpolator with LF IIR filter requires a little less MAC operations than with FIR filter. However, an additional the advantage of this solution is achieving a zero phase shift of the output signal.

VII. CONCLUSION

The LF IIR filters are suitable for applications where a linear-phase is required and long delay time is acceptable.

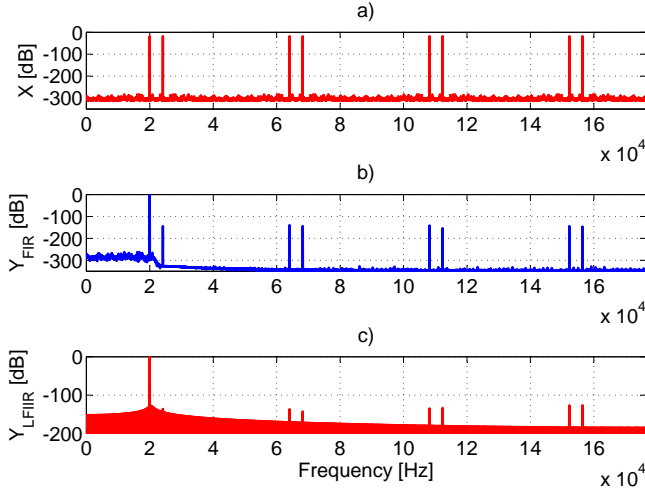


Fig. 10. Spectra of interpolator signals: a) upsampled input signal, b) with FIR filter, c) with LF IIR filter

TABLE IV
COMPARISON OF INTERPOLATORS PARAMETERS

Interpolator type	Filter order	Quantity of MAC operations per sample
	L	N_{MAC}
Parks-McClellan FIR	455	456
Elliptic LF IIR $N=2048, N_{ov}=1024$	$2*8$	320

Much smaller number of arithmetic operations in comparison to FIR filters is an undeniable advantage. The LF IIR filters are suitable especially for audio applications where long delay time is typically acceptable.

By selecting the length of the overlap, it is possible to choose a compromise between the quality of the output signal and the length of additional signal delay equal to $2N_{ov}$.

The LF IIR filter are also suitable for interpolator, however in comparison with very efficient polyphase FIR filter, difference in quantity of MAC operations is not too significant.

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