

Operational Amplifier Stability
Part 11 of 15: Modeling Complex Zo for Op Amps
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Part 11 of this series will venture into the world of complex Zo inside operational amplifiers. As single supply applications of operational amplifiers have become more predominant at the board and systems level, semiconductor manufacturers' have been challenged to create unique op amp topologies to provide rail-to-rail inputs and outputs along with high open loop gains for accuracy and low noise on an ever-dwindling signal range (i.e. from +/-10V down to 0-5V down to 0-3V). These new and unique op amp topologies produce some very interesting and unique Zo characteristics which need to be understood and modeled, in order to guarantee, by design, a stable circuit when driving reactive loads. We will look at a novel approach to modeling the complex output impedance of op amps. The building of an op amp Zo Block will allow us to move the op amp Zo outside of the traditional op amp SPICE macromodel and thereby separate the Aol curve from the effects of Zo interacting with reactive loads on the output. This will allow us, in a future article, to simplify the challenges of stabilizing op amps with complex Zo characteristics.

As with any engineering problem there is more than one possible solution. One may be tempted to construct a Zo Block from only passive components (inductors, resistors, capacitors). That solution has been researched and proven to not be so good as undesired peaking occurs in the frequency areas of L-C resonance. A real op Zo will have smooth, non-resonant frequency transitions like the model in this article produces. The technique for building an external Zo block in SPICE is rather straight-forward and easy to build yielding acceptable results in the minimum amount of time.

One final prologue before we head to the Zo Block. TINA SPICE, used extensively in this article series, as a SPICE simulator has a very good convergence engine. Some other SPICE simulators are not as forgiving and sometimes require either scaling down of large value reactive components or setting of specific Option parameters to easily converge. Our goal in this article is to divulge a quick way to measure Zo on a SPICE macromodel and show the blueprint for an external Zo Block for SPICE stability analysis. If the recommended circuits below cause other simulators to operate a bit rough then re-scaling of reactive components may be necessary.

By now we have realized, through this article series, that the only thing we need to know about an op amp to solve any op amp stability problem is the Aol curve and the Zo characteristics (see Figure 11.1). The Zload will be determined by system demands and we will tailor 1/Beta for stability.

Given: A_{ol} , Z_o , β , Z_{load}

Find: Solution to any op amp stability problem

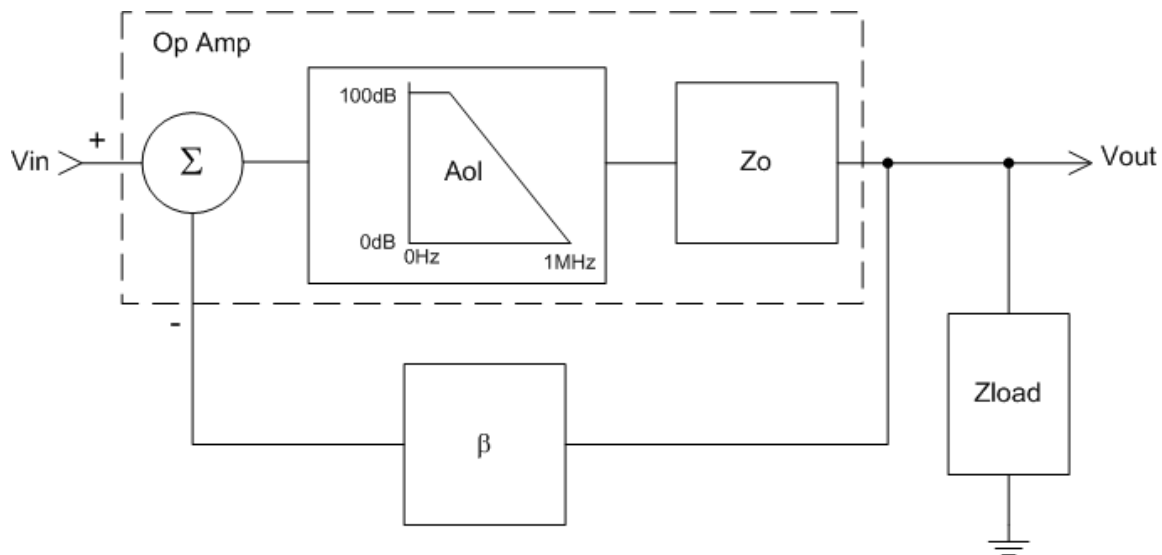


Fig. 11.1: All You Need to Solve Op Amp Stability Problems

To empower us to solve stability problems, when we use op amps with complex Z_o characteristics, we will want to move the Z_o characteristic of the op amp macromodel outside of the op amp (see Figure 11.2). This will be accomplished by creating a standalone Z_o block which will be isolated from the original macromodel by a voltage-controlled-voltage-source with a gain equal to 1. The voltage-controlled-voltage-source acts as an ideal isolation amplifier with a gain of 1 and retains the original macromodel A_{ol} curve and presents no interaction between the op amp macromodel output and the external Z_o block. The final result will be an A_{ol} curve and a Z_o Block with capability of measurement between the A_{ol} curve and Z_o Block.

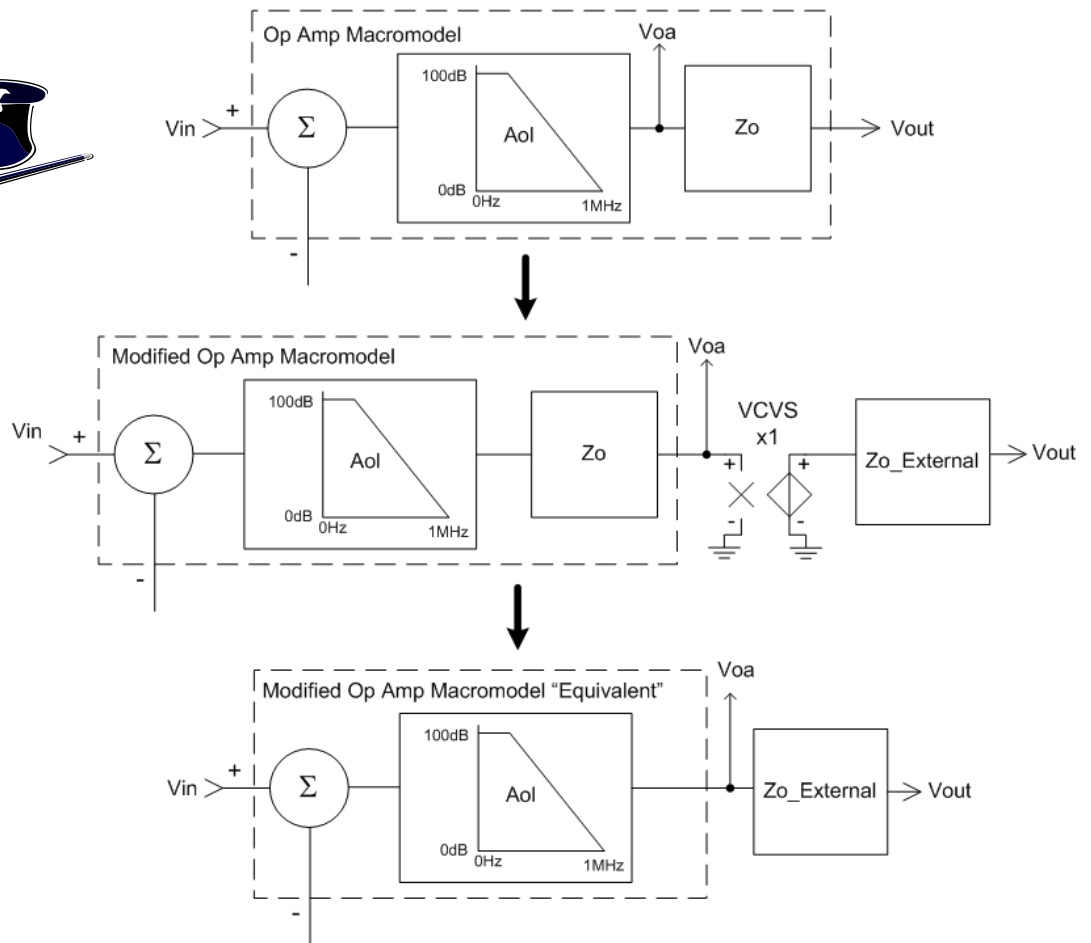


Fig. 11.2: Moving Z_o Outside of the Op Amp Macromodel

Recall from “Part 3 of 15: R_O and R_{OUT} ” how R_O and R_{OUT} are related. R_{OUT} is R_O reduced by loop gain. Figure 11.3 will define the op amp model used for the derivation of R_{OUT} from R_O . This simplified op amp model focuses solely on the basic DC characteristics of an op amp. A high input resistance ($100\text{M}\Omega$ to $\text{G}\Omega$), R_{DIFF} develops an error voltage across it, V_E , due to the voltage differences between $-IN$ and $+IN$. The error voltage, V_E , is amplified by the open loop gain factor A_{ol} and becomes V_O . In series with V_O to the output, V_{OUT} , is R_O , the open loop output resistance. The resultant relationship between R_{OUT} and R_O is shown above with the detailed derivation in this article’s Appendix. We will use the same terminology for complex op amp output impedances. That is Z_o is open loop output impedance and Z_{out} is closed loop output impedance.

R_O = Op Amp **Open Loop** Output Resistance
 R_{OUT} = Op Amp **Closed Loop** Output Resistance

$$R_{OUT} = R_O / (1 + A_{OL}\beta)$$

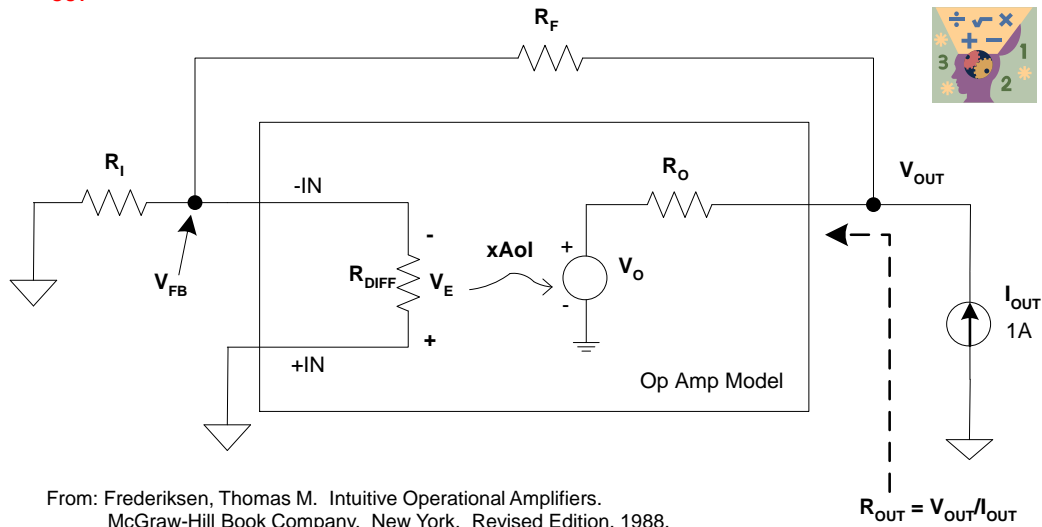


Fig. 11.3: R_O and R_{OUT}

Our op amp of choice for complex Z_o analysis and external Z_o Block build is a CMOS RRIO op amp with specifications as detailed in Fig 11.4. The OPA376 is a low quiescent current (950uA) op amp optimized for single supply operation (2.7V to 5.5V) with beyond rail-to-rail input (greater than 0.1V beyond either supply) and rail-to-rail output ($V_{sat} = 20mV$ @ $I_{out} = 254.8uA$). The OPA376 will also provide output current of 2.7mA at a saturation voltage of 50mV max. In addition the OPA376 has a wide bandwidth of 5.5MHz and a slew rate of 2V/us.

OPA376

Low-Noise, Low Quiescent Current, Precision Operational Amplifier

Input Specs

| | |
|-----------------------------|------------------------|
| Offset Voltage | 25uV max |
| Offset Drift | 1uV/C |
| Input Voltage Range | (V-)-0.1V to (V+)+0.1V |
| Common-Mode Rejection Ratio | 90dB typ |
| Input Bias Current | 10pA max |

Noise

| | |
|-----------------------------|--------------------------|
| Input Voltage Noise | 0.8uVpp, f=0.1Hz to 10Hz |
| Input Voltage Noise Density | 7.5nV/rt-Hz @1kHz |
| Input Current Noise Density | 2fA/rt-Hz |

Output Specs

| | |
|--------------------------------|-----------|
| V_{sat} @ $I_{out} = 54.8uA$ | 20mV max |
| V_{sat} @ $I_{out} = 2.7mA$ | 50m max |
| I_{out} Short Circuit | +30/-50mA |

AC Specs

| | |
|-----------------------------------|------------------|
| Open Loop Gain, $R_L = 10k$ | 134dB typ |
| Open Loop Gain, $R_L = 2k$ | 126dB typ |
| Gain Bandwidth Product | 5.5 MHz |
| Slew Rate | 2V/us |
| Overload Recovery Time | 0.33us |
| Total Harmonic Distortion + Noise | 0.00327%, f=1kHz |
| Settling Time, 0.01% | 2us |

Supply Specs

| | |
|-------------------------|--------------|
| Specified Voltage Range | 2.5V to 5.5V |
| Quiescent Current | 950uA max |
| Over Temperature | 1mA max |

Temperature & Package

| | |
|-----------------|-----------------------|
| Operating Range | -40C to +125C |
| Package options | SC70-5, SOT23-5, SO-8 |

Fig. 11.4: OPA376 Op Amp for External Z_o Block Build

The OPA376 contains a Z_o Curve (Open-Loop Output Resistance vs Frequency Curve) shown in Figure 11.5. This is an exception to the rule, as most op amp data sheets contain only a Closed Loop Output Resistance vs Frequency Curve. It is very difficult to extract a complex Z_o from a closed loop output impedance curve. Fortunately for us, some sort of “ Z_o Wizard” must have had influence on the OPA376 datasheet to include this most valuable curve. Notice that Z_o changes with DC Load current. It will almost always get lower as DC load current increases. Since circuits we design must be stable under all operating conditions we will choose to use the most lightly loaded, or “unloaded” Z_o curve since it will, as a rule of thumb, result in the worst case stability condition especially when driving capacitive loads.

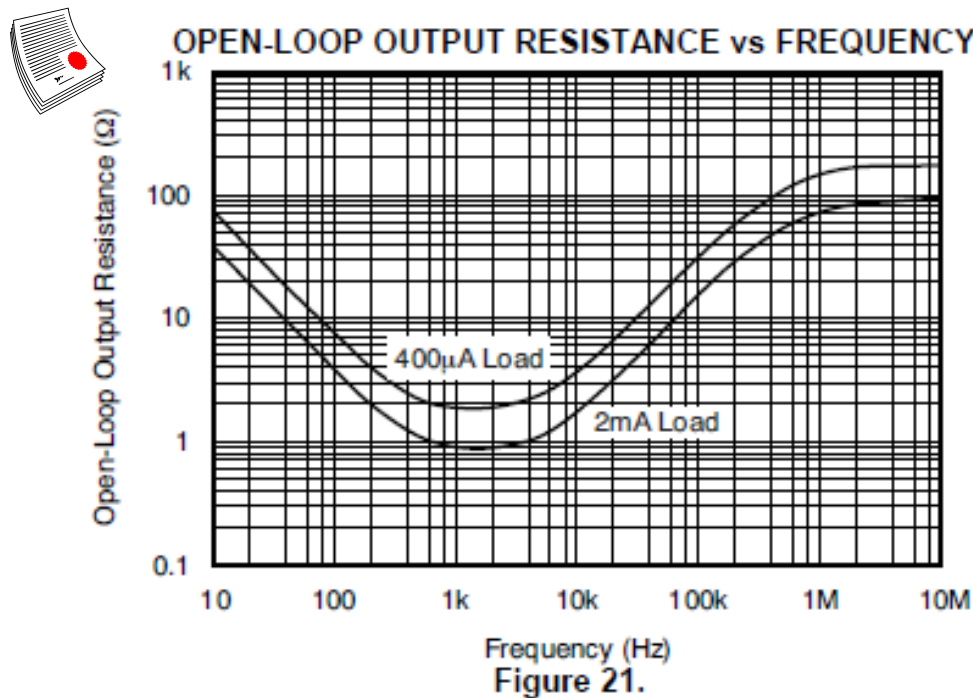


Fig. 11.5: OPA376 Op Amp Data Sheet Z_o Curve

An important tool in our op amp stability toolbox will be a way to measure Z_o on a SPICE op amp macromodel. This measurement will enable us to ensure the op amp macromodel matches either the data sheet Z_o curve or measured Z_o results. With regards to measuring Z_o on an op amp – do not try this at home as it is best left to trained professionals. It requires a gain-phase analyzer, custom circuits and custom software to yield 1kHz to 10MHz accurate measurements. Better to demand the Z_o curve from your semiconductor op amp manufacturer. We can use IC simulated results if real measurements are not available. Comparing the results of a Z_o measured result on a SPICE macromodel to other data sheet characteristics of the op amp can tell us if the op amp macromodel Z_o is believable. The details of this comparison are beyond our immediate focus of this article. Our first step in measuring Z_o is to measure A_{ol} . For single supply op amps we will run all of our AC Tests using dual supplies to eliminate any common mode issue on the input and to eliminate any negative input offset voltages from trying to drive the output below ground (if we used single supply) and saturating the output devices which will not give an accurate AC result. For the A_{ol} Test circuit in Figure 11.6 the inductor, L_T , will act as a short at DC and an open for any frequencies of interest. C_1 will act as an open at DC and as a short for any frequency of interest. R_L can be adjusted to check A_{ol} at different DC load currents if desired by adjusting V_L . Before we run any AC analysis we should run a DC Analysis to ensure the op amp is in a linear region of operation or our AC results will not be valid. In Figure 11.6 we see the op amp output at

-25.38uV for a DC Analysis and thus we are in a linear region of operation (output is not saturated to either rail).

For most single supply op amp AC tests:

Run dual supply to

- eliminate input common mode violations
- eliminate saturation of output devices with zero input times closed loop in DC Analysis (i.e. negative V_{os} trying to driver output less than ground)

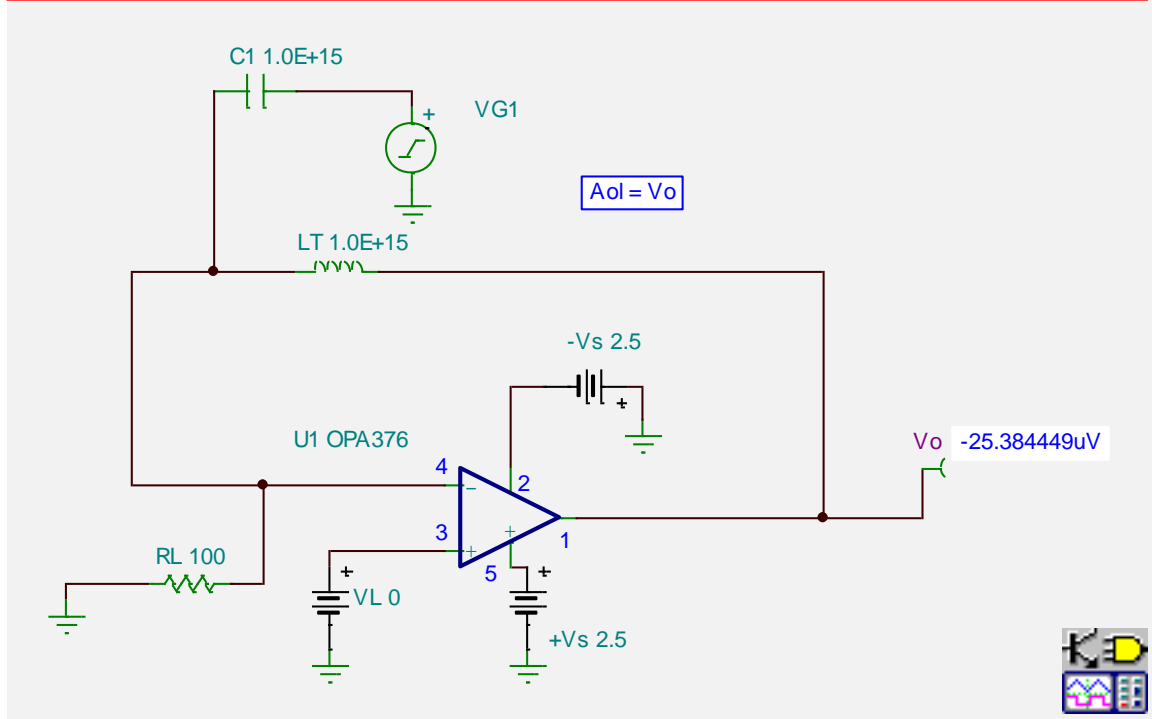


Fig. 11.6: Measuring Z_o in SPICE: Step 1 – Measure A_{ol}

The TINA SPICE results of our A_{ol} Test are shown in Figure 11.7 and we see a low frequency A_{ol} gain of 144.13dB or 16.0879MV/V. We also observe a low frequency pole in the A_{ol} at about 400mHz.

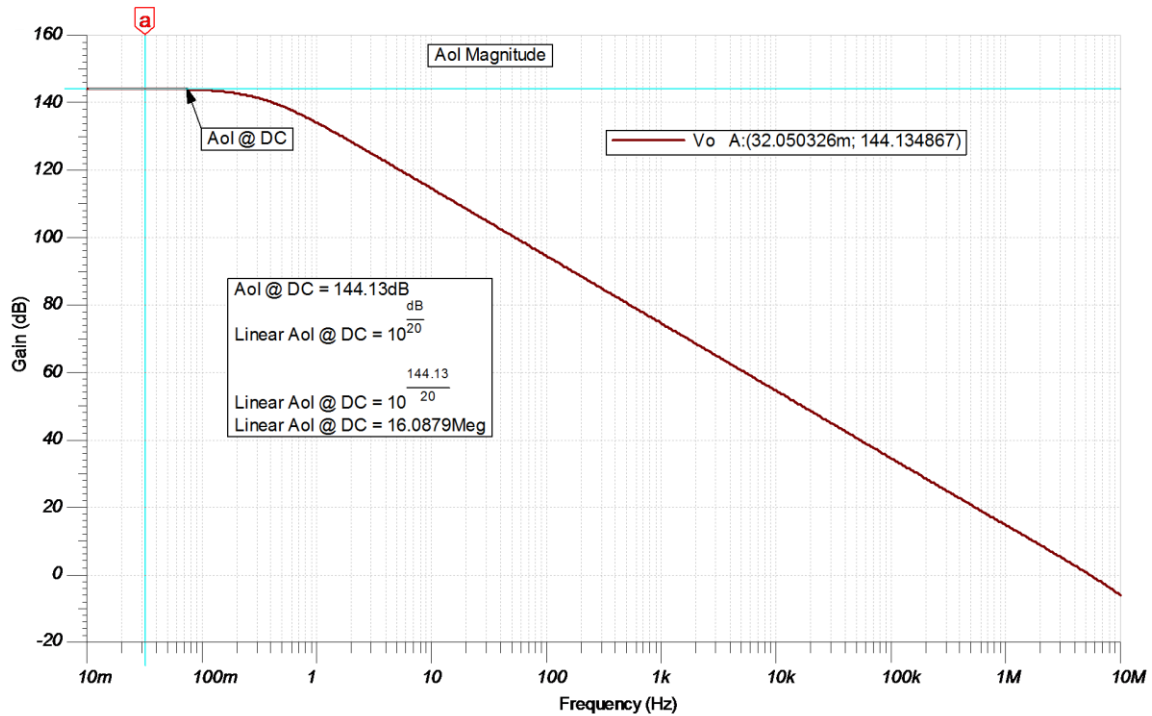


Fig. 11.7: Measuring Zo in SPICE: Step 1 – Measure Aol Results

The test circuit in Figure 11.8 will be constructed to measure Zo once we know the Aol of the amplifier. We want to design the closed loop gain of our Zo test circuit to be greater than the Aol of the amplifier for all frequencies of interest. Such a design will guarantee that when we test for Zo it will be Zo and not Zout. Remember that AolBeta on a dB plot is Aol(dB) – 1/Beta(dB). So if 1/Beta is larger than Aol we will have AolBeta = -?dB or a very small number. Then from $Z_{out} = Z_o / (1 + Aol\beta)$, for $Aol\beta < 0.1$ (or -20dB), Z_{out} approaches Z_o .

Set $R_L = 100$ ohms;
 Low enough for no bias issues and low enough for no Cin issues
 Set Closed Loop Gain = $10 \times (Aol @ DC)$;
 ensure op amp will run in open loop for frequencies of interest
 $R_F = R_L \cdot 10 \cdot (Aol @ DC)$
 $R_F = 100 \cdot 10 \cdot 16.0879M = 16.0879G$
 Select LT for the low est frequency of interest (fz)
 $f_z = \frac{R_F}{LT \cdot 2 \cdot \pi}$
 For our example choose $f_z = 10\mu Hz$
 $f_z = \frac{R_F}{LT \cdot 2 \cdot \pi}$ implies $LT = \frac{R_F}{f_z \cdot 2 \cdot \pi}$
 $LT = \frac{16.0879G}{10\mu Hz \cdot 2 \cdot \pi} = 2.56e14 = 256TH$

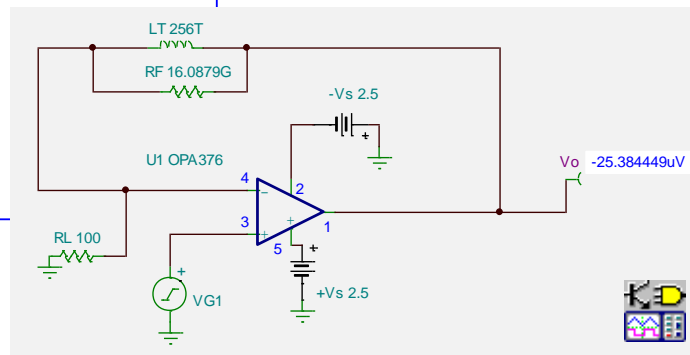


Fig. 11.8: Measuring Zo in SPICE: Step 1 – Configure Closed Loop Gain

Using the design criteria in Figure 11.8 we build a Zo Test Circuit and the TINA SPICE simulation results in Figure 11.9 show us the closed loop gain.

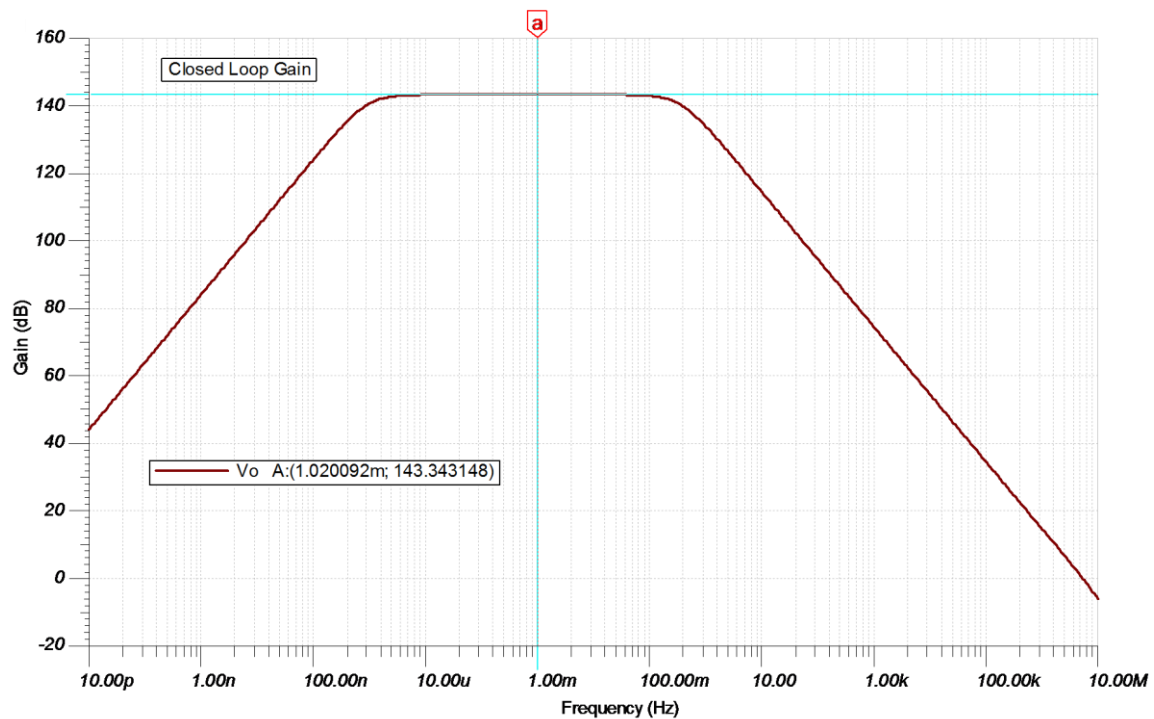


Fig. 11.9: Measuring Zo in SPICE: Step 1 – Configure Closed Loop Gain Results

To check our design procedure for the Zo Test Circuit we will use the circuit in Figure 11.10 to measure $1/\text{Beta}$.

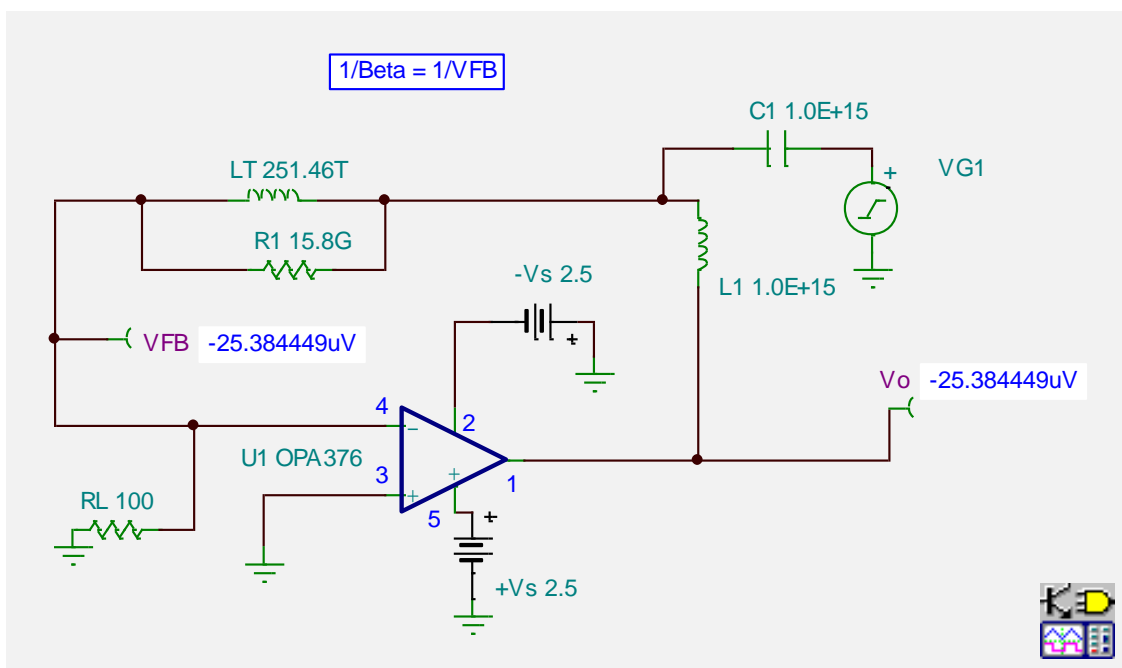


Fig. 11.10: Measuring Zo in SPICE: Step 3 – $1/\text{Beta}$ Test

1/Beta is seen in Figure 11.11 to be greater than our Aol at DC by at least 20dB.

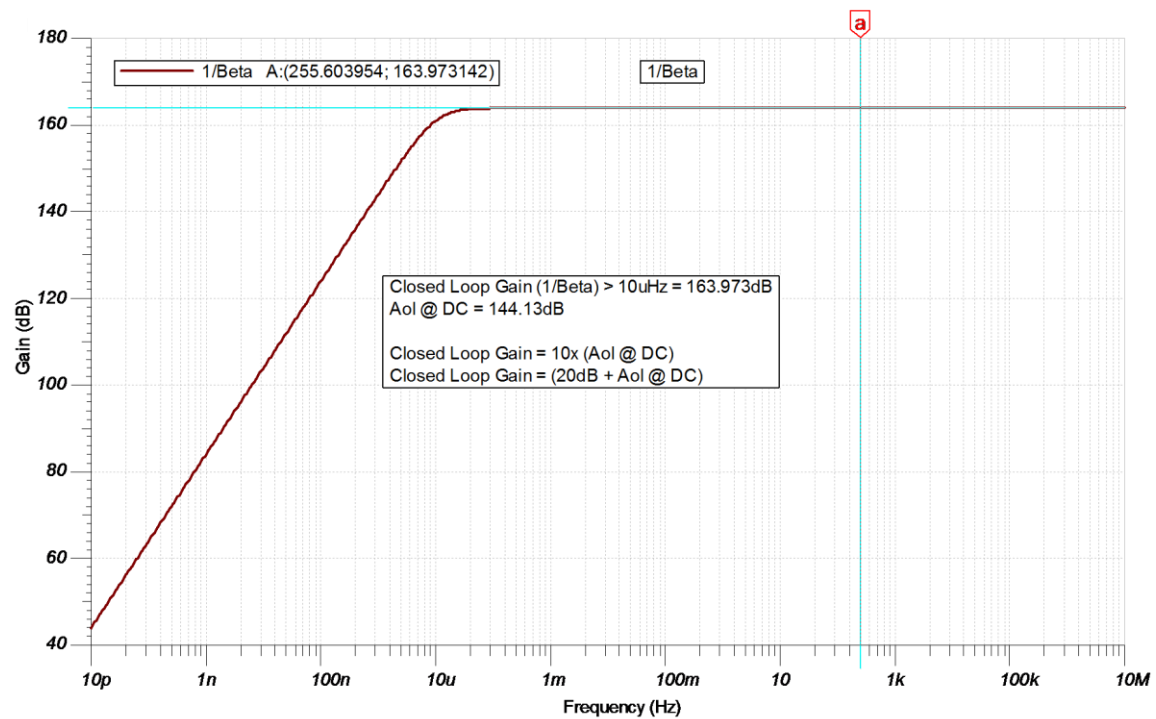


Fig. 11.11: Measuring Zo in SPICE: Step 3 – 1/Beta Test Results

In Figure 11.12 the Zo Test Circuit responses are all plotted on one plot. We see the op amp Aol and our 1/Beta plot that yield the desired closed loop gain we originally designed for. From this figure above we see that any Zo measurements less than 300nHz will not be valid since Aol Beta will approach 0db (or 1V/V) and Zout will not be Zo.

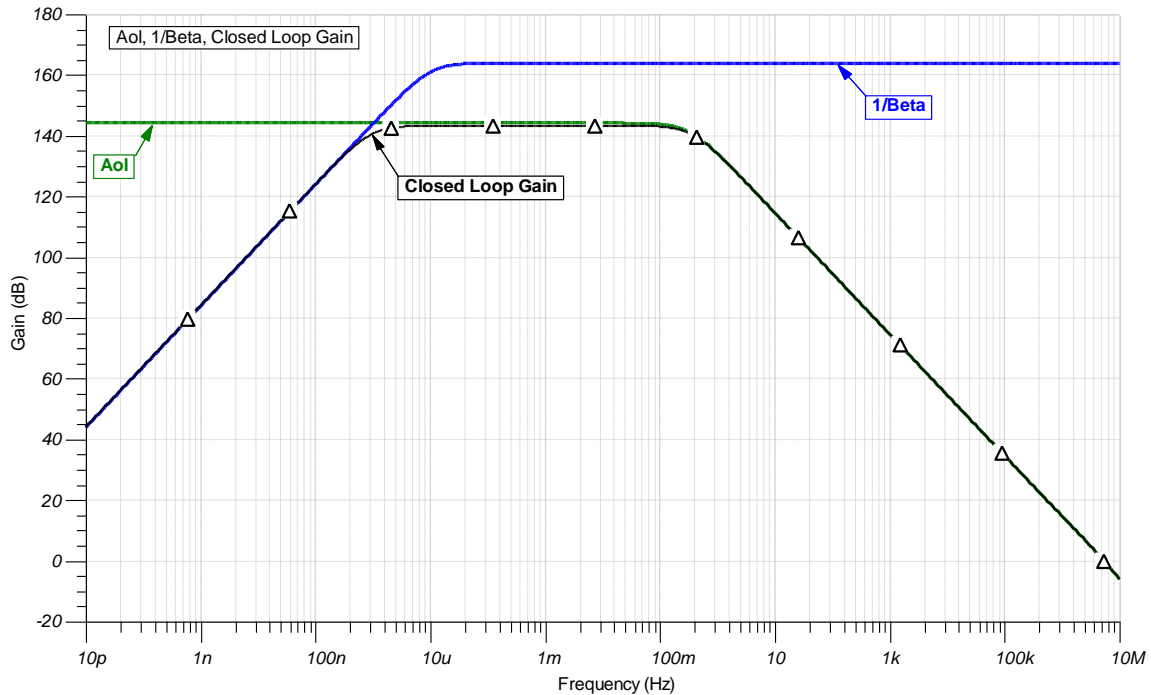


Fig. 11.12: Measuring Zo in SPICE: Step 4 – Aol, 1/Beta, Closed Loop Gain

In Figure 11.13 our final Zo test circuit will use our closed loop gain designed in previous slides to force the op amp to run open loop for test frequencies of interest. We will attach a current generator, IT, on the output of the op amp. The current generator will be set to 0A at DC. This will not affect the DC operating point found by DC Analysis since a 0A DC current source is high impedance by definition. The current generator, IT, will be swept over frequency in an AC Analysis to test for Zo impedance over frequency by dividing Vo by IT.

Op Amp Zo Test

$$Z_o = V_o / I_T$$

Scale Logarithmic to remove $20 \cdot \log(V_o / I_T)$

Log scale $\rightarrow Z_o = V_o$ in ohms

Loaded Zo Test:

$$I_{dc} = V_L / R_L$$

Test Loaded Zo for both +Idc and - Idc

Unloaded Zo Test:

Set $V_L = 0V$

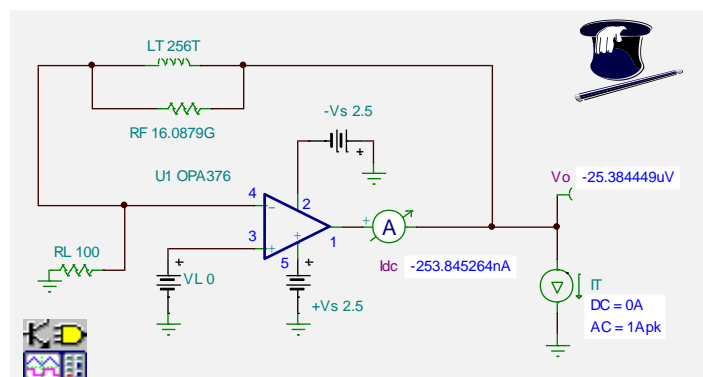


Fig. 11.13: Measuring Zo in SPICE: Step 5 – Final Zo Test Circuit

The results of measuring the OPA376 Zo are shown in this Figure 11.14. Note that we recall any frequencies below f_x (about 300nHz) we do not know Zo since $Aol/Beta$ is not $< -20dB$ (or < 0.1). The results above on the Y-axis labeled Gain(dB) is really V_o/I_T in dB, or Zo.

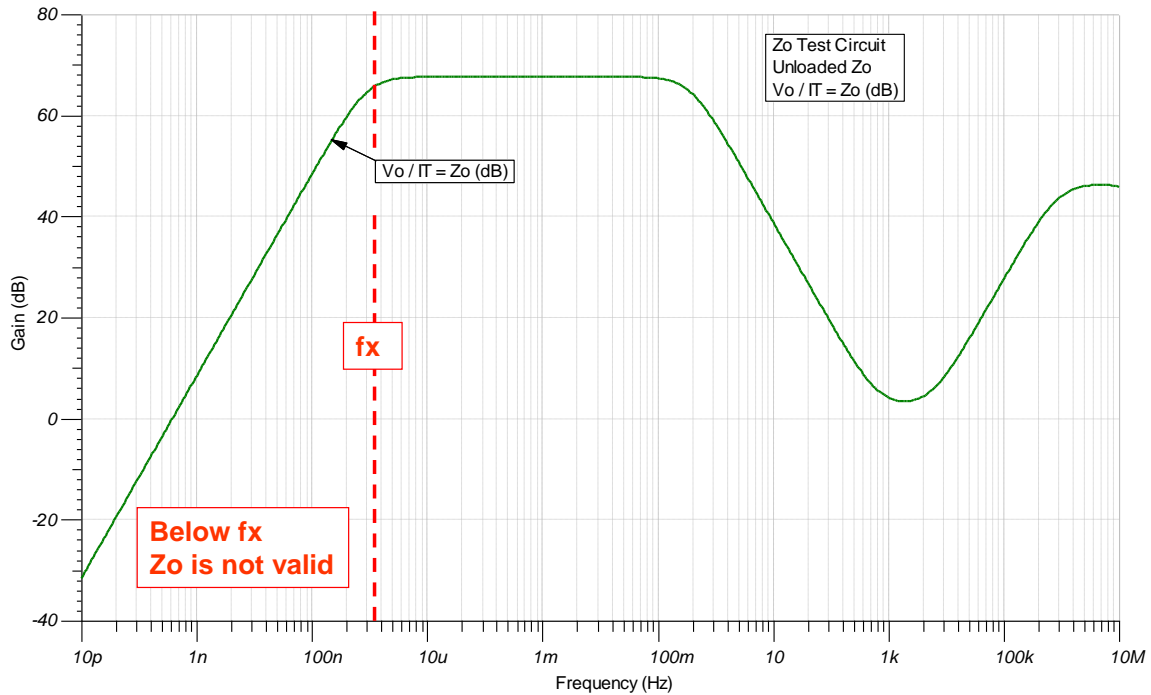


Fig. 11.14: Measuring Zo in SPICE: Step 5 – Final Zo Test Circuit Results (dB)

An easy way to convert our dB AC Analysis results into Zo in ohms is simply to change the Y-axis scaling in SPICE to Logarithmic which will yield Zo in ohms since the Y-axis is the results of V_o/I_T over frequency. The result of our OPA376 Zo Test in Figure 11.15 show that its Zo (starting at 300nHz and going up in frequency) is resistive, then capacitive, then resistive, then inductive then resistive. Not looking much like a simple R_o is it?

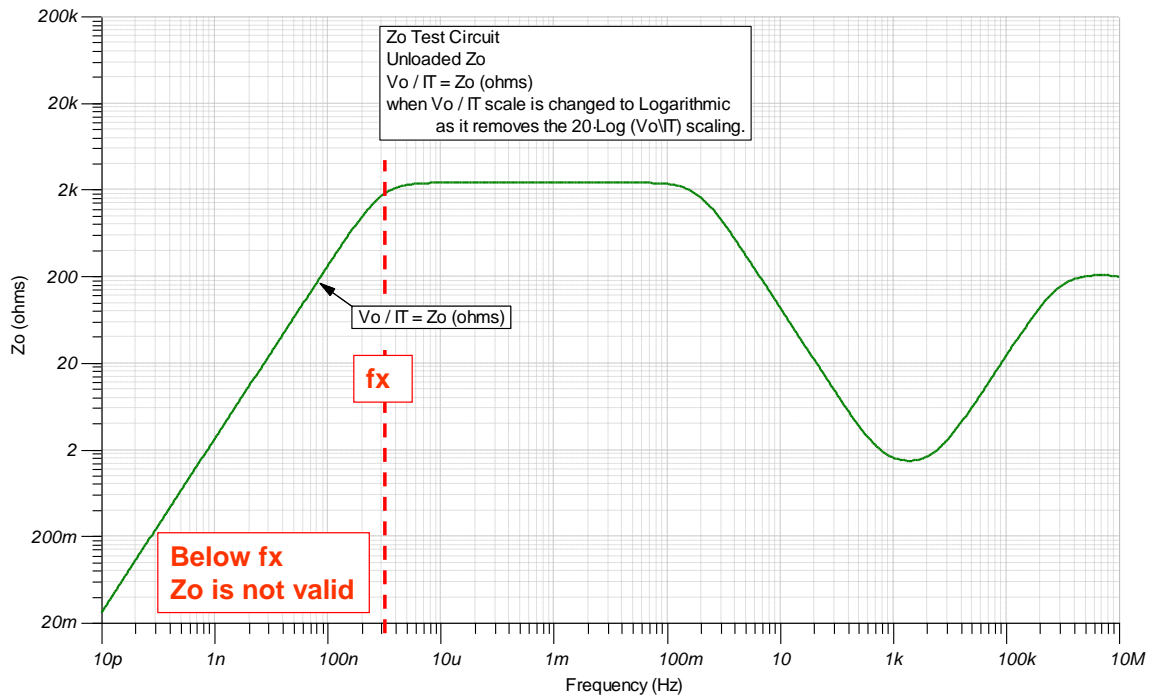


Fig. 11.15: Measuring Zo in SPICE: Step 5 – Final Zo Test Circuit Results (ohms)

For building our Zo Block we will first need frequency breakpoints from our measured Zo. These breakpoints are easily measured by leaving the AC Analysis test results in dB format and measuring for the 3dB points at all frequency transitions as shown in Figure 11.16.

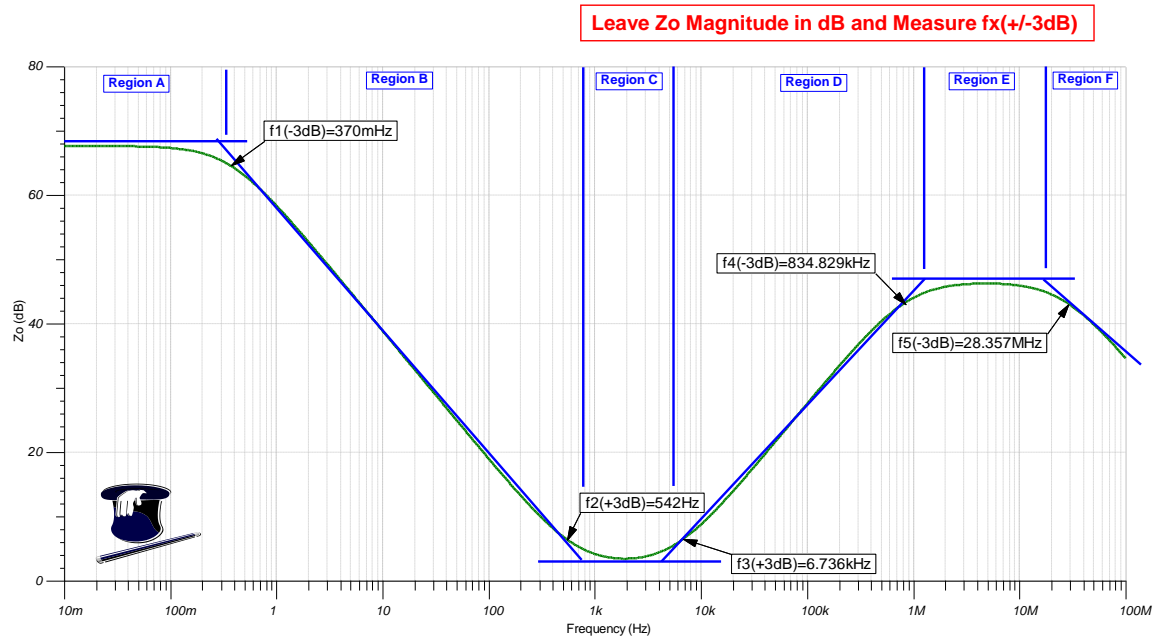


Fig. 11.16: Zo Regions and Frequency Breakpoints

Secondly, to build our Zo block we will need magnitudes in any region where the Zo magnitude is flat. To easily get the magnitudes convert the Y-axis into Logarithmic and measure each region as shown in Figure 11.17.

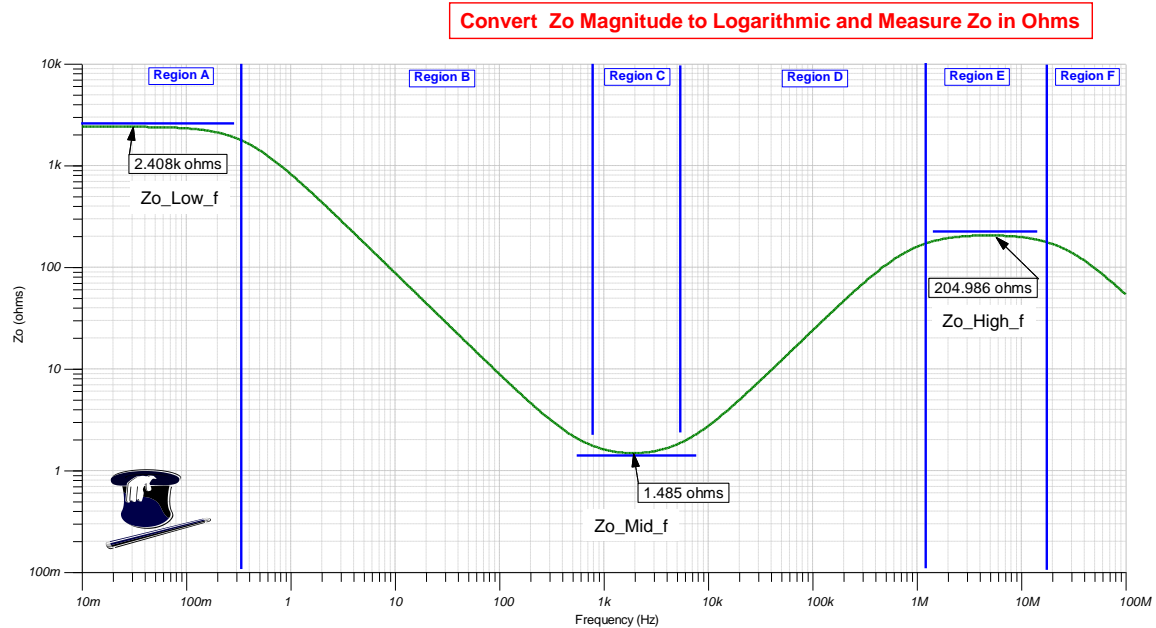


Fig. 11.17: Zo Regions and Magnitudes

The Zo Measurement Steps and results for the OPA376 are shown in Figure 11.18. Now we are ready to proceed forward in building the external Zo block.

Zo Measurement Steps:



- 1) Measure Zo in dB to get frequency breakpoints
- 2) Convert to ohms to get magnitudes in flat regions
- 3) Now ready to build Zo_External?

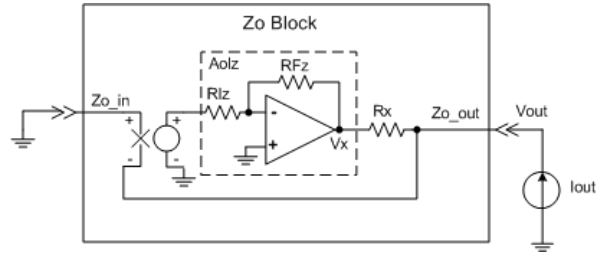
| Summary of Zo Measurements | | | |
|----------------------------|----------|----------|-------------------------------------|
| Region | Freq Min | Freq Max | Zo Magnitude |
| | (Hz) | (Hz) | (ohms) |
| A | 37m | 370m | 2.408k |
| B | 370m | 542 | x1/10 decrease per decade frequency |
| C | 542 | 6.736k | 1.485 |
| D | 6.736k | 834.829k | x10 increase per decade frequency |
| E | 834.829k | 28.357M | 204.986 |
| F | 28.357M | 28.357M | x1/10 decrease per decade frequency |

Fig. 11.18: Zo Measurement Steps

There is probably more than one way to design an external Zo Block. Real inductors, capacitors, resistors in combination do not work easily since their combinations will cause peaking at resonances which do not exist in the real op amp Zo. The technique shown in Figure 11.19 is simple and straight forward to use once it is understood. A fixed Rx will be used inside the closed loop of an op amp whose Aol (Aolz in Figure 11.19) will be designed to create a Closed Loop Zout looking back into the op amp with feedback around Rx. The varying shape of the op amp Aol curve will yield the varying shape of the measured Zout of the configuration which will then be used as a Zo block. In Figure 11.19 we are measuring the Zo Block from its output and so we will call this "Zo_reverse". Since this arrangement is identical to our familiar "Rout vs Ro Derivation" we know what Zo_reverse is as shown in Figure 11.19. Note that if there is a ZL on the output of the Zo Block then the Zo_reverse will be Zo in parallel with this ZL. If $ZL \gg Zo_{rev}$ then $Zo_{rev} = Rx/(1+Aolz)$.

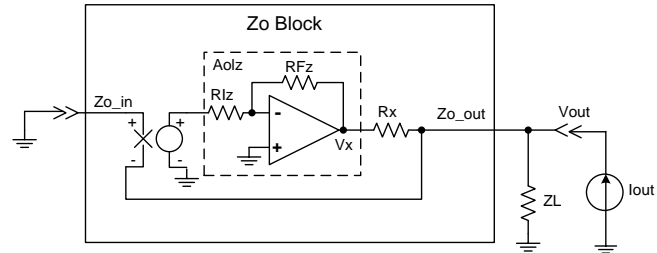
Zo_reverse without ZL

$$Z_{o_rev} = \frac{R_x}{1 + A_{olz}}$$



Zo_reverse with ZL

$$Z_{o_rev_ZL} = \frac{(Z_{o_rev} \cdot Z_L)}{Z_{o_rev} + Z_L}$$



Note for Zo_rev_ZL :

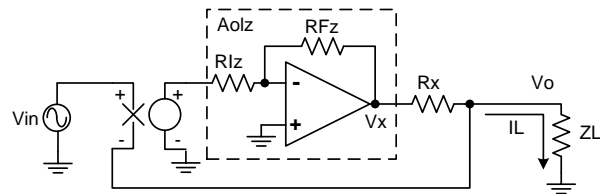
If $Z_L \gg Z_{o_rev}$; Then $Z_{o_rev_ZL} = Z_{o_rev} = R_x / (1 + A_{olz})$

Fig. 11.19: Zo “Reverse” Equations

For the Zo Block to give accurate simulation result for stability analysis the Zo must look the same in both directions. That is Zo_forward must be the same as Zo_reverse. Zo Forward is defined in Figure 11.20. The derivation for Zo_fwd is given in the Appendix of this article. Zo_fwd is whatever Vin is put into the Zo block divided by the current, IL, which comes out of the block as shown above. Note the effect of ZL on Zo_fwd. If $Z_L \ll R_x$ then $Z_{o_fwd} = R_x / A_{olz}$.

Zo_forward with ZL

$$Z_{o_fwd} = \frac{R_x + Z_L}{A_{olz}}$$



$$Z_{o_fwd} = (V_{in} - V_o) / I_L$$

Note:

If $Z_L \gg R_x$ then Z_{o_fwd} is dominated by Z_L . Z_L can be very large for a capacitive load in the middle to low frequency regions. This will yield erroneous stability analysis for Zo interacting with a capacitive load.

Note for Zo_fwd:

If $Z_L \ll R_x$; Then $Z_{o_fwd} = R_x / A_{olz}$

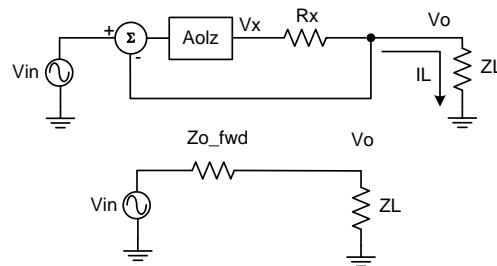


Fig. 11.20: Zo “Forward” Equations

The example in Figure 11.21 shows how an improper design of the Zo Block can yield erroneous results with a capacitive load. This example places a 100pF capacitor directly on the output of a Zo block whose $R_x = 400k$. Notice that on the “Zo_forward open, 100p” curve that until Z_L (X_c of the 100pF capacitive load) becomes $< \frac{1}{2}$ of R_x (about 162k) that the “Desired Zo” curve is not met. Other curves are shown with varying resistive loads in parallel with the capacitive load of

100pF. In conclusion we see that the larger ZL is in comparison to Rx the more in error Zo_fwd is from our Desired Zo in this improperly designed Zo Block.

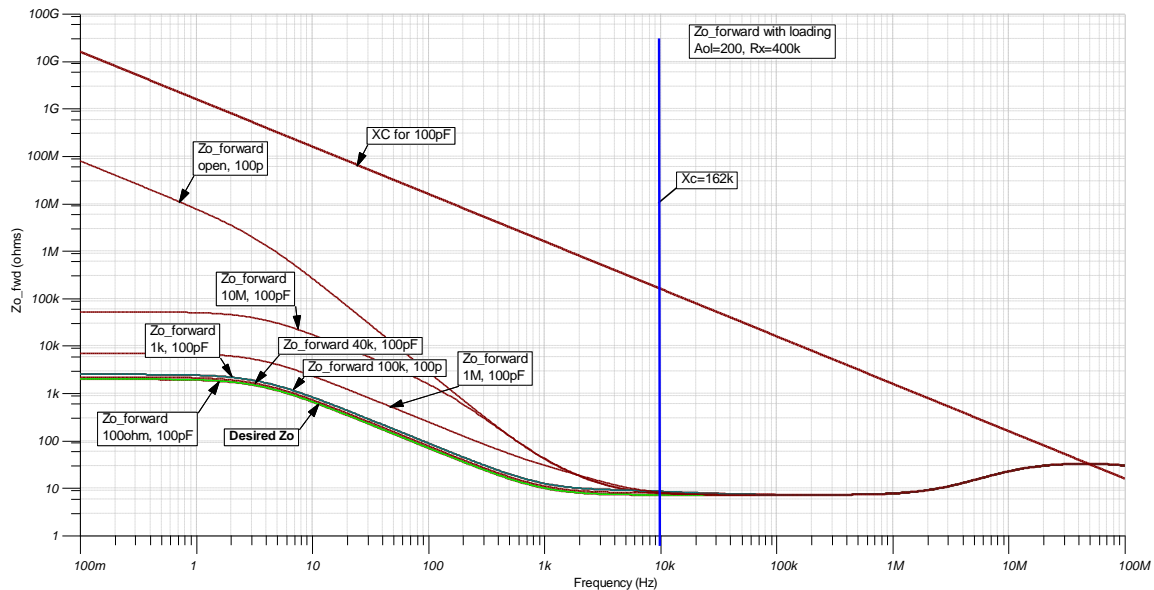


Fig. 11.21: Improper Zo Design – ZL >> Rx

Figure 11.22 shows how to fix our problem of ZL larger than Rx creating errors in our Desired Zo by adding an Rdummy inside of our Zo Block to keep the “effective ZL” at a minimum value regardless of how high in value the actual load external to the Zo Block becomes. Note also that because of the special arrangement and use of VCVS1 (gain =-1) the gain computation for Aolz is simply the respective $(-Z_F/Z_I) * (-1)$ or just Z_F/Z_I where ZF and ZI represent the total impedance in the feedback (ZF) or input (ZI) path of our closed loop op amp arrangement.

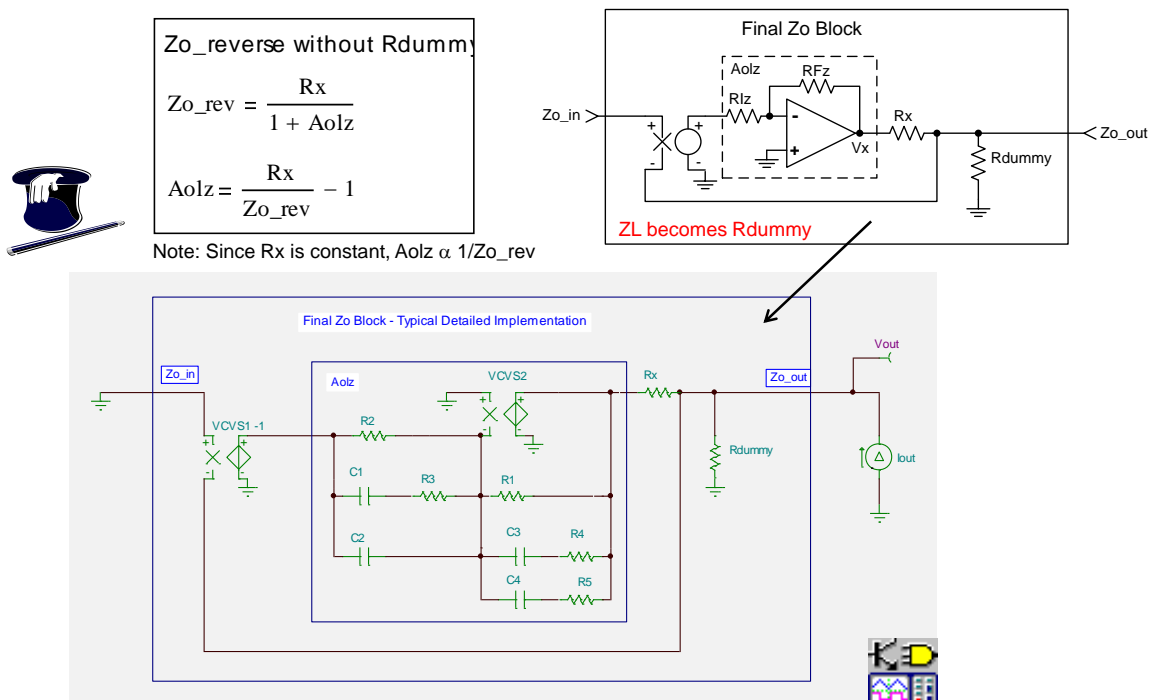


Fig. 11.22: Final Zo Block Architecture

Now let's zoom out and evaluate our design trade-offs for the Zo Block as shown in Figure 11.23. We note that no matter how we design the Zo Block using our proposed architecture it will never have $Zo_{fwd} = Zo_{rev}$. However, we will see as we proceed forward that it will yield acceptable closeness between Zo_{fwd} and Zo_{rev} to make this external Zo Block a powerful tool in our stability analysis toolbox. Based on many, many Zo Blocks built, with this architecture, we see in Figure 11.23 a few design rules of thumb, to optimize the Zo Block design on first pass.

Note Design Compromise:

Zo_{fwd} : If $R_{dummy} \ll R_x$; Then **$Zo_{fwd} = R_x/Aolz$**

Zo_{rev} : If $R_{dummy} \gg Zo_{rev}$; Then **$Zo_{rev} = R_x/(1+Aolz)$**

Final Zo Block Design Guidelines:

- 1) $R_{dummy} > 50 * Zo_{Low_f}$
(or $50 * \text{Highest } Zo \text{ value} - \text{usually } Zo_{Low_f}$)
- 2) $R_x > 10 * R_{dummy}$
- 3) $VCVS2 > 10 * (\text{Largest } Aolz \text{ Magnitude})$



Fig. 11.23: Zo Block Design Guidelines

Before we begin our OPA376 Zo Block final design we might want to consider designing a little Excel spreadsheet calculator to help speed the computation process. As shown in Figure 11.24 a “3dB Frequency” calculator will allow us to enter a frequency and an R value and compute the necessary C value. The “R1 or R2 from Req Parallel” calculator allows us to enter Req (parallel resistance desired) and either R1 or R2 and compute the other parallel resistor value, either R2 or R1, to yield Req. The “Req Parallel” calculator accepts R1 and R2 and computes Req or the parallel combination of R1 and R2.

| 3dB Frequency | | | | Comment |
|-----------------------------------|------|------------|--------|------------------------------|
| Enter | f = | 3.2800E+04 | Hz | $f = 1/(2*\pi*R*C)$ |
| | R= | 1.4000E+02 | ohms | |
| Compute | C= | 3.4659E-08 | Farads | $C = 1 / (f*2*\pi*R)$ |
| Enter | f = | 8.1100E+05 | Hz | |
| | C= | 1.5900E-08 | Farads | |
| Compute | R= | 1.2342E+01 | ohms | $R = 1 / (f*2*\pi*C)$ |
| | | | | |
| R1 or R2 from Req Parallel | | | | |
| Enter | Req= | 1.2277E+02 | ohms | $Req = (R1*R2) / (R1+R2)$ |
| | R1= | 1.0000E+03 | ohms | |
| Compute | R2= | 1.3995E+02 | ohms | $R2 = (Req*R1) / (R1 - Req)$ |
| Enter | Req= | 1.6363E+04 | ohms | $Req = (R1*R2) / (R1+R2)$ |
| | R1= | 2.5641E+04 | ohms | |
| Compute | R2= | 4.5221E+04 | ohms | $R1 = (Req*R2) / (R2 - Req)$ |
| | | | | |
| Req Parallel | | | | |
| Enter | R1= | 2.0000E+03 | ohms | |
| | R2= | 2.0000E+03 | ohms | |
| Compute | Req= | 1.0000E+03 | ohms | $Req = (R1*R2) / (R1+R2)$ |

Fig. 11.24: Excel Calculator Ideas for Ease of Zo Block Build

Three more calculators in Excel will help speed the Zo Block building process. As shown in Figure 11.25 the “Gain RI Calculator” allows us to enter a desired Gain and RF and compute the required RI. The “Gain RF Calculator” allows us to enter a desired Gain and RI and compute the required RF. The “Aolz Calculator” requires we enter the desired Zo_rev and fixed value of Rx so it can compute the required Aolz.

| Gain RI | | | | Comment |
|------------------------|----------|------------|------|---------------------------|
| Enter | Gain = | 8.0800E+05 | V/V | $Gain = RF / RI$ |
| | RF= | 4.9734E+04 | ohms | |
| Compute | RI= | 6.1552E-02 | ohms | $RI = RF / Gain$ |
| | | | | |
| Gain RF | | | | |
| Enter | Gain = | 4.9734E+02 | V/V | $Gain = RF / RI$ |
| | RI = | 1.0000E+02 | ohms | |
| Compute | RF= | 4.9734E+04 | ohms | $RF = Gain * RI$ |
| | | | | |
| Aolz Calculator | | | | Comment |
| Enter | Zo_rev = | 2.05E+02 | ohms | |
| | Rx = | 1.20E+06 | ohms | |
| Compute | Aolz = | 5.8531E+03 | V/V | $Aolz = (Rx/Zo_rev) - 1$ |

Zo_reverse without Rdumm;

$$Zo_rev = \frac{Rx}{1 + Aolz}$$

$$Aolz = \frac{Rx}{Zo_rev} - 1$$

Fig. 11.25: Excel Calculator Ideas for Ease of Zo Block Build (cont.)

The real trick to building the Zo Block is shown in Figure 11.26. Notice that we have added two new regions, Region Lo-DC and Region DC, to the OPA376 Zo Curve. When we go to use our Zo Block in SPICE simulations, SPICE will run a DC Analysis before it runs an AC Analysis. We need to make sure that the DC Operating Point can be found correctly without our External Zo Block adding any errors or preventing DC Convergence. To guarantee this find we need a low value at DC for our Zo Block. For example if the OPA376, in a final SPICE application circuit, needs 3mA for the DC Operating point to be correct then a Zo Block with a flat curve at DC of 2.408kohms would drop >6V across our Zo Block from a 5V single supply on the OPA376. The DC Operating Point would not be found. From the OPA376 we know it can drive 3mA with only about a 10mV drop from the 5V rail. This is the difference between the real op amp and the original SPICE macromodel of the OPA376, and our External Zo Block. Our External Zo Block does not model the DC or Large Signal Zo behavior of the op amp. The focus of our Zo Block is for AC Stability Analysis. There are other, more complicated ways, to add a DC and Large Signal Zo operation of an External Zo Block, but they are not required for our end goal and are beyond the focus of this article. In Figure 11.26 we see our desired Zo curve over frequency. Remember that the relationship between Zo_rev and Aolz is a reciprocal one for a fixed Rx. So once we know the desired Zo_rev the necessary Aolz, for a fixed Rx, is simply the inverse shape of it. Shown in Figure 11.26 are the measured frequency points and desired magnitudes from the OPA376 Zo testing in the Zo_rev curve. The Aolz curve gets its frequency points from the Zo_rev Curve and its magnitudes from the magnitude values in the Zo_rev Curve, Rx, and using the Aolz Calculator.

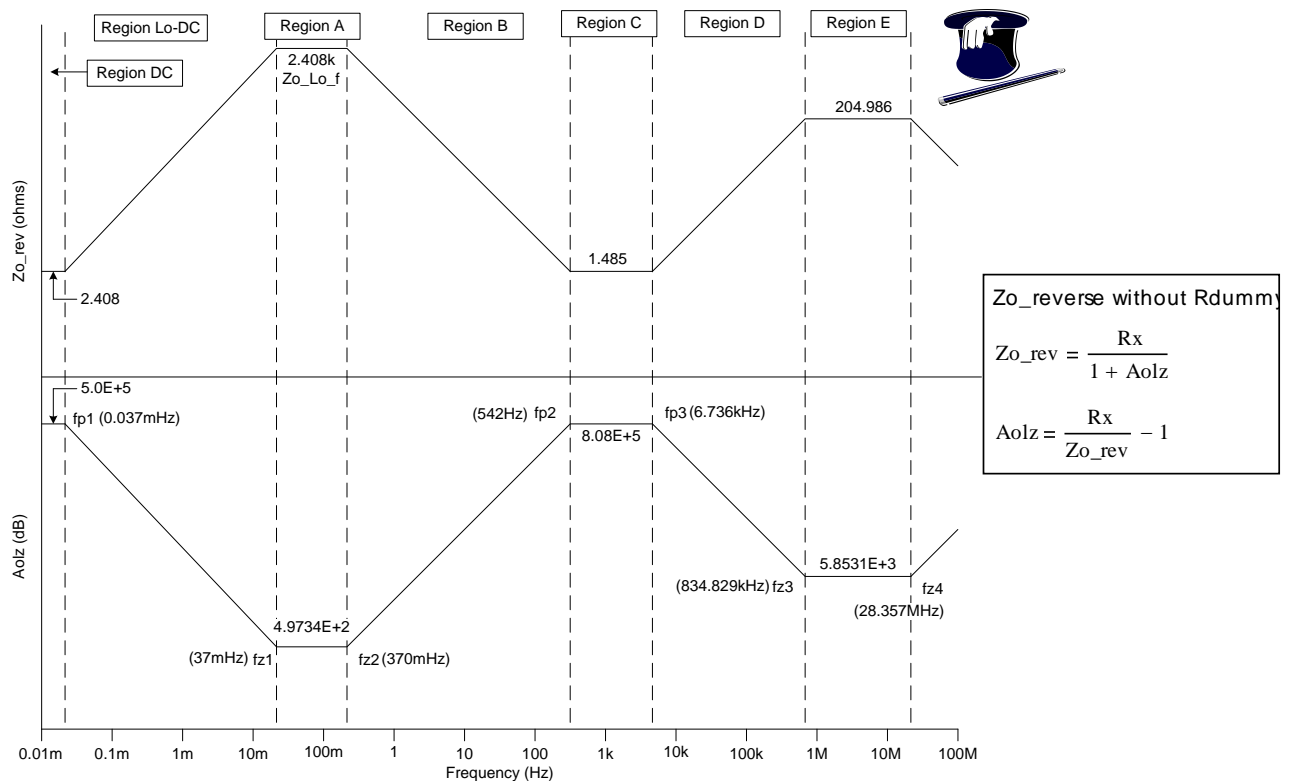


Fig. 11.26: Final OPA376 Zo Block Design

To achieve the desired Aolz for a fixed Rx we will need the circuit arrangement in Figure 11.27. Now we are ready to compute values for this circuit. From our design guidelines in Figure 11.23 we choose Rx=1.2Meg, Rdummy=120k, and VCVS2 = 5M.

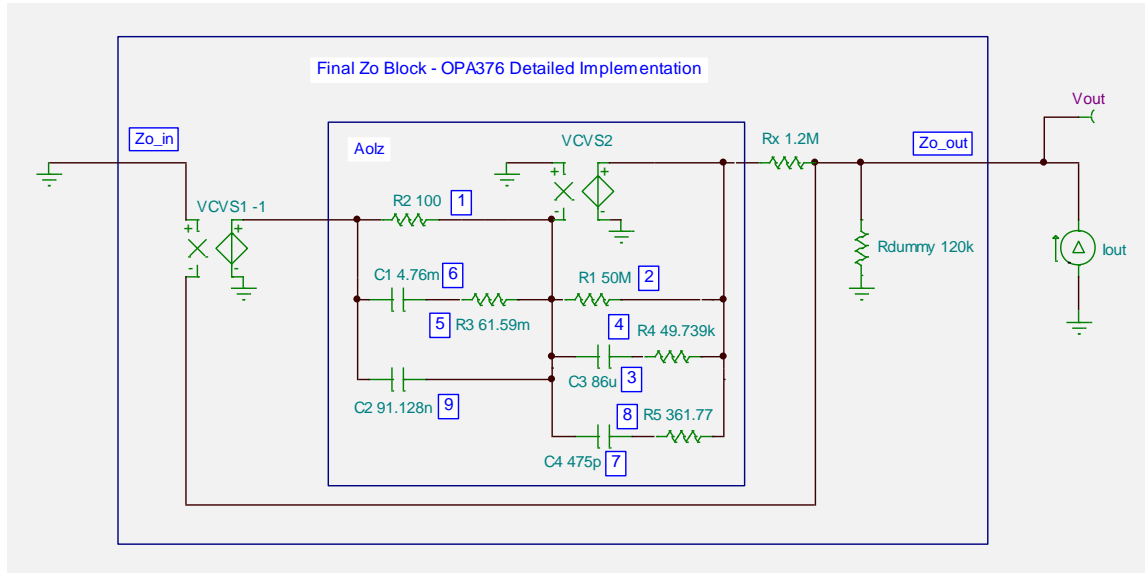


Fig. 11.27: Final OPA376 Zo Block Design – Zo_rev Test

Figure 11.26 and Figure 11.27 should be referred to as we walk through our OPA376 Zo Block design using the table in Figure 11.28. We will start from Region DC and work our way from low frequency to high frequency in the design process. Refer to Figure 11.26. There are Pole Eq and Zero Eq equations which are estimates as to the respective pole and zero locations. These estimates will use the dominant components which set the pole or zero and will turn out to be accurate enough for our external Zo Block. If one is concerned about exact accuracy he can derive and use more exact equations for the pole/zero locations. Remember to use our Excel calculators to simplify the choosing of values for R and/or C to design the respective Aolz. You will notice in the table in Figure 11.28 that we do not compute every pole and zero since the ones we do not compute will occur as a result of the +/-20dB/decade slopes in the Aolz curve along with its flat regions. Step 1 is to set a starting point by assigning $R2=100$ ohms to keep $R1$ reasonable. Step 2 is in Region DC where gain is set by $R1/R2$ and we need $Aolz=5.0e+5$ which yields $R1=50M\Omega$ for $R2=100\Omega$. Step 3 is at the start of the Region Lo-DC where we need $fp1=0.037mHz$ which is predominantly set by $R1$ and $C3$. $C3$ is chosen to yield $fp1$ given $R1$. Step 4 uses Region A where we need $Aolz=4.9734e+2$. This gain is set by $(R1//R4)/R2$. We already know $R1$ and $R2$ so we solve for $R4$. Step 5 jumps ahead to Region C where gain is set by $(R1//R4) / (R2//R3)$. Since we know $R1$, $R2$, and $R4$ we can solve for $R3$ to get $Aolz=8.08e+5$. Step 6 computes the pole $fp2$ in Region B by computing $C1$ based on $R3$ which we now know. Step 7 computes $fp3$ at the beginning of Region D. $fp3$ is determined by $(R1//R4)$ and $C4$. Since we know $R1$ and $R4$, $C4$ is computed to yield $fp3$. Step 8 computes $Aolz=5.8531e+3$ by $[R5//((R1//R4))]/[R2//R3]$. Since we already have $R1$, $R2$, $R3$, and $R4$ we can compute $R5$. Finally Step 9 will give us $fz4$ by $(R2//R3)$ and $C2$. Since we know $R2$ and $R3$ we will compute for $C2$. The external Zo Block for OPA376 is now completely built and ready for testing.

| Zo Final Build Data | | | | | | | | | | | |
|---------------------|----------|----------|-------------------------------------|-------------|---------------|--------|-------------------------|----------|-------------------------|--------------------|--------|
| Region | Freq Min | Freq Max | Zo Magnitude | Aolz | Aolz Eq | Pole | Pole Eq | Zero | Zero Eq | Solve | Step |
| | (Hz) | (Hz) | (ohms) | (V/V) | | (Hz) | (estimate) | (Hz) | (estimate) | | |
| DC | DC | 0.037m | 2.408 | 5.0000E+05 | R1/R2 | | | | | R2=100Ω R1=50MΩ | 1 2 |
| Lo-DC | 0.037m | 37m | x10 increase per decade frequency | -20db slope | | 0.037m | fp1= 1/(2*pi*R1*C3) | | | C3=86uF | 3 |
| A | 37m | 370m | 2.408k | 4.9734E+02 | Z14/R2 | | | | | R4=49.739kΩ | 4 |
| C | 542 | 6.736k | 1.485 | 8.0800E+05 | Z14/Z23 | | | | | R3=61.59mΩ | 5 |
| B | 370m | 542 | x1/10 decrease per decade frequency | +20db slope | | 542 | fp2= 1/(2*pi*R3*C1) | | | C1=4.76mF | 6 |
| D | 6.736k | 834.829k | x10 increase per decade frequency | -20db slope | | 6.736k | fp3= 1/(2*pi*Z14*C4) | 834.829k | | C4=475pF | 7 |
| E | 834.829k | 28.357M | 204.986 | 5.8531E+03 | (R5//Z14)/Z23 | | | | | R5=361.77Ω | 8 |
| F | 28.357M | 28.357M | x1/10 decrease per decade frequency | +20db slope | | | | 28.357M | fz4= 1/(2*pi*Z23*C2) | C2=91.128nF | 9 |

Rdummy = 120kohm, Rx = 1.2Mohm, R2 = 1kohm, Z14=R1//R4, Z23=R2//R3

Fig. 11.28: Final OPA376 Zo Block Design – Equations

The final build of our OPA376 Zo Block is tested for Zo_rev and the results shown in the Figure 11.29 where we leave the results in dB and measure the frequency 3dB points.

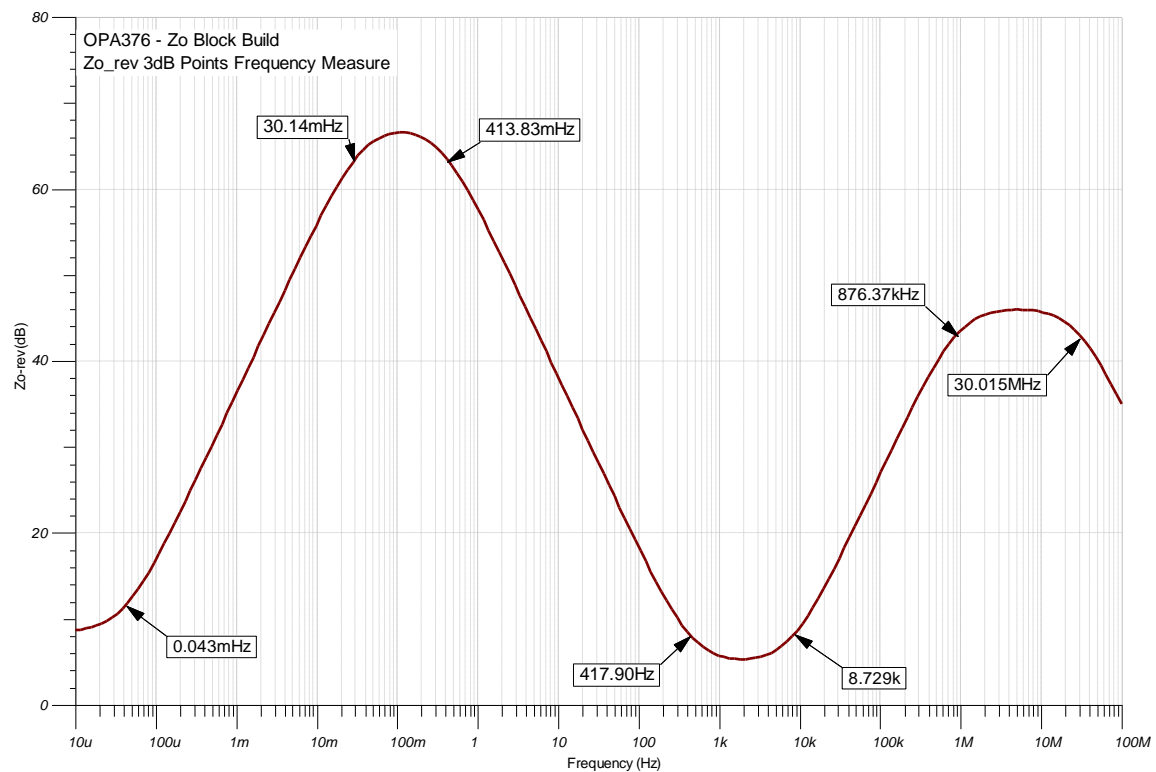


Fig. 11.29: Final OPA376 Zo Block Design – Zo_rev Curve (dB)

In Figure 11.30 our final build of our OPA376 Zo Block is tested for Zo_rev and the magnitude results on the Y-axis are converted to Logarithmic so we can read Zo_rev magnitude directly in ohms.

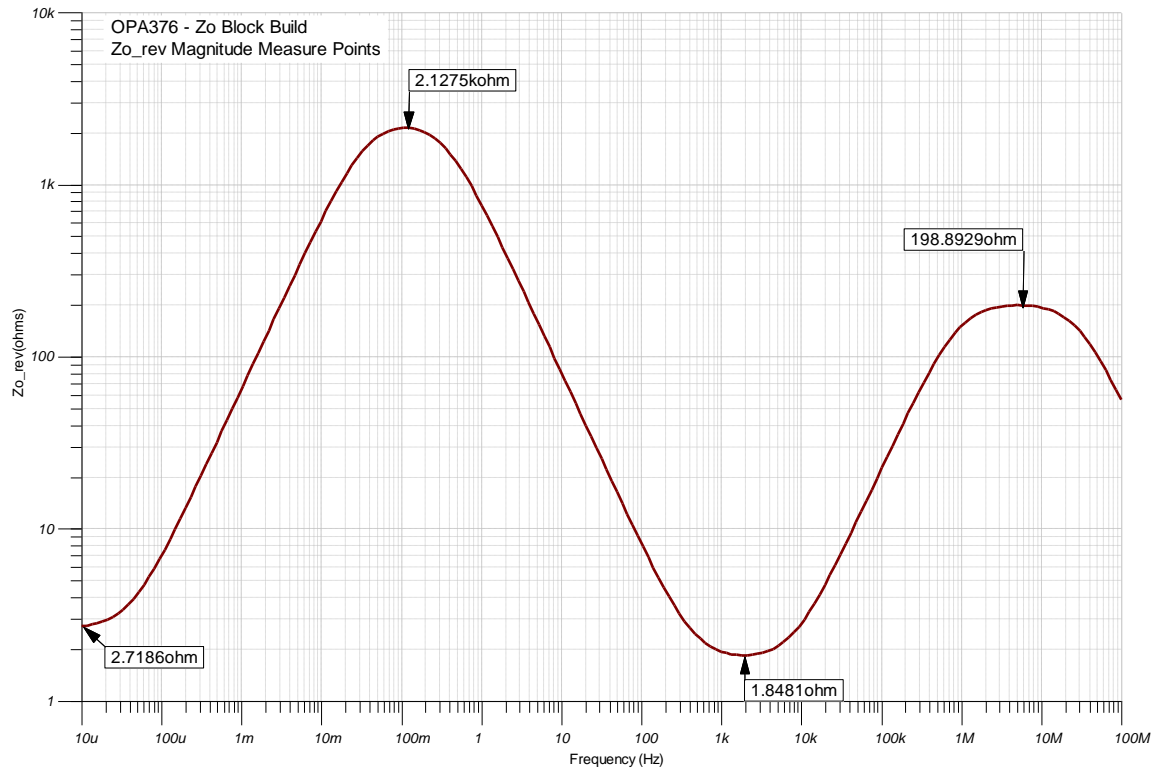


Fig. 11.30: Final OPA376 Zo Block Design – Zo_rev Curve (ohms)

In Figure 11.31 we compile the results of Zo_rev to compare both frequency break points and magnitude flat regions of the final OPA376 Zo Block build and the original design goal for each respective data point. Our final frequency breakpoints are going to be bounded by +20dB/decade or -20dB/decade slopes with intercepts at the flat magnitude portions of the Zo_rev curve. As we compare Design Frequency points versus Final Frequency points we see 10%-20% variance from Desired versus Final Frequency points. In the magnitude comparison we see 3%-24% variance from Desired versus Final Magnitude points. This is an acceptable result without spending any more design time on the Zo Block. Real silicon typically has capacitors that vary +/-20% over process and temperature along with resistors that vary +/-30% over process and temperature. So yes, Zo, will vary from part to part but with our stability techniques in place using decade rules-of-thumb we will have good design margin.

| Zo_rev | Design | Final | Design | Final |
|-----------|-----------|-----------|-----------|-----------|
| | Frequency | Frequency | Magnitude | Magnitude |
| Parameter | (Hz) | (Hz) | (ohms) | (ohms) |
| Region DC | | | 2.408 | 2.7186 |
| fp1 | 0.037m | 0.043m | | |
| fz1 | 37m | 30.14m | | |
| Region A | | | 2.408k | 2.1275k |
| fz2 | 370m | 413.83m | | |
| fp2 | 542 | 417.90 | | |
| Region C | | | 1.485 | 1.8481 |
| fp3 | 6.736k | 8.729k | | |
| fz3 | 834.829k | 876.37k | | |
| Region E | | | 204.986 | 198.8929 |
| fz4 | 28.357M | 30.015M | | |



Fig. 11.31: Final OPA376 Zo Block Design – Build vs Design Goal

Our Final Zo Block for the OPA376 will now be tested for Zo_fwd using the test circuit in Figure 11.32 in TINA SPICE.

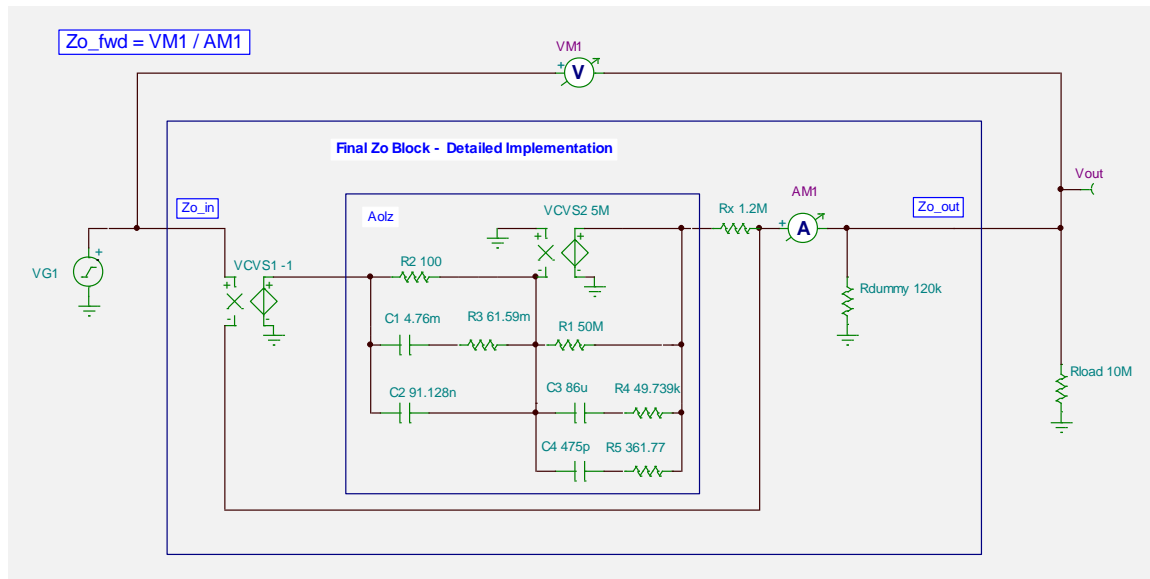


Fig. 11.32: Final OPA376 Zo Block Design – Zo_fwd Test

The TINA SPICE results of our testing Zo_fwd versus Zo_rev is shown in Figure 11.33. We see the Zo_fwd versus Zo_rev does have some minor discrepancies as we would expect since the equations for Zo_fwd versus Zo_rev have a small difference. Recall from Figure 11.23: $Zo_fwd = R_x / Aolz$ and $Zo_rev = R_x / (1 + Aolz)$. The frequency differences are so small they are not of any concern. Figure 11.34 shows the results of our testing Zo_fwd versus Zo_rev with the Y-axis set to Logarithmic scale so we can compare the magnitude differences directly in ohms. Magnitude comparison shows Zo_rev and Zo_fwd within 12% at all flat magnitude portions of the curve. Again, no major concerns here.

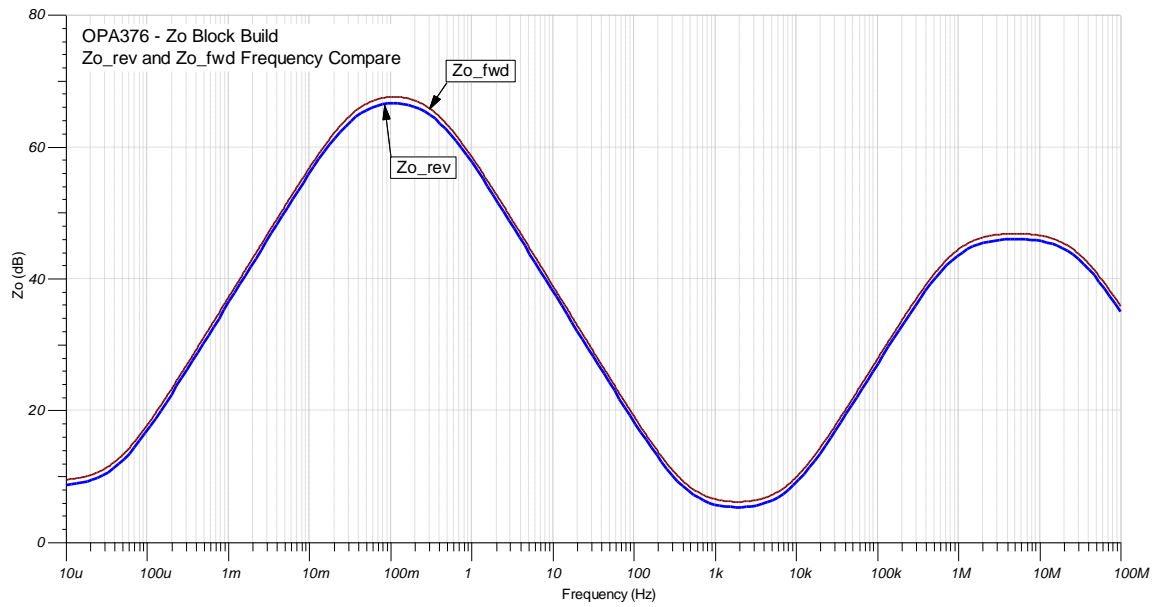


Fig. 11.33: Final OPA376 Zo Block Design – Zo_fwd and Zo_rev Frequency Compare

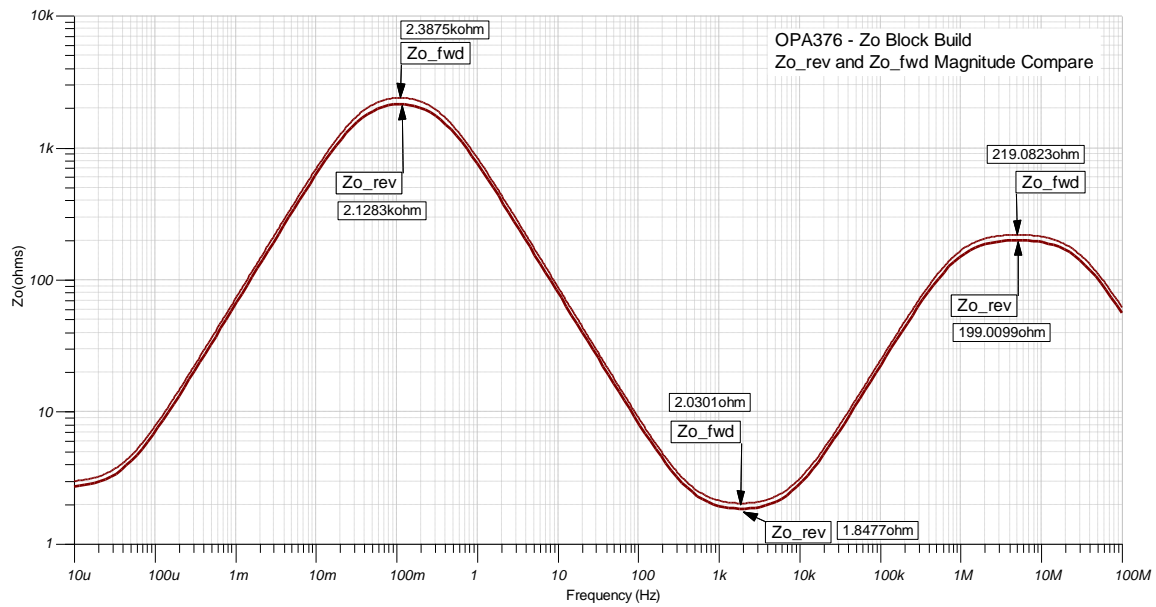


Fig. 11.34: Final OPA376 Zo Block Design – Zo_fwd and Zo_rev Magnitude Compare

In Figure 11.35 we add a side note on building Zo External Blocks. Sometimes the Zo_Lo_f value is not available from the data sheet as the curve may still be rising at the rate of 20dB/decade as it approaches lower frequencies. Sometimes the op amp Electrical Characteristics table will contain two different test conditions for the Open Loop Gain or Aol specification as shown in Figure 11.35. Based on these measurements one can compute the Zo_Lo_f value as shown in Figure 11.35. The detailed derivation of this equation is included in this article's Appendix.

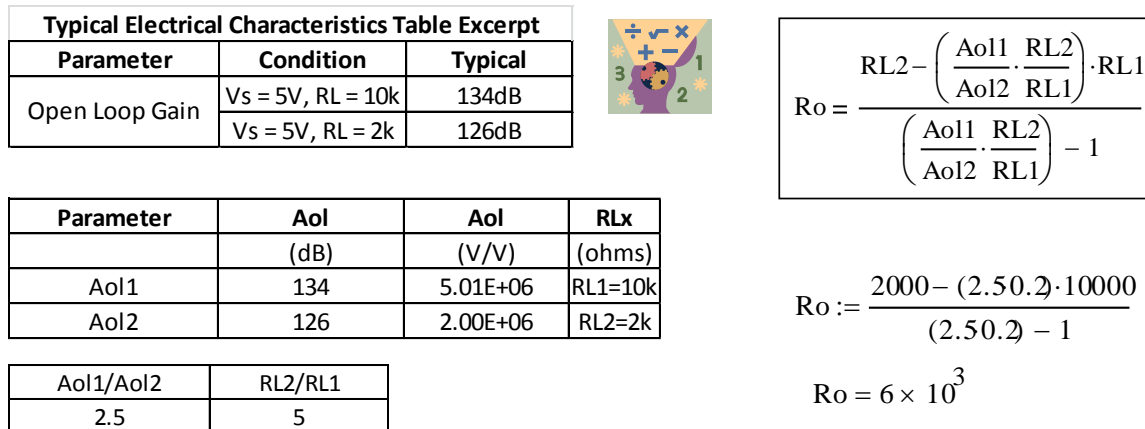


Fig. 11.35: Zo_Lo_f Calculation from Datasheet Aol Tests

One final trick for our External Zo Block is shown in Figure 11.36. When the External Zo Block is used in an op amp SPICE circuit it is advisable to bound the possible voltages coming out of the External Zo Block to slightly above (typically +/-0.7V) or below the positive and negative supplies used on the respective op amp for which the External Zo Block was created. This helps prevent convergence issues in SPICE especially since VCVS2 in Figure 11.36 can potentially produce very large voltages. D1 and D2 are diode clamps to keep the output of the External Zo Block within a reasonable range close to the positive and negative supplies used for the simulated application circuit. Vp_clmp and Vm_clmp DC Sources in Figure 11.36 can be adjusted to move Vout clamp points as close as desired to the supplies used on the op amp for which the External Zo Block was created.

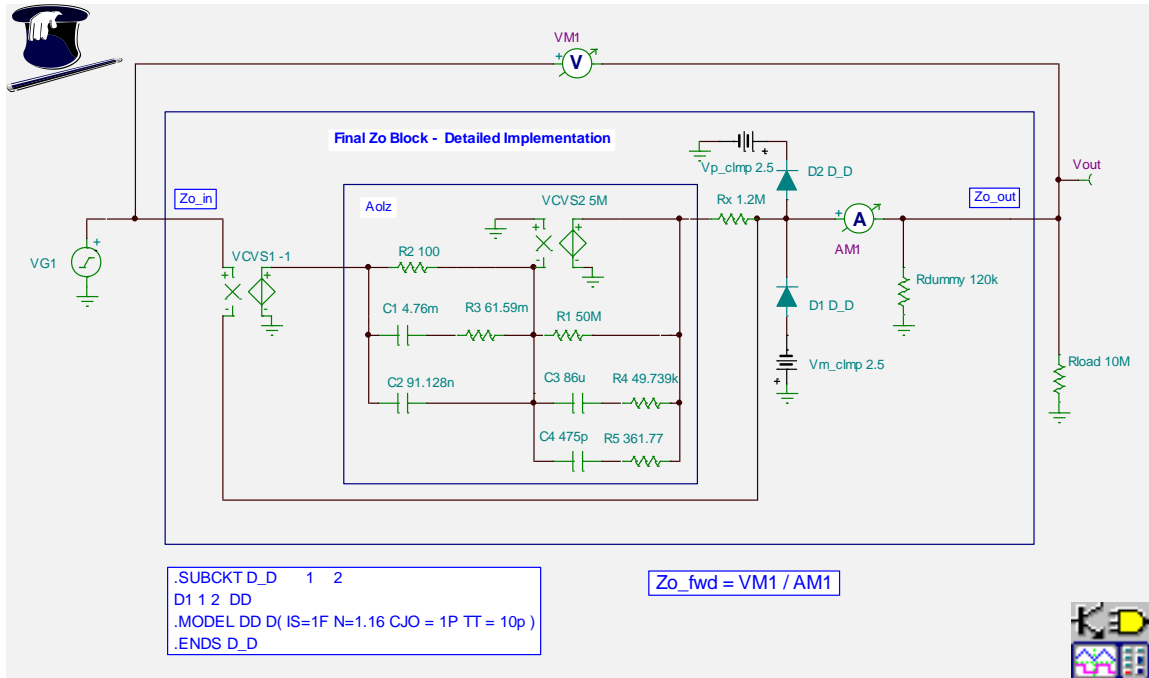


Fig. 11.36: Final Zo Block with Added Diode Clamps for SPICE Convergence

A transient TINA SPICE simulation of our final Zo Block with Added Clamp Diodes is shown in Figure 11.37 and illustrates the point that Vout is clamped within $\pm 1.1V$ of the $\pm 2.5V$ supplies (Vm_clmp and Vp_clmp) used to set the clamp levels. Vm_clmp and Vp_clmp may be adjusted as desired to raise or lower these clamping levels.

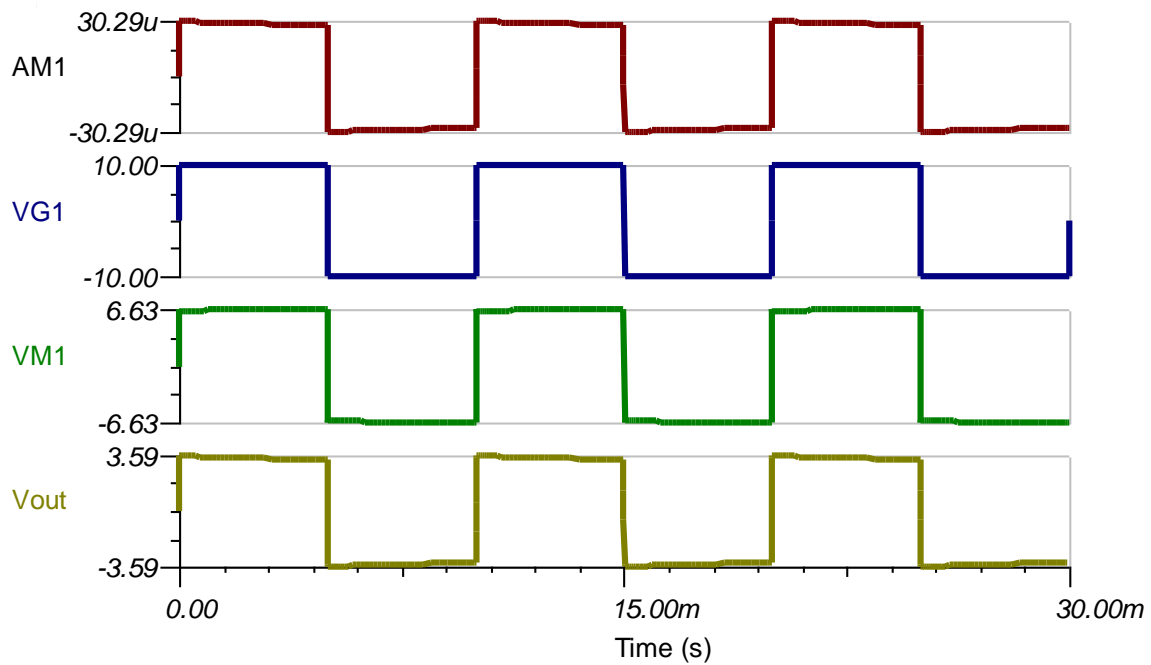
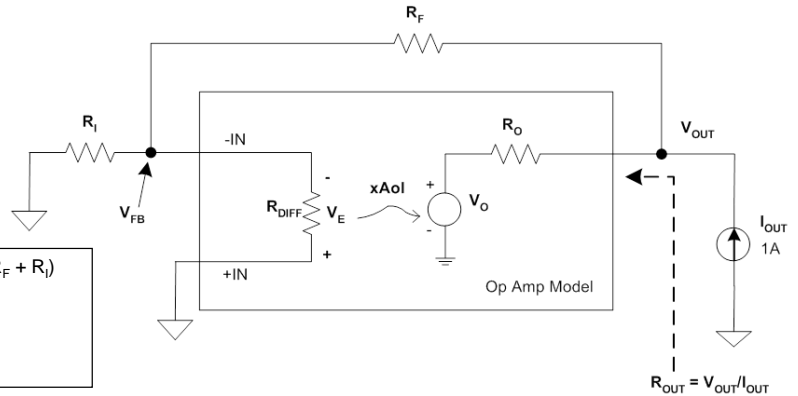


Fig. 11.37: Final Zo Block with Added Diode Clamps for SPICE Convergence Test

APPENDIX: DETAILED DERIVATIONS

R_O and R_{OUT} Derivation

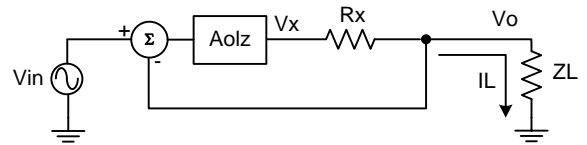


- 1) $\beta = V_{FB} / V_{OUT} = [V_{OUT} (R_i / (R_F + R_i))] / V_{OUT} = R_i / (R_F + R_i)$
- 2) $R_{OUT} = V_{OUT} / I_{OUT}$
- 3) $V_O = -V_E A_{ol}$
- 4) $V_E = V_{OUT} [R_i / (R_F + R_i)]$

- 5) $V_{OUT} = V_O + I_{OUT} R_O$
- 6) $V_{OUT} = -V_E A_{ol} + I_{OUT} R_O$ Substitute 3) into 5) for V_O
- 7) $V_{OUT} = -V_{OUT} [R_i / (R_F + R_i)] A_{ol} + I_{OUT} R_O$ Substitute 4) into 6) for V_E
- 8) $V_{OUT} + V_{OUT} [R_i / (R_F + R_i)] A_{ol} = I_{OUT} R_O$ Rearrange 7) to get V_{OUT} terms on left
- 9) $V_{OUT} = I_{OUT} R_O / \{1 + [R_i A_{ol} / (R_F + R_i)]\}$ Divide in 8) to get V_{OUT} on left
- 10) $R_{OUT} = V_{OUT} / I_{OUT} = [I_{OUT} R_O / \{1 + [R_i A_{ol} / (R_F + R_i)]\}] / I_{OUT}$
Divide both sides of 9) by I_{OUT} to get R_{OUT} [from 2)] on left
- 11) $R_{OUT} = R_O / (1 + A_{ol} \beta)$ Substitute 1) into 10) ←

From: Frederiksen, Thomas M. Intuitive Operational Amplifiers.
McGraw-Hill Book Company. New York. Revised Edition. 1988.

Z_o forward Derivation



Z_o Forward Derivation

Eq1: $V_x = (V_{in} - V_o) \cdot A_{olz}$

Eq2: $V_o = \frac{V_x Z_L}{R_x + Z_L}$

Eq3: $I_L = \frac{V_o}{Z_L}$

Eq4: $Z_{o_fwd} = \frac{V_{in} - V_o}{I_L}$

Solve for V_x

Substitute Eq2 into Eq1

$$V_x = \left(V_{in} - \frac{V_x Z_L}{R_x + Z_L} \right) \cdot A_{olz}$$

$$\frac{V_x}{A_{olz}} = V_{in} - \frac{(V_x Z_L)}{R_x + Z_L}$$

$$\frac{V_x}{A_{olz}} + \frac{V_x Z_L}{R_x + Z_L} = V_{in}$$

Eq5: $V_x = \frac{V_{in}}{\frac{1}{A_{olz}} + \frac{Z_L}{R_x + Z_L}}$

Solve for V_o

Substitute Eq5 into Eq2

Eq6: $V_o = \frac{\left(V_{in} \cdot \frac{Z_L}{R_x + Z_L} \right)}{\frac{1}{A_{olz}} + \frac{Z_L}{R_x + Z_L}}$

Zo_forward Derivation (cont.)

Solve for Zo_fwd
Substitute Eq3 into Eq4

$$\text{Eq7: } Z_{o_fwd} = \frac{V_{in} - V_o}{\left(\frac{V_o}{Z_L}\right)}$$



Solve for Zo_fwd
Substitute Eq6 into Eq7

$$Z_{o_fwd} = \frac{\left[V_{in} - \frac{\left(V_{in} \cdot \frac{Z_L}{R_x + Z_L} \right)}{\frac{1}{A_{olz}} + \frac{Z_L}{R_x + Z_L}} \right]}{\left[\frac{\left(V_{in} \cdot \frac{Z_L}{R_x + Z_L} \right)}{\frac{1}{A_{olz}} + \frac{Z_L}{R_x + Z_L}} \right] \cdot \frac{1}{Z_L}}$$

Divide numerator and denominator by

$$Z_{o_fwd} = \frac{\left[1 - \frac{\left(\frac{Z_L}{R_x + Z_L} \right)}{\frac{1}{A_{olz}} + \frac{Z_L}{R_x + Z_L}} \right]}{\left[\frac{\left(\frac{Z_L}{R_x + Z_L} \right)}{\frac{1}{A_{olz}} + \frac{Z_L}{R_x + Z_L}} \right] \cdot \frac{1}{Z_L}}$$

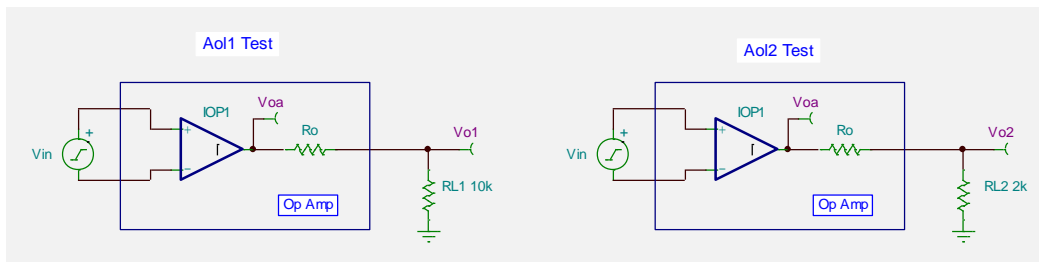
Multiply numerator and denominator by $[1/A_{olz} + Z_L/(R_x + Z_L)]$

$$Z_{o_fwd} = \frac{\frac{1}{A_{olz}} + \frac{Z_L}{R_x + Z_L} - \left(\frac{Z_L}{R_x + Z_L} \right)}{\left(\frac{Z_L}{R_x + Z_L} \right) \cdot \frac{1}{Z_L}}$$

$$Z_{o_fwd} = \frac{\frac{1}{A_{olz}}}{\left(\frac{1}{R_x + Z_L} \right)}$$

$$\text{Eq8: } Z_{o_fwd} = \frac{R_x + Z_L}{A_{olz}}$$

Zo_Lo_f Calculation from Aol Tests Derivation



$$\text{Eq1.1: } V_{o1} = \frac{V_{oa} \cdot R_{L1}}{R_o + R_{L1}}$$

$$\text{Eq 2.1: } A_{ol1} = \frac{V_{o1}}{V_{in}}$$

$$\text{Eq 3.1: } V_{oa} = V_{o1} \cdot \frac{R_o + R_{L1}}{R_{L1}} \quad \text{From Eq 1.1}$$

$$\text{Eq 4.1: } V_{o1} = A_{ol1} \cdot V_{in} \quad \text{From Eq 2.1}$$

$$\text{Eq 5.1: } V_{oa} = (A_{ol1} \cdot V_{in}) \cdot \frac{R_o + R_{L1}}{R_{L1}} \quad \text{Substitute EQ 4.1 into EQ 3.1}$$

$$\text{Eq 6.1: } \frac{V_{oa}}{V_{in}} = A_{ol1} \cdot \frac{R_o + R_{L1}}{R_{L1}} \quad \text{Divide by } V_{in}$$

$$\text{Eq1.2 } V_{o2} = \frac{V_{oa} \cdot R_{L2}}{R_o + R_{L2}}$$

$$\text{Eq 2.2: } A_{ol2} = \frac{V_{o2}}{V_{in}}$$

$$\text{Eq 3.2: } V_{oa} = V_{o2} \cdot \frac{R_o + R_{L2}}{R_{L2}} \quad \text{From Eq 1.2}$$

$$\text{Eq 4.2: } V_{o2} = A_{ol2} \cdot V_{in} \quad \text{From Eq 2.2}$$

$$\text{Eq 5.2: } V_{oa} = (A_{ol2} \cdot V_{in}) \cdot \frac{R_o + R_{L2}}{R_{L2}} \quad \text{Substitute EQ 4.2 into EQ 3.2}$$

$$\text{Eq 6.2: } \frac{V_{oa}}{V_{in}} = A_{ol2} \cdot \frac{R_o + R_{L2}}{R_{L2}} \quad \text{Divide by } V_{in}$$



Zo_Lo_f Calculation from Aol Tests **Derivation** (cont.)

Set EQ 6.1 = EQ 6.2

$$\text{EQ 7: } A_{ol1} \cdot \frac{R_o + R_{L1}}{R_{L1}} = A_{ol2} \cdot \frac{R_o + R_{L2}}{R_{L2}}$$

$$\frac{A_{ol1}}{A_{ol2}} = \frac{R_o + R_{L2}}{R_{L2}} \cdot \frac{R_{L1}}{R_o + R_{L1}}$$

$$\text{EQ 8: } \frac{A_{ol1}}{A_{ol2}} \cdot \frac{R_{L2}}{R_{L1}} = \frac{R_o + R_{L2}}{R_o + R_{L1}}$$

$$\text{EQ 9: } X = \frac{A_{ol1}}{A_{ol2}} \cdot \frac{R_{L2}}{R_{L1}}$$

$$\text{Eq 10: } X = \frac{R_o + R_{L2}}{R_o + R_{L1}}$$

Substitute Eq 9 into Eq 8

$$\text{Eq 11: } R_o = \frac{R_{L2} - X \cdot R_{L1}}{X - 1}$$

Solve Eq 10 for Ro



Eq 12:

$$R_o = \frac{R_{L2} - \left(\frac{A_{ol1}}{A_{ol2}} \cdot \frac{R_{L2}}{R_{L1}} \right) \cdot R_{L1}}{\left(\frac{A_{ol1}}{A_{ol2}} \cdot \frac{R_{L2}}{R_{L1}} \right) - 1}$$

Substitute Eq 9 into Eq 11

Acknowledgements:

For his countless years of technical dialogue and discussions with the “Wizard of Zo” (aka the author) about Zo - Bill Sands, Analog & RF Models, Purveyor of fine SPICE Macromodels of anything Analog. For SPICE Macromodel Development contact Bill through <http://www.home.earthlink.net/~wksands/>

For his being the first IC Design Engineer to work cooperatively with the author on Zo at the IC simulation level and measure without argument, question, or protest Zo on his op amp - Stephen Sanchez, Manager Design Engineering Apex Precision Power, a division of Cirrus Logic, Inc.

For their easy to use SPICE Simulator – DesignSoft, TINA SPICE link at <http://www.tina.com/>

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After earning a BSEE from the University of Arizona, in 1981, Tim Green has worked as an analog and mixed signal board/system level design engineer for over 30 years, including brushless motor control, aircraft jet engine control, missile systems, power op amps, data acquisition systems, CCD cameras, and analog/mixed signal semiconductors. Tim's recent experience is focused on Power Audio for the automotive market. He is currently a Senior Staff Systems Engineer for Audio Power Products located in Tucson, Arizona at Apex Precision Power, a division of Cirrus Logic, Inc. Tim may be reached at tim.green@cirrus.com