

**Operational Amplifier Stability**  
**Part 11 of 15: Modeling Complex Zo for Op Amps**

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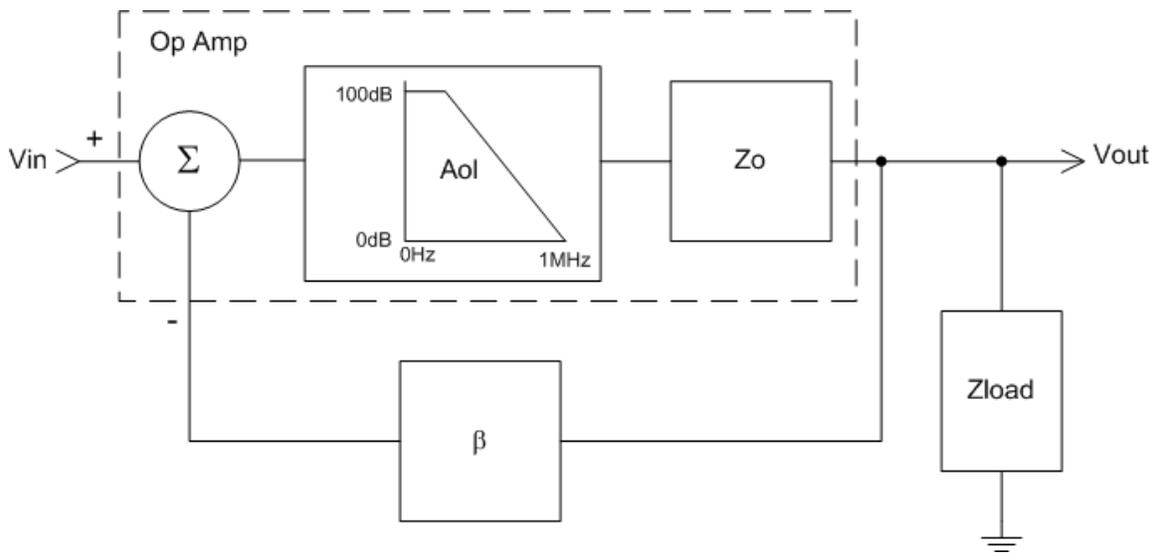
Part 11 of this series will venture into the world of complex Zo inside operational amplifiers. As single supply applications of operational amplifiers have become more predominant at the board and systems level, semiconductor manufacturers' have been challenged to create unique op amp topologies to provide rail-to-rail inputs and outputs along with high open loop gains for accuracy and low noise on an ever-dwindling signal range (i.e. from +/-10V down to 0-5V down to 0-3V). These new and unique op amp topologies produce some very interesting and unique Zo characteristics which need to be understood and modeled, in order to guarantee, by design, a stable circuit when driving reactive loads. We will look at a novel approach to modeling the complex output impedance of op amps. The building of an op amp Zo Block will allow us to move the op amp Zo outside of the traditional op amp SPICE macromodel and thereby separate the Aol curve from the effects of Zo interacting with reactive loads on the output. This will allow us, in a future article, to simplify the challenges of stabilizing op amps with complex Zo characteristics.

As with any engineering problem there is more than one possible solution. One may be tempted to construct a Zo Block from only passive components (inductors, resistors, capacitors). That solution has been researched and proven to not be so good as undesired peaking occurs in the frequency areas of L-C resonance. A real op Zo will have smooth, non-resonant frequency transitions like the model in this article produces. The technique for building an external Zo block in SPICE is rather straight-forward and easy to build yielding acceptable results in the minimum amount of time.

One final prologue before we head to the Zo Block. TINA SPICE, used extensively in this article series, as a SPICE simulator has a very good convergence engine. Some other SPICE simulators are not as forgiving and sometimes require either scaling down of large value reactive components or setting of specific Option parameters to easily converge. Our goal in this article is to divulge a quick way to measure Zo on a SPICE macromodel and show the blueprint for an external Zo Block for SPICE stability analysis. If the recommended circuits below cause other simulators to operate a bit rough then re-scaling of reactive components may be necessary.

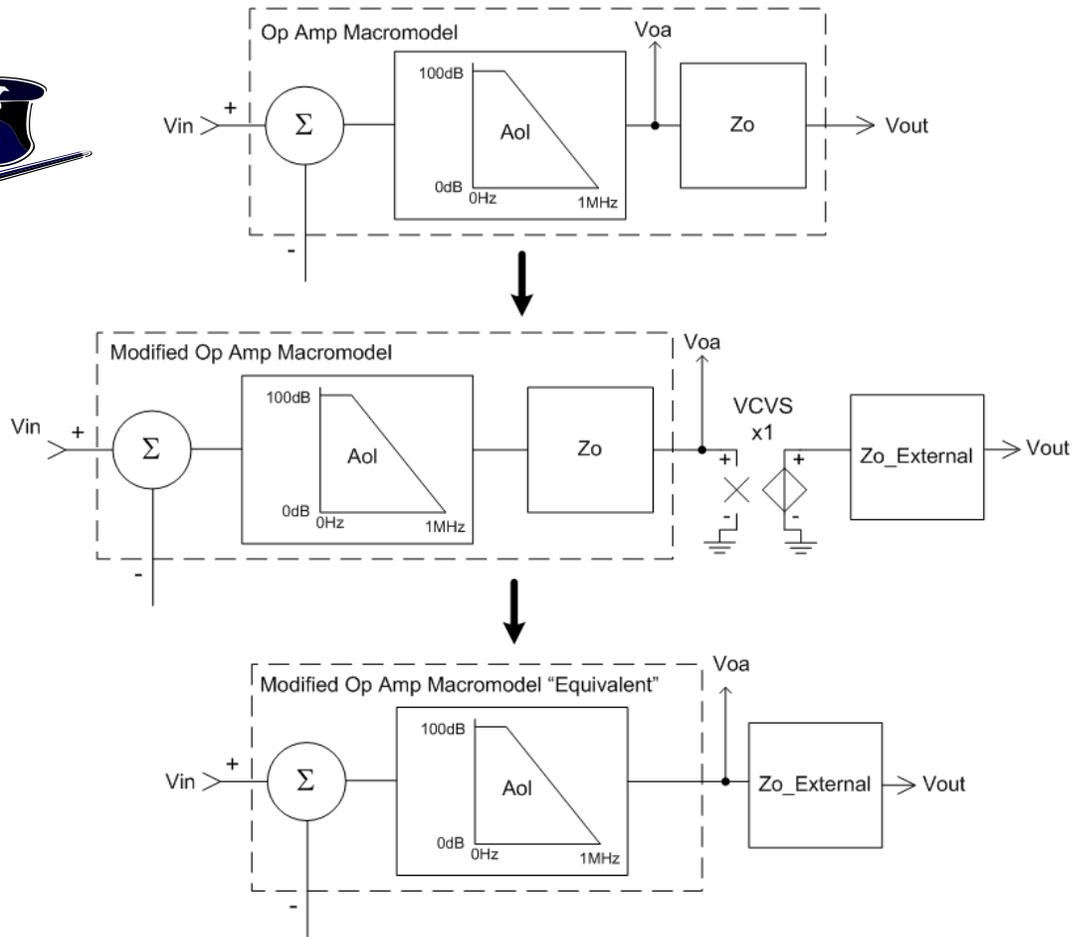
By now we have realized, through this article series, that the only thing we need to know about an op amp to solve any op amp stability problem is the Aol curve and the Zo characteristics (see Figure 11.1). The Zload will be determined by system demands and we will tailor 1/Beta for stability.

Given:  $A_{ol}$ ,  $Z_o$ ,  $\beta$ ,  $Z_{load}$   
 Find: Solution to any op amp stability problem



**Fig. 11.1: All You Need to Solve Op Amp Stability Problems**

To empower us to solve stability problems, when we use op amps with complex  $Z_o$  characteristics, we will want to move the  $Z_o$  characteristic of the op amp macromodel outside of the op amp (see Figure 11.2). This will be accomplished by creating a standalone  $Z_o$  block which will be isolated from the original macromodel by a voltage-controlled-voltage-source with a gain equal to 1. The voltage-controlled-voltage-source acts as an ideal isolation amplifier with a gain of 1 and retains the original macromodel  $A_{ol}$  curve and presents no interaction between the op amp macromodel output and the external  $Z_o$  block. The final result will be an  $A_{ol}$  curve and a  $Z_o$  Block with capability of measurement between the  $A_{ol}$  curve and  $Z_o$  Block.

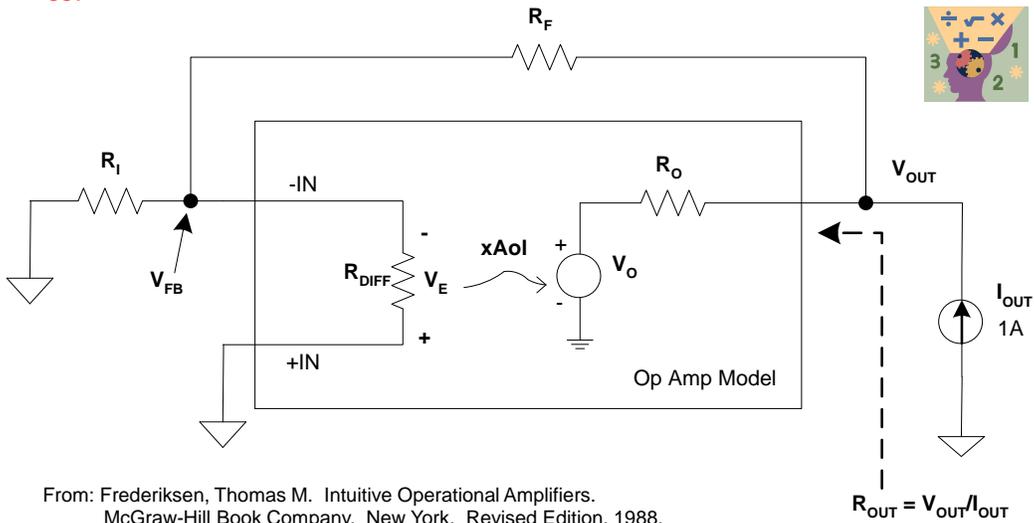


**Fig. 11.2: Moving  $Z_o$  Outside of the Op Amp Macromodel**

Recall from “Part 3 of 15:  $R_O$  and  $R_{OUT}$ ” how  $R_O$  and  $R_{OUT}$  are related.  $R_{OUT}$  is  $R_O$  reduced by loop gain. Figure 11.3 will define the op amp model used for the derivation of  $R_{OUT}$  from  $R_O$ . This simplified op amp model focuses solely on the basic DC characteristics of an op amp. A high input resistance ( $100M\Omega$  to  $G\Omega$ ),  $R_{DIFF}$  develops an error voltage across it,  $V_E$ , due to the voltage differences between  $-IN$  and  $+IN$ . The error voltage,  $V_E$ , is amplified by the open loop gain factor  $A_{ol}$  and becomes  $V_O$ . In series with  $V_O$  to the output,  $V_{OUT}$ , is  $R_O$ , the open loop output resistance. The resultant relationship between  $R_{OUT}$  and  $R_O$  is shown above with the detailed derivation in this article’s Appendix. We will use the same terminology for complex op amp output impedances. That is  $Z_o$  is open loop output impedance and  $Z_{out}$  is closed loop output impedance.

$R_O$  = Op Amp **Open Loop** Output Resistance  
 $R_{OUT}$  = Op Amp **Closed Loop** Output Resistance

$$R_{OUT} = R_O / (1 + A_{OL}\beta)$$



From: Frederiksen, Thomas M. Intuitive Operational Amplifiers.  
 McGraw-Hill Book Company. New York. Revised Edition. 1988.

**Fig. 11.3:  $R_O$  and  $R_{OUT}$**

Our op amp of choice for complex  $Z_o$  analysis and external  $Z_o$  Block build is a CMOS RRIO op amp with specifications as detailed in Fig 11.4. The OPA376 is a low quiescent current (950uA) op amp optimized for single supply operation (2.7V to 5.5V) with beyond rail-to-rail input (greater than 0.1V beyond either supply) and rail-to-rail output ( $V_{sat} = 20mV @ I_{out} = 254.8uA$ ). The OPA376 will also provide output current of 2.7mA at a saturation voltage of 50mV max. In addition the OPA376 has a wide bandwidth of 5.5MHz and a slew rate of 2V/us.

**OPA376**

**Low-Noise, Low Quiescent Current, Precision Operational Amplifier**

**Input Specs**

Offset Voltage	25uV max
Offset Drift	1uV/C
Input Voltage Range	(V-)-0.1V to (V+)+0.1V
Common-Mode Rejection Ratio	90dB typ
Input Bias Current	10pA max

**Noise**

Input Voltage Noise	0.8uVpp, f=0.1Hz to 10Hz
Input Voltage Noise Density	7.5nV/rt-Hz @1kHz
Input Current Noise Density	2fA/rt-Hz

**Output Specs**

$V_{sat} @ I_{out} = 54.8uA$	20mV max
$V_{sat} @ I_{out} = 2.7mA$	50m max
$I_{out}$ Short Circuit	+30/-50mA

**AC Specs**

Open Loop Gain, $R_L = 10k$	134dB typ
Open Loop Gain, $R_L = 2k$	126dB typ
Gain Bandwidth Product	5.5 MHz
Slew Rate	2V/us
Overload Recovery Time	0.33us
Total Harmonic Distortion + Noise	0.00327%, f=1kHz
Settling Time, 0.01%	2us

**Supply Specs**

Specified Voltage Range	2.5V to 5.5V
Quiescent Current	950uA max
Over Temperature	1mA max

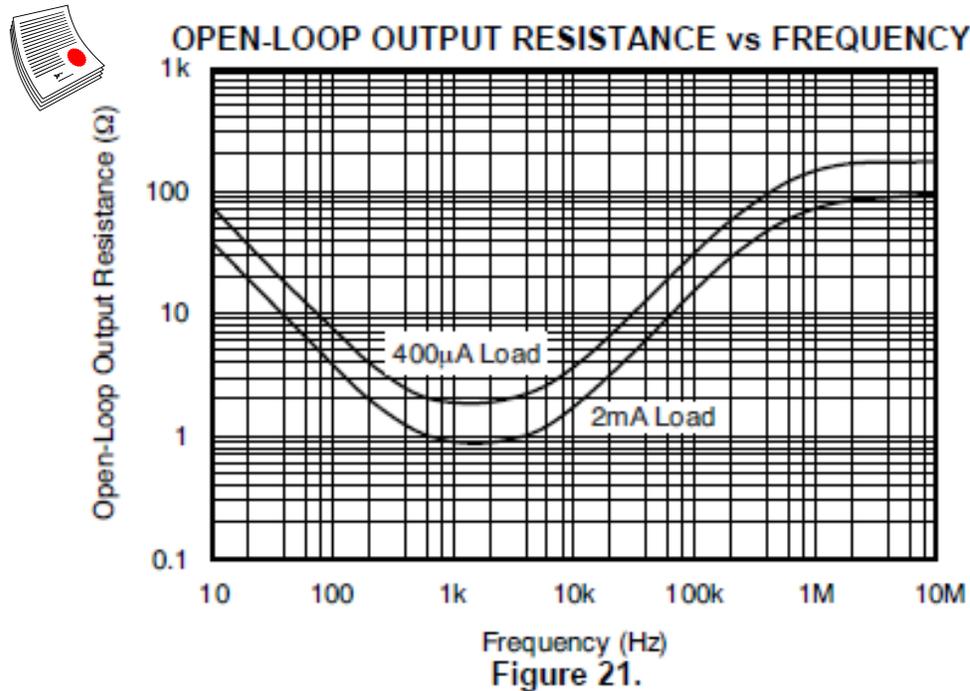
**Temperature & Package**

Operating Range	-40C to +125C
Package options	SC70-5, SOT23-5, SO-8



**Fig. 11.4: OPA376 Op Amp for External  $Z_o$  Block Build**

The OPA376 contains a  $Z_o$  Curve (Open-Loop Output Resistance vs Frequency Curve) shown in Figure 11.5. This is an exception to the rule, as most op amp data sheets contain only a Closed Loop Output Resistance vs Frequency Curve. It is very difficult to extract a complex  $Z_o$  from a closed loop output impedance curve. Fortunately for us, some sort of “ $Z_o$  Wizard” must have had influence on the OPA376 datasheet to include this most valuable curve. Notice that  $Z_o$  changes with DC Load current. It will almost always get lower as DC load current increases. Since circuits we design must be stable under all operating conditions we will choose to use the most lightly loaded, or “unloaded”  $Z_o$  curve since it will, as a rule of thumb, result in the worst case stability condition especially when driving capacitive loads.



**Fig. 11.5: OPA376 Op Amp Data Sheet  $Z_o$  Curve**

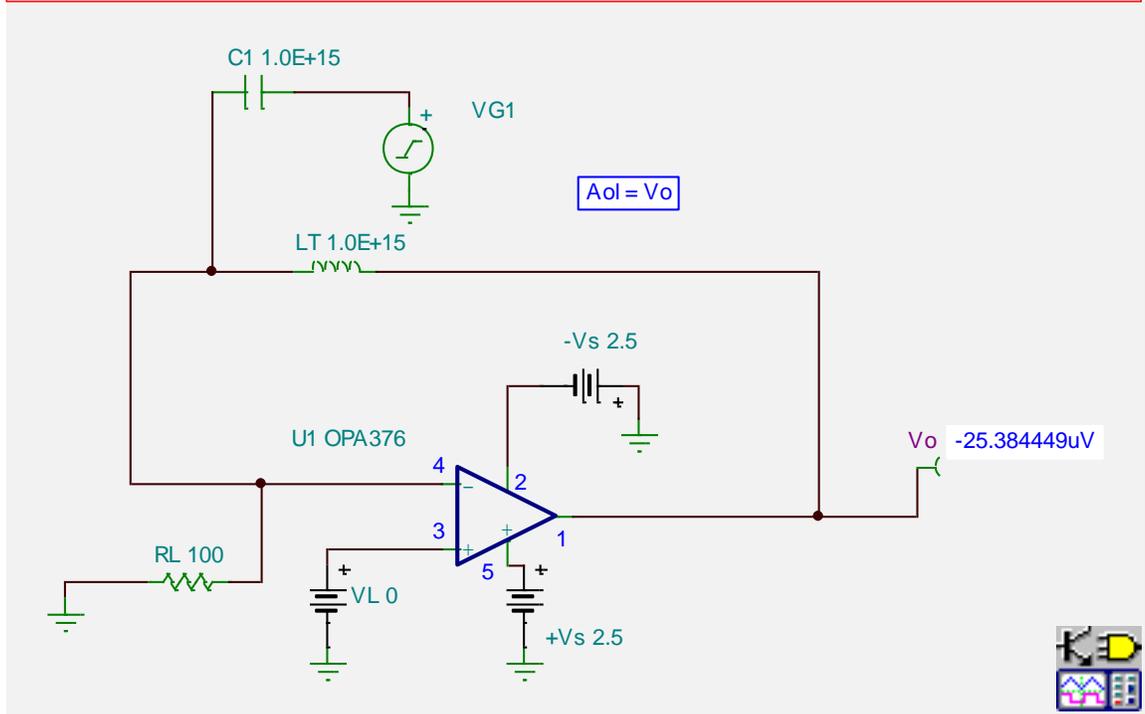
An important tool in our op amp stability toolbox will be a way to measure  $Z_o$  on a SPICE op amp macromodel. This measurement will enable us to ensure the op amp macromodel matches either the data sheet  $Z_o$  curve or measured  $Z_o$  results. With regards to measuring  $Z_o$  on an op amp – do not try this at home as it is best left to trained professionals. It requires a gain-phase analyzer, custom circuits and custom software to yield 1kHz to 10MHz accurate measurements. Better to demand the  $Z_o$  curve from your semiconductor op amp manufacturer. We can use IC simulated results if real measurements are not available. Comparing the results of a  $Z_o$  measured result on a SPICE macromodel to other data sheet characteristics of the op amp can tell us if the op amp macromodel  $Z_o$  is believable. The details of this comparison are beyond our immediate focus of this article. Our first step in measuring  $Z_o$  is to measure  $A_{ol}$ . For single supply op amps we will run all of our AC Tests using dual supplies to eliminate any common mode issue on the input and to eliminate any negative input offset voltages from trying to drive the output below ground (if we used single supply) and saturating the output devices which will not give an accurate AC result. For the  $A_{ol}$  Test circuit in Figure 11.6 the inductor,  $L_T$ , will act as a short at DC and an open for any frequencies of interest.  $C_1$  will act as an open at DC and as a short for any frequency of interest.  $R_L$  can be adjusted to check  $A_{ol}$  at different DC load currents if desired by adjusting  $V_L$ . Before we run any AC analysis we should run a DC Analysis to ensure the op amp is in a linear region of operation or our AC results will not be valid. In Figure 11.6 we see the op amp output at

-25.38uV for a DC Analysis and thus we are in a linear region of operation (output is not saturated to either rail).

**For most single supply op amp AC tests:**

**Run dual supply to**

- eliminate input common mode violations
- eliminate saturation of output devices with zero input times closed loop in DC Analysis (i.e. negative  $V_{os}$  trying to driver output less than ground)



**Fig. 11.6: Measuring Zo in SPICE: Step 1 – Measure Aol**

The TINA SPICE results of our Aol Test are shown in Figure 11.7 and we see a low frequency Aol gain of 144.13dB or 16.0879MV/V. We also observe a low frequency pole in the Aol at about 400mHz.

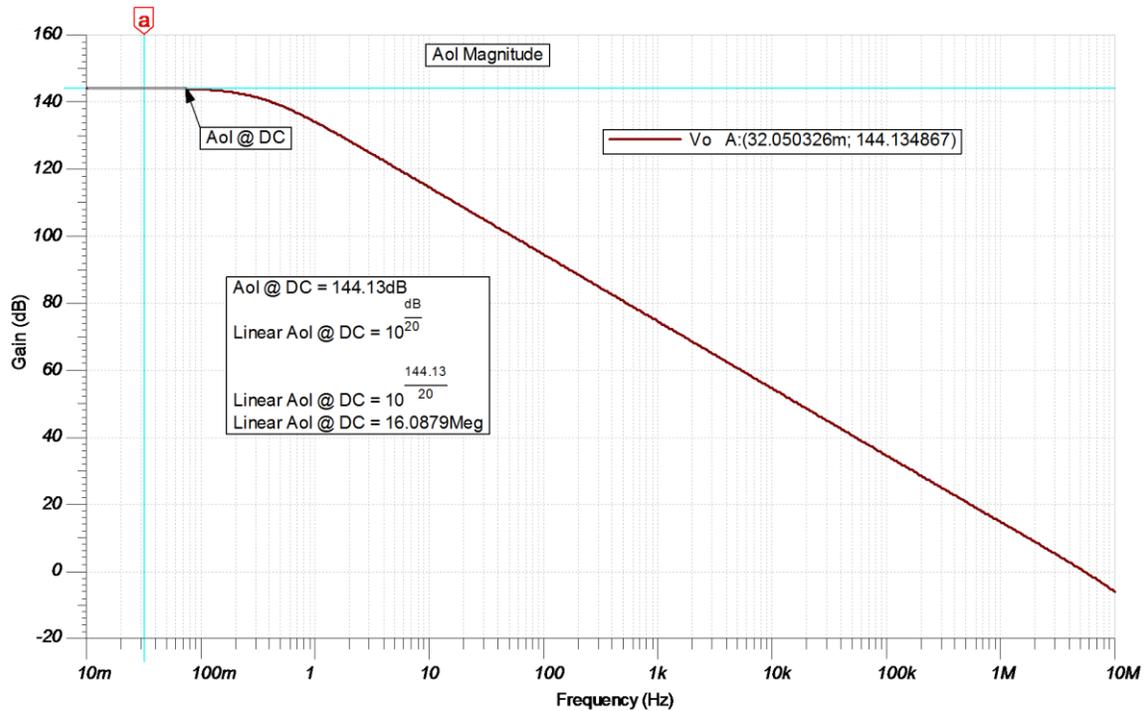


Fig. 11.7: Measuring Zo in SPICE: Step 1 – Measure Aol Results

The test circuit in Figure 11.8 will be constructed to measure Zo once we know the Aol of the amplifier. We want to design the closed loop gain of our Zo test circuit to be greater than the Aol of the amplifier for all frequencies of interest. Such a design will guarantee that when we test for Zo it will be Zo and not Zout. Remember that AolBeta on a dB plot is Aol(dB) – 1/Beta(dB). So if 1/Beta is larger than Aol we will have AolBeta = -?dB or a very small number. Then from  $Z_{out} = Z_o / (1 + Aol\beta)$ , for  $Aol\beta < 0.1$  (or -20dB),  $Z_{out}$  approaches  $Z_o$ .

Set  $R_L = 100$  ohms;  
 Low enough for no bias issues and low enough for no  $C_{in}$  issues  
 Set Closed Loop Gain =  $10 \times (Aol @ DC)$ ;  
 ensure op amp will run in open loop for frequencies of interest  
 $R_F = R_L \cdot 10 \cdot (Aol @ DC)$   
 $R_F = 100 \cdot 10 \cdot 16.0879M = 16.0879G$

Select LT for the low est frequency of interest (fz)

$$f_z = \frac{R_F}{L_T \cdot 2 \cdot \pi}$$

For our example choose  $f_z = 10 \mu Hz$

$$f_z = \frac{R_F}{L_T \cdot 2 \cdot \pi} \text{ implies } L_T = \frac{R_F}{f_z \cdot 2 \cdot \pi}$$

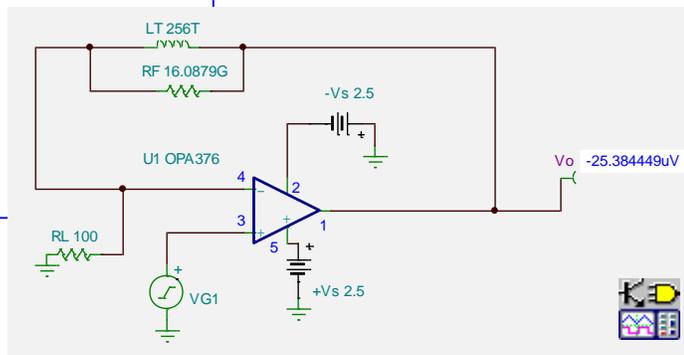
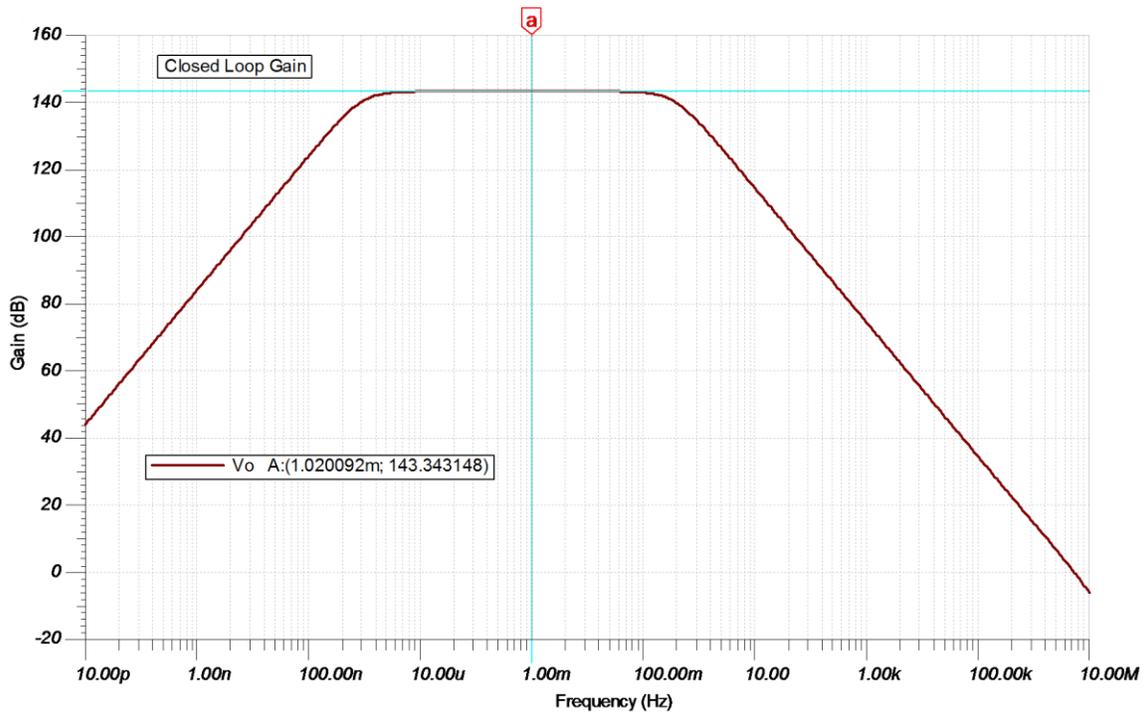
$$L_T = \frac{16.0879G}{10 \mu Hz \cdot 2 \cdot \pi} = 2.56e14 = 256TH$$


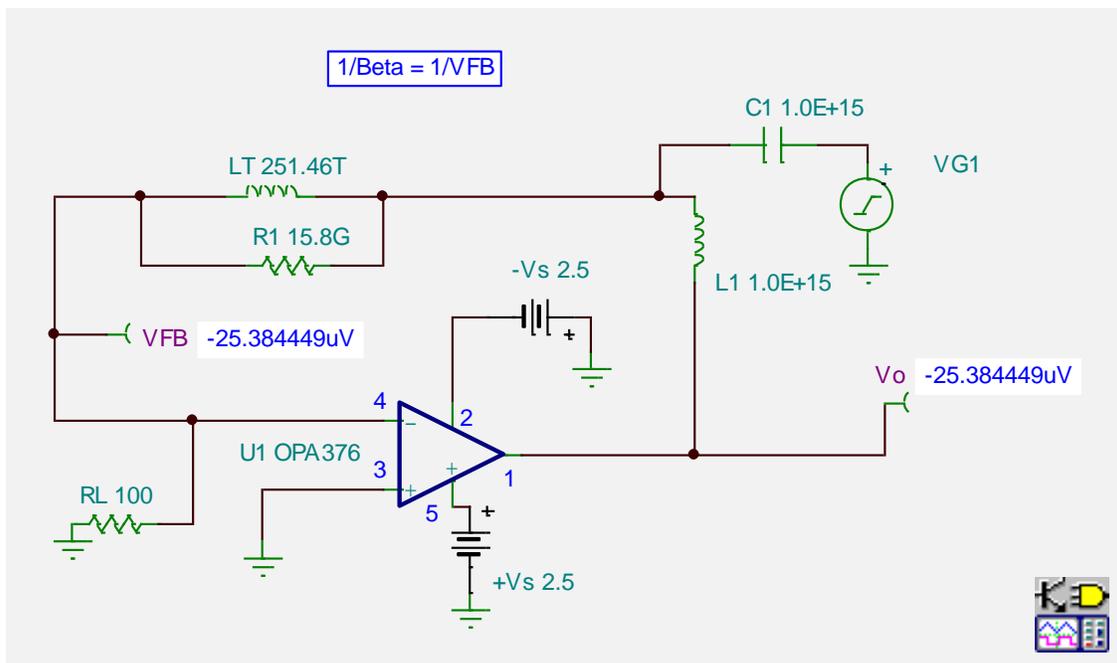
Fig. 11.8: Measuring Zo in SPICE: Step 1 – Configure Closed Loop Gain

Using the design criteria in Figure 11.8 we build a Zo Test Circuit and the TINA SPICE simulation results in Figure 11.9 show us the closed loop gain.



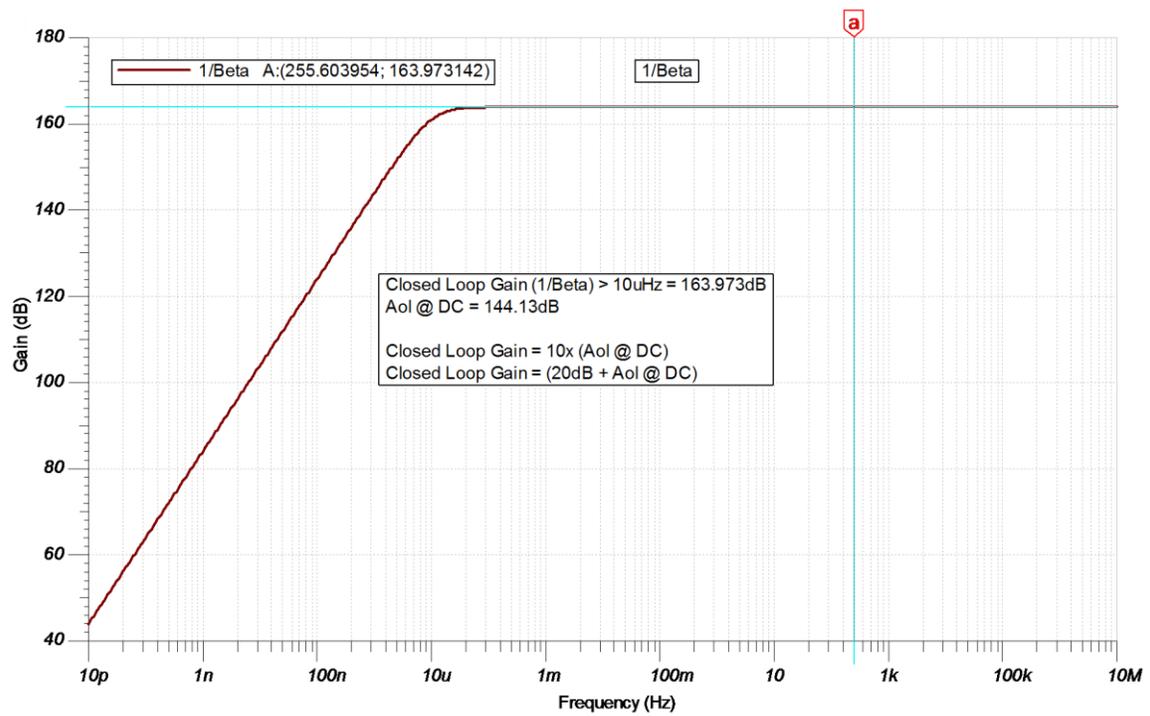
**Fig. 11.9: Measuring Zo in SPICE: Step 1 – Configure Closed Loop Gain Results**

To check our design procedure for the Zo Test Circuit we will use the circuit in Figure 11.10 to measure 1/Beta.



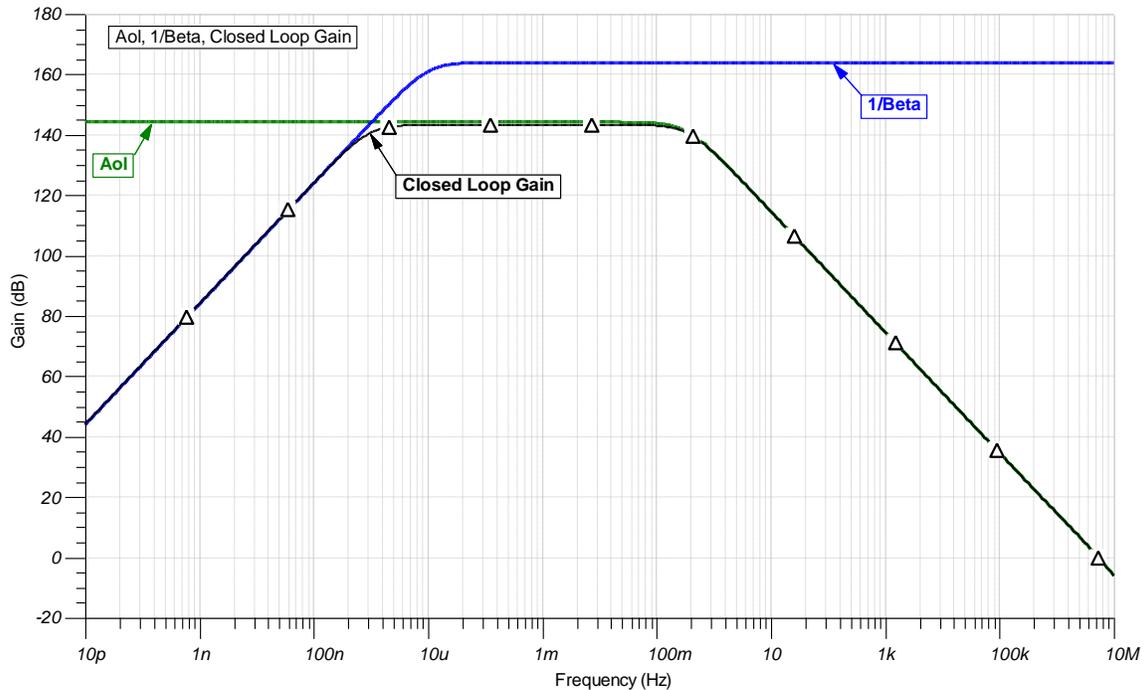
**Fig. 11.10: Measuring Zo in SPICE: Step 3 – 1/Beta Test**

1/Beta is seen in Figure 11.11 to be greater than our Aol at DC by at least 20dB.



**Fig. 11.11: Measuring Zo in SPICE: Step 3 – 1/Beta Test Results**

In Figure 11.12 the Zo Test Circuit responses are all plotted on one plot. We see the op amp Aol and our 1/Beta plot that yield the desired closed loop gain we originally designed for. From this figure above we see that any Zo measurements less than 300nHz will not be valid since Aol Beta will approach 0db (or 1V/V) and Zout will not be Zo.



**Fig. 11.12: Measuring Zo in SPICE: Step 4 – Aol, 1/Beta, Closed Loop Gain**

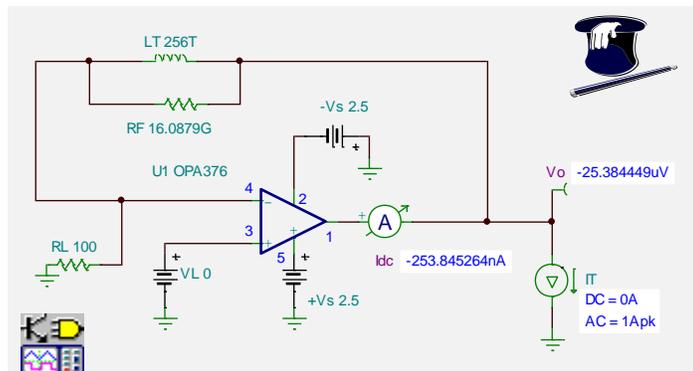
In Figure 11.13 our final Zo test circuit will use our closed loop gain designed in previous slides to force the op amp to run open loop for test frequencies of interest. We will attach a current generator, IT, on the output of the op amp. The current generator will be set to 0A at DC. This will not affect the DC operating point found by DC Analysis since a 0A DC current source is high impedance by definition. The current generator, IT, will be swept over frequency in an AC Analysis to test for Zo impedance over frequency by dividing Vo by IT.

**Op Amp Zo Test**

$Z_o = V_o / I_T$   
 Scale Logarithmic to remove  $20 \cdot \log(V_o / I_T)$   
 Log scale  $\rightarrow Z_o = V_o$  in ohms

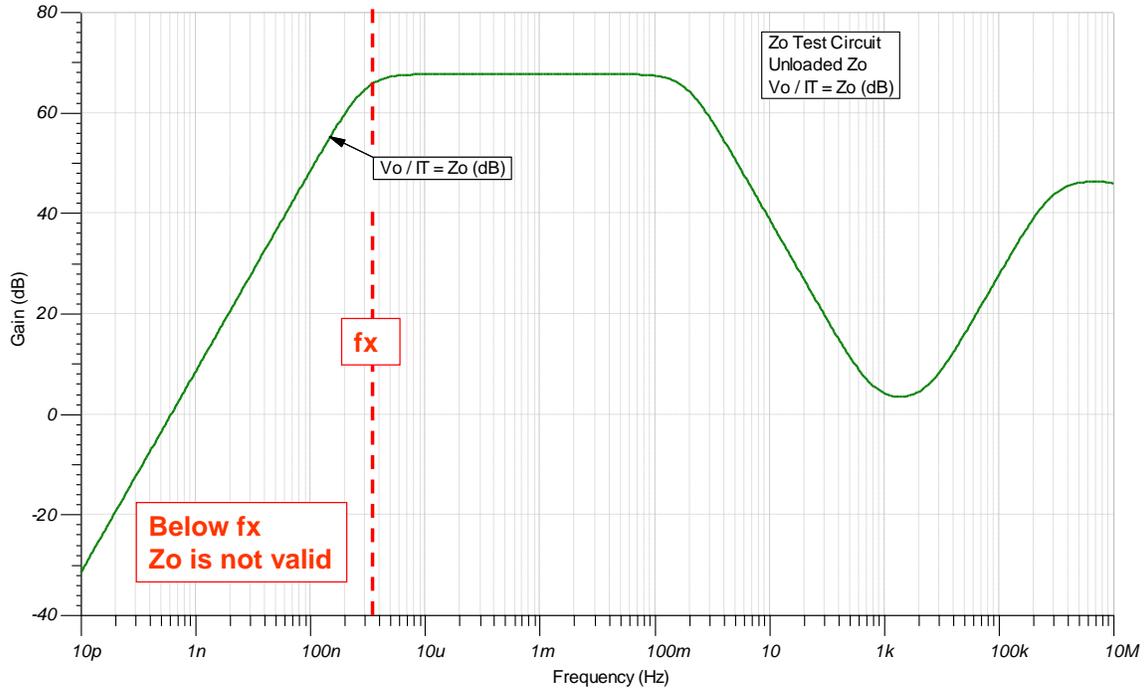
**Loaded Zo Test:**  
 $I_{dc} = V_L / R_L$   
 Test Loaded Zo for both +I<sub>dc</sub> and - I<sub>dc</sub>

**Unloaded Zo Test:**  
 Set  $V_L = 0V$



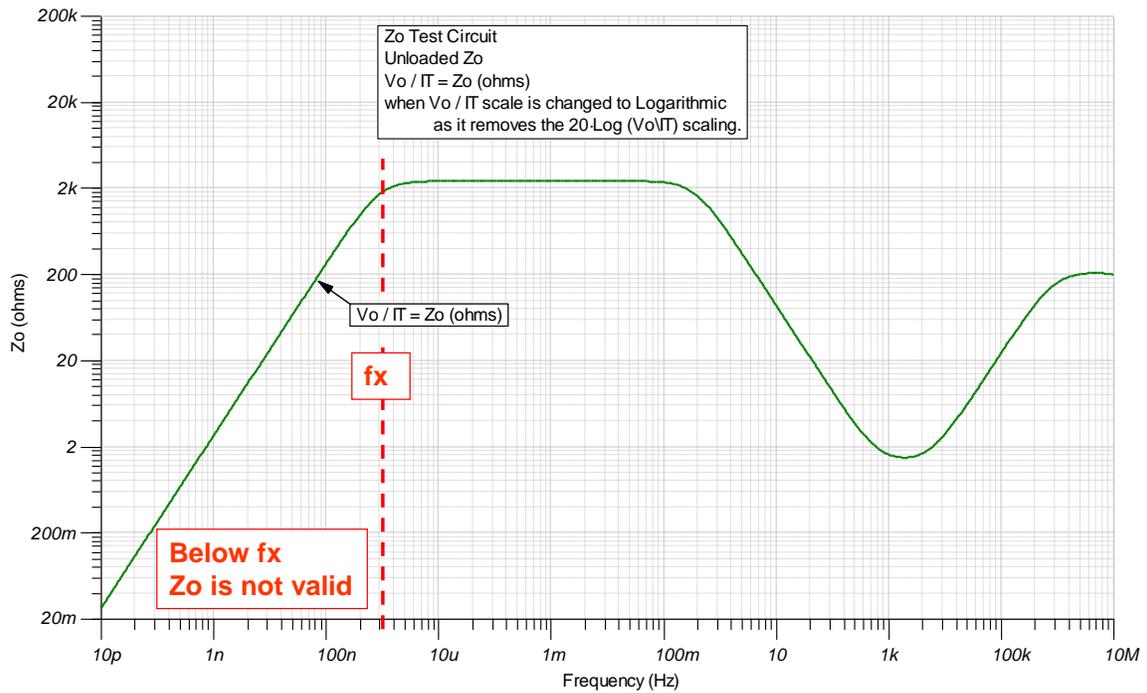
**Fig. 11.13: Measuring Zo in SPICE: Step 5 – Final Zo Test Circuit**

The results of measuring the OPA376 Zo are shown in this Figure 11.14. Note that we recall any frequencies below  $f_x$  (about 300nHz) we do not know Zo since  $A_o/Beta$  is not  $<-20dB$  (or  $<0.1$ ). The results above on the Y-axis labeled Gain(dB) is really  $V_o/I_T$  in dB, or Zo.



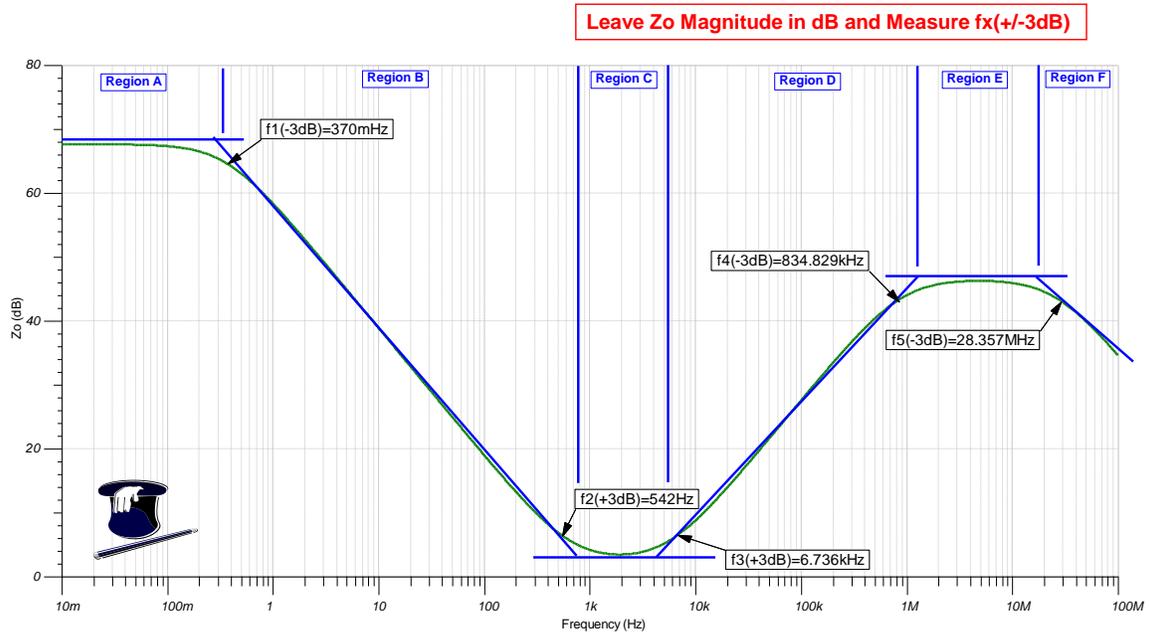
**Fig. 11.14: Measuring Zo in SPICE: Step 5 – Final Zo Test Circuit Results (dB)**

An easy way to convert our dB AC Analysis results into Zo in ohms is simply to change the Y-axis scaling in SPICE to Logarithmic which will yield Zo in ohms since the Y-axis is the results of  $V_o/I_T$  over frequency. The result of our OPA376 Zo Test in Figure 11.15 show that its Zo (starting at 300nHz and going up in frequency) is resistive, then capacitive, then resistive, then inductive then resistive. Not looking much like a simple  $R_o$  is it?



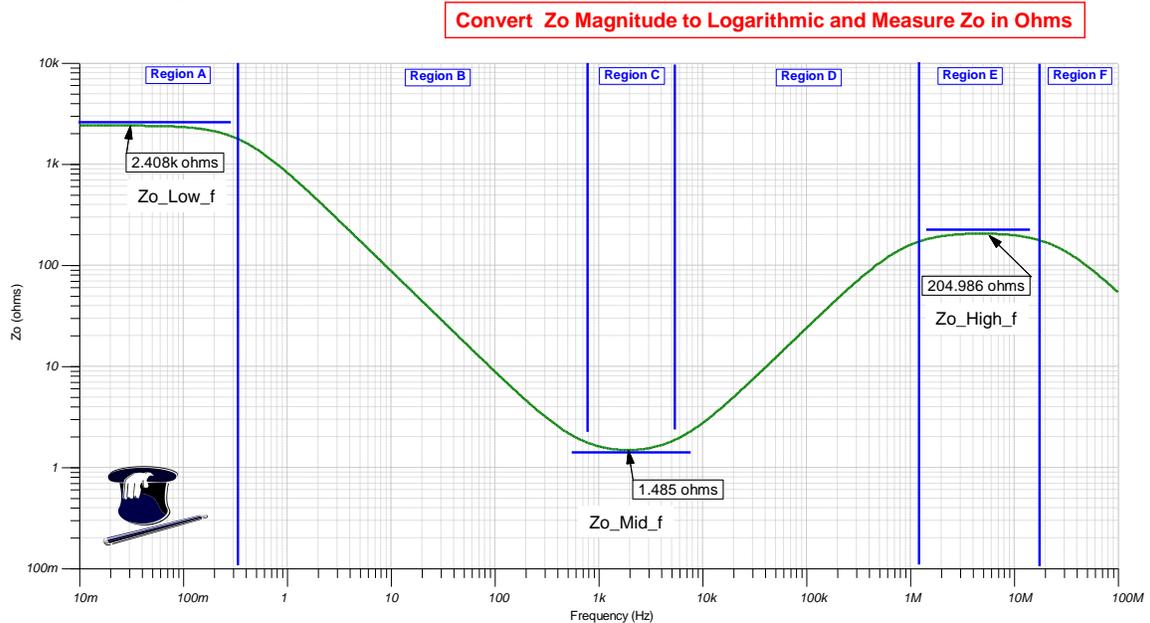
**Fig. 11.15: Measuring Zo in SPICE: Step 5 – Final Zo Test Circuit Results (ohms)**

For building our Zo Block we will first need frequency breakpoints from our measured Zo. These breakpoints are easily measured by leaving the AC Analysis test results in dB format and measuring for the 3dB points at all frequency transitions as shown in Figure 11.16.



**Fig. 11.16: Zo Regions and Frequency Breakpoints**

Secondly, to build our Zo block we will need magnitudes in any region where the Zo magnitude is flat. To easily get the magnitudes convert the Y-axis into Logarithmic and measure each region as shown in Figure 11.17.



**Fig. 11.17: Zo Regions and Magnitudes**

The Zo Measurement Steps and results for the OPA376 are shown in Figure 11.18. Now we are ready to proceed forward in building the external Zo block.

Zo Measurement Steps:



- 1) Measure Zo in dB to get frequency breakpoints
- 2) Convert to ohms to get magnitudes in flat regions
- 3) Now ready to build Zo\_External?

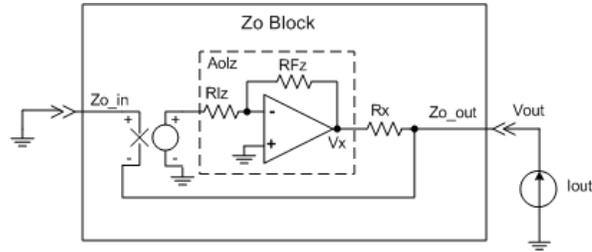
Summary of Zo Measurements			
Region	Freq Min	Freq Max	Zo Magnitude
	(Hz)	(Hz)	(ohms)
A	37m	370m	2.408k
B	370m	542	x1/10 decrease per decade frequency
C	542	6.736k	1.485
D	6.736k	834.829k	x10 increase per decade frequency
E	834.829k	28.357M	204.986
F	28.357M	28.357M	x1/10 decrease per decade frequency

**Fig. 11.18: Zo Measurement Steps**

There is probably more than one way to design an external Zo Block. Real inductors, capacitors, resistors in combination do not work easily since their combinations will cause peaking at resonances which do not exist in the real op amp Zo. The technique shown in Figure 11.19 is simple and straight forward to use once it is understood. A fixed Rx will be used inside the closed loop of an op amp whose Aol (Aolz in Figure 11.19) will be designed to create a Closed Loop Zout looking back into the op amp with feedback around Rx. The varying shape of the op amp Aol curve will yield the varying shape of the measured Zout of the configuration which will then be used as a Zo block. In Figure 11.19 we are measuring the Zo Block from its output and so we will call this "Zo\_reverse". Since this arrangement is identical to our familiar "Rout vs Ro Derivation" we know what Zo\_reverse is as shown in Figure 11.19. Note that if there is a ZL on the output of the Zo Block then the Zo\_reverse will be Zo in parallel with this ZL. If  $ZL \gg Zo_{rev}$  then  $Zo_{rev} = Rx/(1+Aolz)$ .

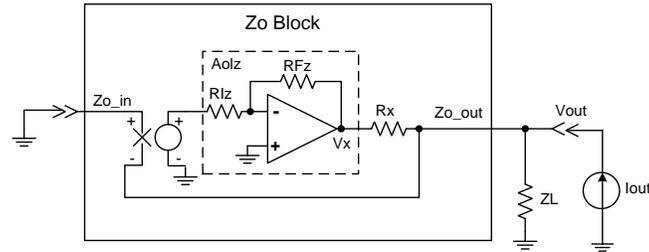
Zo\_reverse without ZL

$$Z_{o\_rev} = \frac{R_x}{1 + A_{olz}}$$



Zo\_reverse with ZL

$$Z_{o\_rev\_ZL} = \frac{(Z_{o\_rev} \cdot Z_L)}{Z_{o\_rev} + Z_L}$$



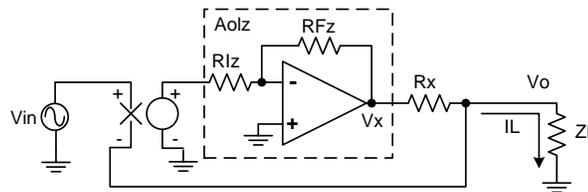
**Note for Zo\_rev\_ZL :**  
If  $Z_L \gg Z_{o\_rev}$ ; Then  $Z_{o\_rev\_ZL} = Z_{o\_rev} = R_x / (1 + A_{olz})$

**Fig. 11.19: Zo “Reverse” Equations**

For the Zo Block to give accurate simulation result for stability analysis the Zo must look the same in both directions. That is Zo\_forward must be the same as Zo\_reverse. Zo Forward is defined in Figure 11.20. The derivation for Zo\_fwd is given in the Appendix of this article. Zo\_fwd is whatever Vin is put into the Zo block divided by the current, IL, which comes out of the block as shown above. Note the effect of ZL on Zo\_fwd. If  $Z_L \ll R_x$  then  $Z_{o\_fwd} = R_x / A_{olz}$ .

Zo\_forward with ZL

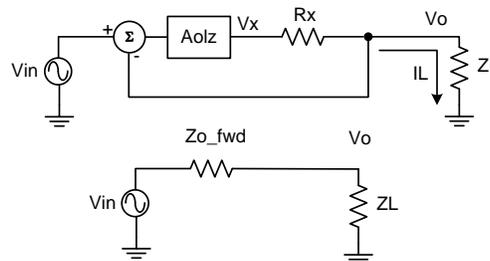
$$Z_{o\_fwd} = \frac{R_x + Z_L}{A_{olz}}$$



$$Z_{o\_fwd} = (V_{in} - V_o) / I_L$$

**Note:**  
If  $Z_L \gg R_x$  then  $Z_{o\_fwd}$  is dominated by  $Z_L$ .  $Z_L$  can be very large for a capacitive load in the middle to low frequency regions. This will yield erroneous stability analysis for Zo interacting with a capacitive load.

**Note for Zo\_fwd:**  
If  $Z_L \ll R_x$ ; Then  $Z_{o\_fwd} = R_x / A_{olz}$



**Fig. 11.20: Zo “Forward” Equations**

The example in Figure 11.21 shows how an improper design of the Zo Block can yield erroneous results with a capacitive load. This example places a 100pF capacitor directly on the output of a Zo block whose  $R_x = 400k$ . Notice that on the “Zo\_forward open, 100p” curve that until  $Z_L$  ( $X_c$  of the 100pF capacitive load) becomes  $< 1/2$  of  $R_x$  (about 162k) that the “Desired Zo” curve is not met. Other curves are shown with varying resistive loads in parallel with the capacitive load of

100pF. In conclusion we see that the larger ZL is in comparison to Rx the more in error Zo\_fwd is from our Desired Zo in this improperly designed Zo Block.

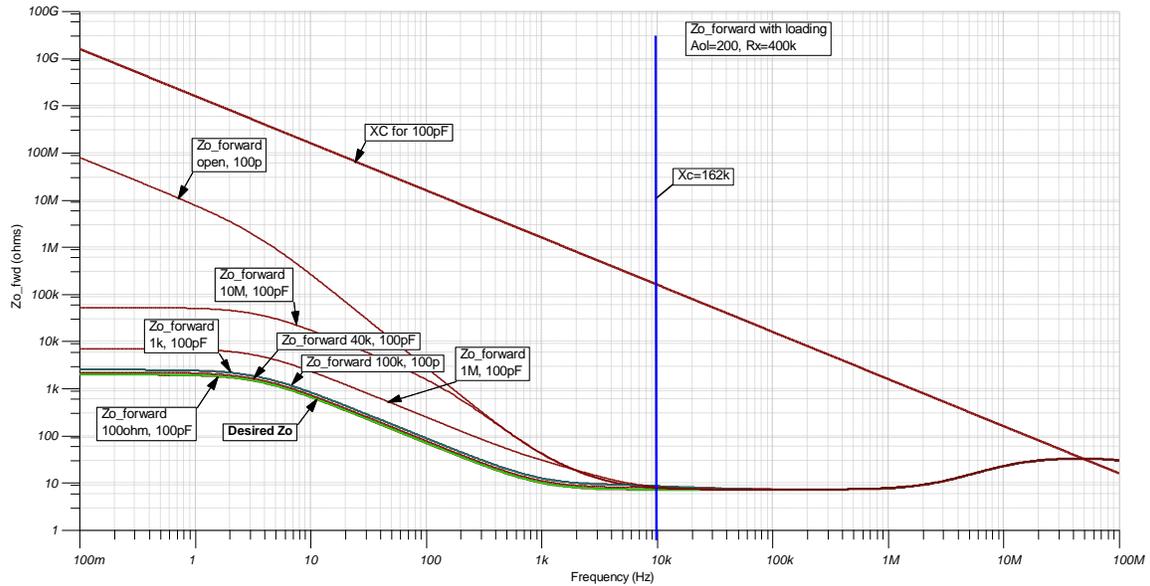


Fig. 11.21: Improper Zo Design – ZL >>Rx

Figure 11.22 shows how to fix our problem of ZL larger than Rx creating errors in our Desired Zo by adding an Rdummy inside of our Zo Block to keep the “effective ZL” at a minimum value regardless of how high in value the actual load external to the Zo Block becomes. Note also that because of the special arrangement and use of VCVS1 (gain =-1) the gain computation for Aolz is simply the respective  $(-Z_F/Z_I) * (-1)$  or just  $Z_F/Z_I$  where ZF and ZI represent the total impedance in the feedback (ZF) or input (ZI) path of our closed loop op amp arrangement.

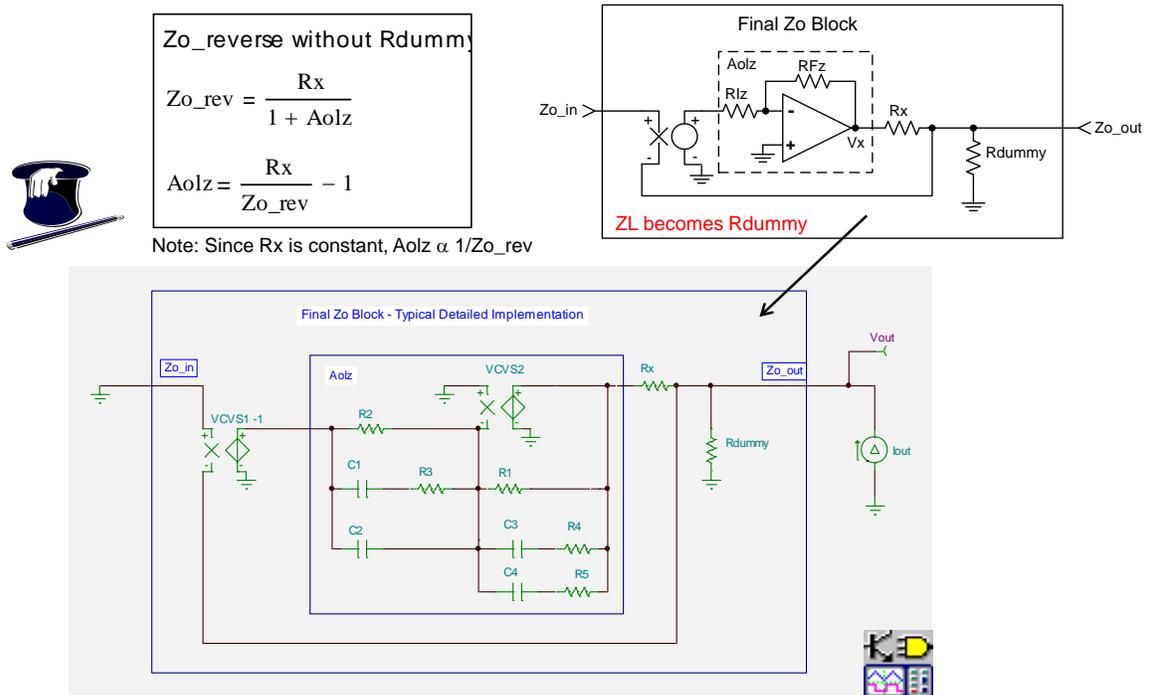


Fig. 11.22: Final Zo Block Architecture

Now let's zoom out and evaluate our design trade-offs for the Zo Block as shown in Figure 11.23. We note that no matter how we design the Zo Block using our proposed architecture it will never have  $Zo_{fwd} = Zo_{rev}$ . However, we will see as we proceed forward that it will yield acceptable closeness between  $Zo_{fwd}$  and  $Zo_{rev}$  to make this external Zo Block a powerful tool in our stability analysis toolbox. Based on many, many Zo Blocks built, with this architecture, we see in Figure 11.23 a few design rules of thumb, to optimize the Zo Block design on first pass.

**Note Design Compromise:**

$Zo_{fwd}$ : If  $R_{dummy} \ll R_x$ ; Then  **$Zo_{fwd} = R_x/Aolz$**

$Zo_{rev}$ : If  $R_{dummy} \gg Zo_{rev}$ ; Then  **$Zo_{rev} = R_x/(1+Aolz)$**

**Final Zo Block Design Guidelines:**

- 1)  $R_{dummy} > 50 * Zo_{Low\_f}$   
(or  $50 * \text{Highest } Zo \text{ value} - \text{usually } Zo_{Low\_f}$ )
- 2)  $R_x > 10 * R_{dummy}$
- 3)  $VCVS2 > 10 * (\text{Largest } Aolz \text{ Magnitude})$



**Fig. 11.23: Zo Block Design Guidelines**

Before we begin our OPA376 Zo Block final design we might want to consider designing a little Excel spreadsheet calculator to help speed the computation process. As shown in Figure 11.24 a “3dB Frequency” calculator will allow us to enter a frequency and an R value and compute the necessary C value. The “R1 or R2 from Req Parallel” calculator allows us to enter Req (parallel resistance desired) and either R1 or R2 and compute the other parallel resistor value, either R2 or R1, to yield Req. The “Req Parallel” calculator accepts R1 and R2 and computes Req or the parallel combination of R1 and R2.

3dB Frequency				Comment
Enter	f =	3.2800E+04	Hz	f = 1/(2*pi*R*C)
	R=	1.4000E+02	ohms	
Compute	C=	3.4659E-08	Farads	C = 1 / (f*2*pi*R)
Enter	f =	8.1100E+05	Hz	
	C=	1.5900E-08	Farads	
Compute	R=	1.2342E+01	ohms	R = 1 / (f*2*pi*C)
<b>R1 or R2 from Req Parallel</b>				
Enter	Req=	1.2277E+02	ohms	Req = (R1*R2) / (R1+R2)
	R1=	1.0000E+03	ohms	
Compute	R2=	1.3995E+02	ohms	R2 = (Req*R1) / (R1 - Req)
Enter	Req=	1.6363E+04	ohms	Req = (R1*R2) / (R1+R2)
	R1=	2.5641E+04	ohms	
Compute	R2=	4.5221E+04	ohms	R1 = (Req*R2) / (R2 - Req)
<b>Req Parallel</b>				
Enter	R1=	2.0000E+03	ohms	
	R2=	2.0000E+03	ohms	
Compute	Req=	1.0000E+03	ohms	Req = (R1*R2) / (R1+R2)

Fig. 11.24: Excel Calculator Ideas for Ease of Zo Block Build

Three more calculators in Excel will help speed the Zo Block building process. As shown in Figure 11.25 the "Gain RI Calculator" allows us to enter a desired Gain and RF and compute the required RI. The "Gain RF Calculator" allows us to enter a desired Gain and RI and compute the required RF. The "Aolz Calculator" requires we enter the desired Zo\_rev and fixed value of Rx so it can compute the required Aolz.

Gain RI				Comment
Enter	Gain =	8.0800E+05	V/V	Gain = RF / RI
	RF=	4.9734E+04	ohms	
Compute	RI=	6.1552E-02	ohms	RI = RF / Gain
<b>Gain RF</b>				
Enter	Gain =	4.9734E+02	V/V	Gain = RF / RI
	RI =	1.0000E+02	ohms	
Compute	RF=	4.9734E+04	ohms	RF = Gain * RI
<b>Aolz Calculator</b>				
Enter	Zo_rev =	2.05E+02	ohms	
	Rx =	1.20E+06	ohms	
Compute	Aolz =	5.8531E+03	V/V	Aolz = (Rx/Zo_rev) - 1

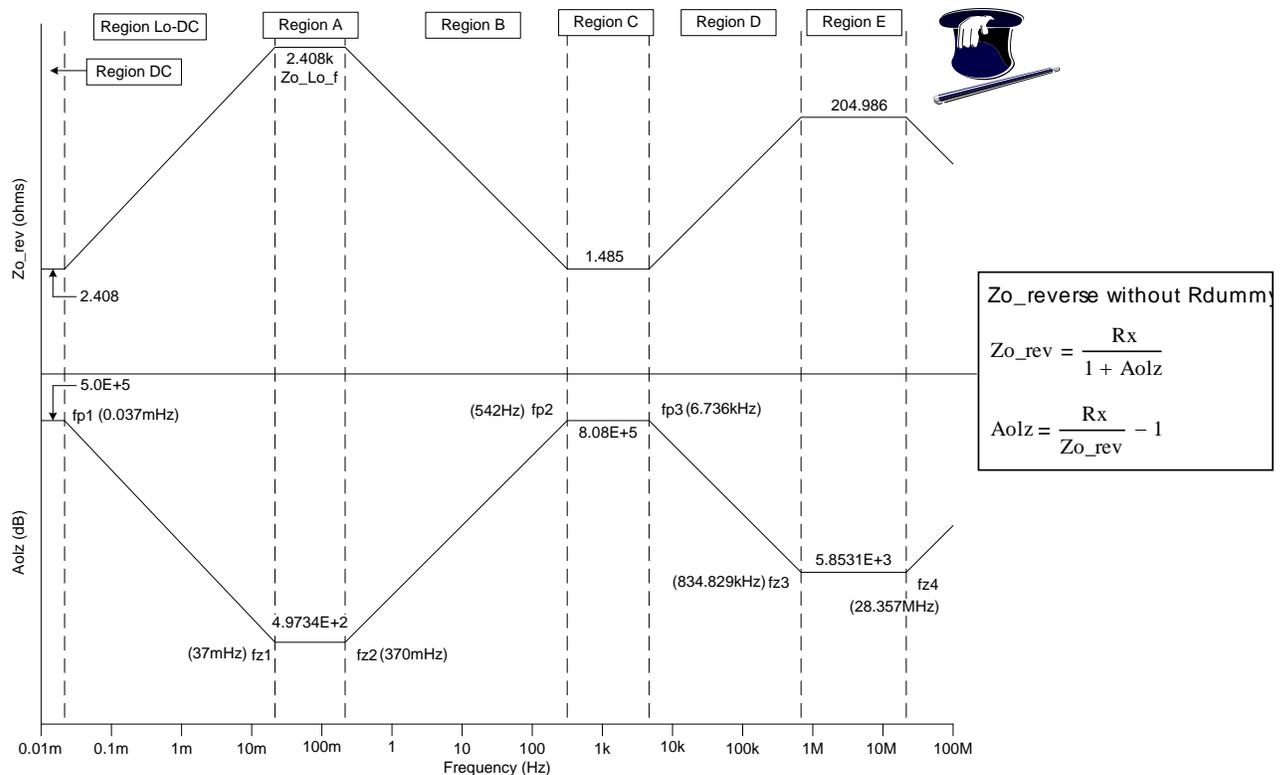
Zo\_reverse without Rdummy

$$Zo_{rev} = \frac{Rx}{1 + Aolz}$$

$$Aolz = \frac{Rx}{Zo_{rev}} - 1$$

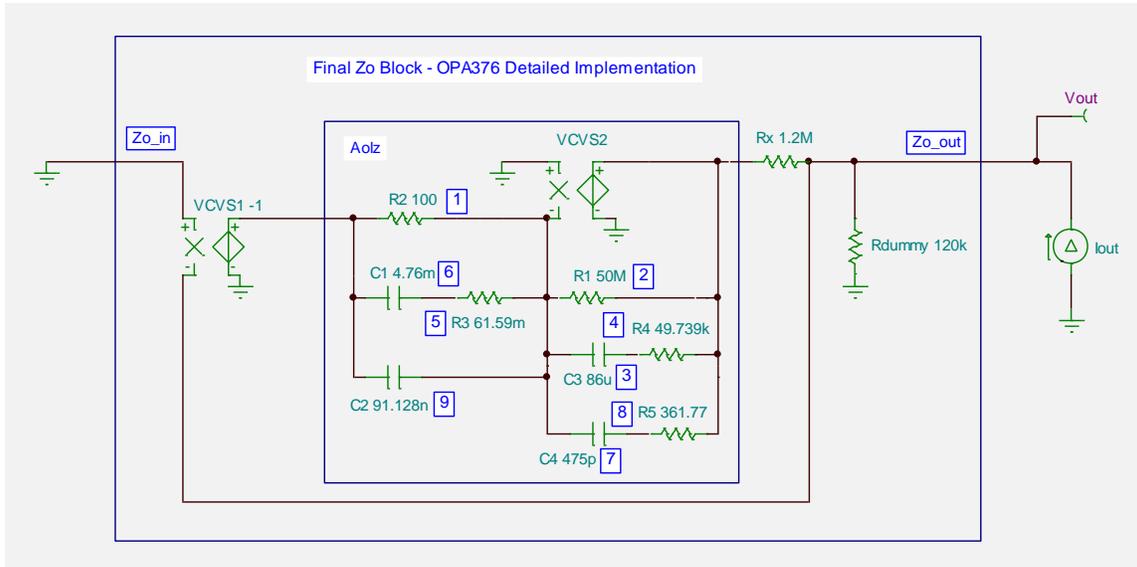
Fig. 11.25: Excel Calculator Ideas for Ease of Zo Block Build (cont.)

The real trick to building the Zo Block is shown in Figure 11.26. Notice that we have added two new regions, Region Lo-DC and Region DC, to the OPA376 Zo Curve. When we go to use our Zo Block in SPICE simulations, SPICE will run a DC Analysis before it runs an AC Analysis. We need to make sure that the DC Operating Point can be found correctly without our External Zo Block adding any errors or preventing DC Convergence. To guarantee this find we need a low value at DC for our Zo Block. For example if the OPA376, in a final SPICE application circuit, needs 3mA for the DC Operating point to be correct then a Zo Block with a flat curve at DC of 2.408kohms would drop >6V across our Zo Block from a 5V single supply on the OPA376. The DC Operating Point would not be found. From the OPA376 we know it can drive 3mA with only about a 10mV drop from the 5V rail. This is the difference between the real op amp and the original SPICE macromodel of the OPA376, and our External Zo Block. Our External Zo Block does not model the DC or Large Signal Zo behavior of the op amp. The focus of our Zo Block is for AC Stability Analysis. There are other, more complicated ways, to add a DC and Large Signal Zo operation of an External Zo Block, but they are not required for our end goal and are beyond the focus of this article. In Figure 11.26 we see our desired Zo curve over frequency. Remember that the relationship between Zo\_rev and Aolz is a reciprocal one for a fixed Rx. So once we know the desired Zo\_rev the necessary Aolz, for a fixed Rx, is simply the inverse shape of it. Shown in Figure 11.26 are the measured frequency points and desired magnitudes from the OPA376 Zo testing in the Zo\_rev curve. The Aolz curve gets its frequency points from the Zo\_rev Curve and its magnitudes from the magnitude values in the Zo\_rev Curve, Rx, and using the Aolz Calculator.



**Fig. 11.26: Final OPA376 Zo Block Design**

To achieve the desired Aolz for a fixed Rx we will need the circuit arrangement in Figure 11.27. Now we are ready to compute values for this circuit. From our design guidelines in Figure 11.23 we choose Rx=1.2Meg, Rdummy=120k, and VCVS2 = 5M.



**Fig. 11.27: Final OPA376 Zo Block Design – Zo\_rev Test**

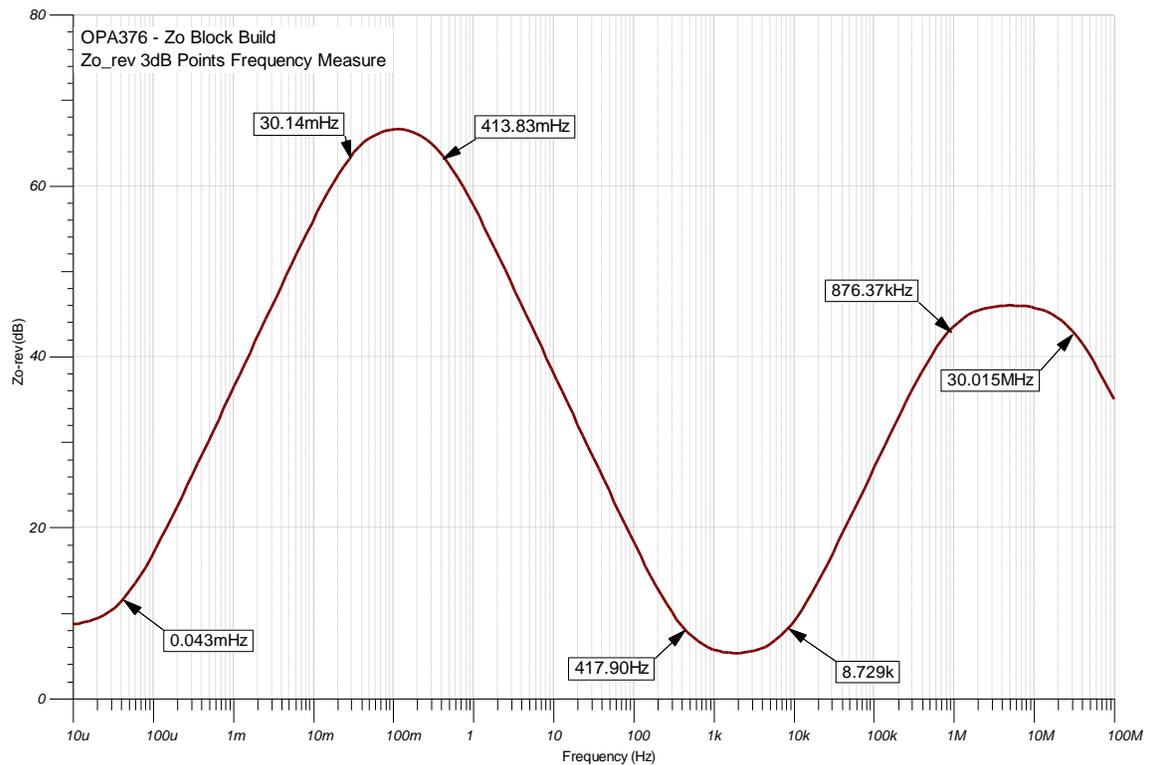
Figure 11.26 and Figure 11.27 should be referred to as we walk through our OPA376 Zo Block design using the table in Figure 11.28. We will start from Region DC and work our way from low frequency to high frequency in the design process. Refer to Figure 11.26. There are Pole Eq and Zero Eq equations which are estimates as to the respective pole and zero locations. These estimates will use the dominant components which set the pole or zero and will turn out to be accurate enough for our external Zo Block. If one is concerned about exact accuracy he can derive and use more exact equations for the pole/zero locations. Remember to use our Excel calculators to simplify the choosing of values for R and/or C to design the respective Aolz. You will notice in the table in Figure 11.28 that we do not compute every pole and zero since the ones we do not compute will occur as a result of the +/-20dB/decade slopes in the Aolz curve along with its flat regions. Step 1 is to set a starting point by assigning R2=100 ohms to keep R1 reasonable. Step 2 is in Region DC where gain is set by R1/R2 and we need Aolz=5.0e+5 which yields R1=50Mohm for R2=100ohm. Step 3 is at the start of the Region Lo-DC where we need fp1=0.037mHz which is predominantly set by R1 and C3. C3 is chosen to yield fp1 given R1. Step 4 uses Region A where we need Aolz=4.9734e+2. This gain is set by (R1//R4)/R2. We already know R1 and R2 so we solve for R4. Step 5 jumps ahead to Region C where gain is set by (R1//R4) / (R2//R3). Since we know R1, R2, and R4 we can solve for R3 to get Aolz=8.08e+5. Step 6 computes the pole fp2 in Region B by computing C1 based on R3 which we now know. Step 7 computes fp3 at the beginning of Region D. fp3 is determined by (R1//R4) and C4. Since we know R1 and R4, C4 is computed to yield fp3. Step 8 computes Aolz=5.8531e+3 by [R5//((R1//R4))]/[R2//R3]. Since we already have R1, R2, R3, and R4 we can compute R5. Finally Step 9 will give us fz4 by (R2//R3) and C2. Since we know R2 and R3 we will compute for C2. The external Zo Block for OPA376 is now completely built and ready for testing.

Zo Final Build Data											
Region	Freq Min (Hz)	Freq Max (Hz)	Zo Magnitude (ohms)	Aolz (V/V)	Aolz Eq	Pole (Hz)	Pole Eq (estimate)	Zero (Hz)	Zero Eq (estimate)	Solve	Step
DC	DC	0.037m	2.408	5.0000E+05	R1/R2					R2=100Ω R1=50MΩ	1 2
Lo-DC	0.037m	37m	x10 increase per decade frequency	-20db slope		0.037m	fp1= 1/(2*pi*R1*C3)			C3=86uF	3
A	37m	370m	2.408k	4.9734E+02	Z14/R2					R4=49.739kΩ	4
C	542	6.736k	1.485	8.0800E+05	Z14/Z23					R3=61.59mΩ	5
B	370m	542	x1/10 decrease per decade frequency	+20db slope		542	fp2= 1/(2*pi*R3*C1)			C1=4.76mF	6
D	6.736k	834.829k	x10 increase per decade frequency	-20db slope		6.736k	fp3= 1/(2*pi*Z14*C4)	834.829k		C4=475pF	7
E	834.829k	28.357M	204.986	5.8531E+03	(R5//Z14)/Z23					R5=361.77Ω	8
F	28.357M	28.357M	x1/10 decrease per decade frequency	+20db slope				28.357M	fz4= 1/(2*pi*Z23*C2)	C2=91.128nF	9

Rdummy = 120kohm, Rx = 1.2Mohm, R2 = 1kohm, Z14=R1//R4, Z23=R2//R3

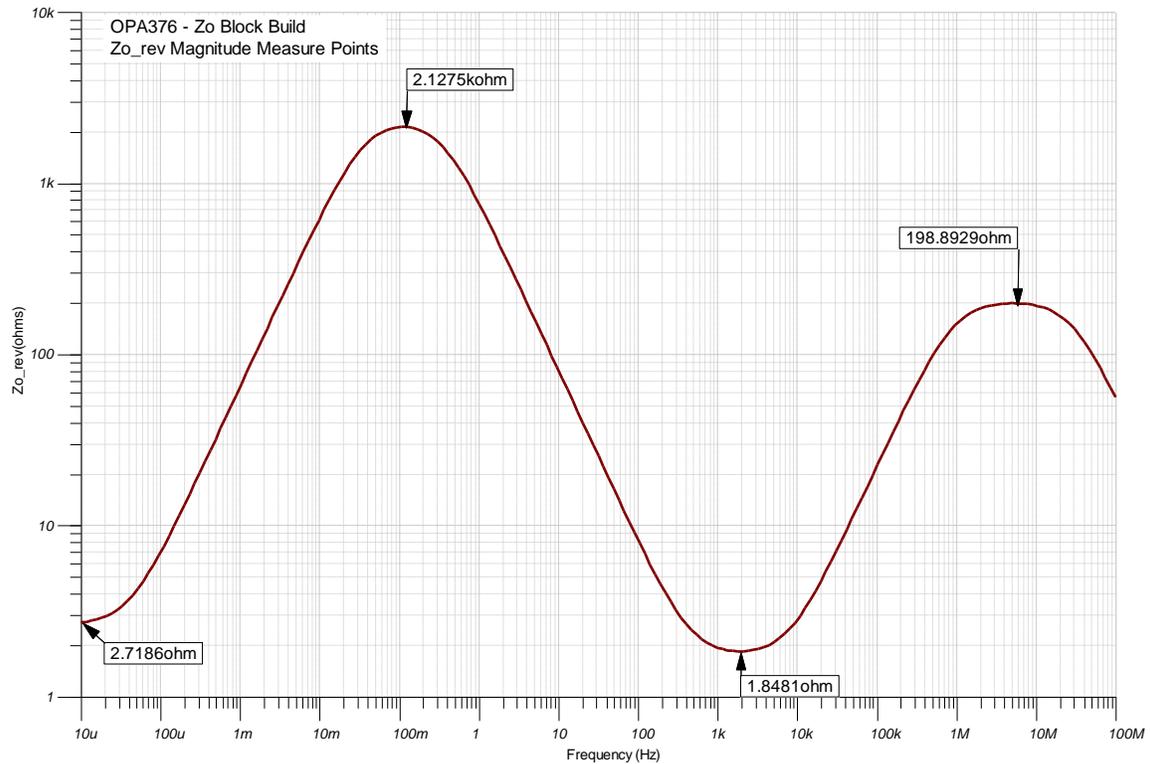
**Fig. 11.28: Final OPA376 Zo Block Design – Equations**

The final build of our OPA376 Zo Block is tested for Zo\_rev and the results shown in the Figure 11.29 where we leave the results in dB and measure the frequency 3dB points.



**Fig. 11.29: Final OPA376 Zo Block Design – Zo\_rev Curve (dB)**

In Figure 11.30 our final build of our OPA376 Zo Block is tested for Zo\_rev and the magnitude results on the Y-axis are converted to Logarithmic so we can read Zo\_rev magnitude directly in ohms.



**Fig. 11.30: Final OPA376 Zo Block Design – Zo\_rev Curve (ohms)**

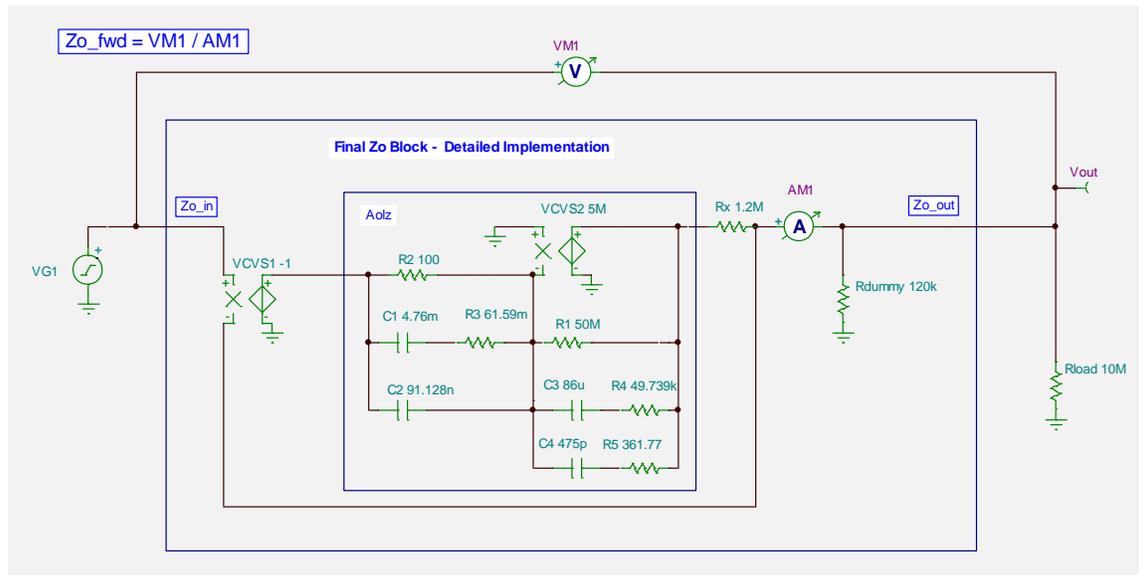
In Figure 11.31 we compile the results of Zo\_rev to compare both frequency break points and magnitude flat regions of the final OPA376 Zo Block build and the original design goal for each respective data point. Our final frequency breakpoints are going to be bounded by +20dB/decade or -20dB/decade slopes with intercepts at the flat magnitude portions of the Zo\_rev curve. As we compare Design Frequency points versus Final Frequency points we see 10%-20% variance from Desired versus Final Frequency points. In the magnitude comparison we see 3%-24% variance from Desired versus Final Magnitude points. This is an acceptable result without spending any more design time on the Zo Block. Real silicon typically has capacitors that vary +/-20% over process and temperature along with resistors that vary +/-30% over process and temperature. So yes, Zo, will vary from part to part but with our stability techniques in place using decade rules-of-thumb we will have good design margin.

Zo_rev	Design	Final	Design	Final
	Frequency	Frequency	Magnitude	Magnitude
Parameter	(Hz)	(Hz)	(ohms)	(ohms)
Region DC			2.408	2.7186
fp1	0.037m	0.043m		
fz1	37m	30.14m		
Region A			2.408k	2.1275k
fz2	370m	413.83m		
fp2	542	417.90		
Region C			1.485	1.8481
fp3	6.736k	8.729k		
fz3	834.829k	876.37k		
Region E			204.986	198.8929
fz4	28.357M	30.015M		



**Fig. 11.31: Final OPA376 Zo Block Design – Build vs Design Goal**

Our Final Zo Block for the OPA376 will now be tested for Zo\_fwd using the test circuit in Figure 11.32 in TINA SPICE.



**Fig. 11.32: Final OPA376 Zo Block Design – Zo\_fwd Test**

The TINA SPICE results of our testing Zo\_fwd versus Zo\_rev is shown in Figure 11.33. We see the Zo\_fwd versus Zo\_rev does have some minor discrepancies as we would expect since the equations for Zo\_fwd versus Zo\_rev have a small difference. Recall from Figure 11.23:  $Zo\_fwd = Rx/Aolz$  and  $Zo\_rev = Rx/(1+Aolz)$ . The frequency differences are so small they are not of any concern. Figure 11.34 shows the results of our testing Zo\_fwd versus Zo\_rev with the Y-axis set to Logarithmic scale so we can compare the magnitude differences directly in ohms. Magnitude comparison shows Zo\_rev and Zo\_fwd within 12% at all flat magnitude portions of the curve. Again, no major concerns here.

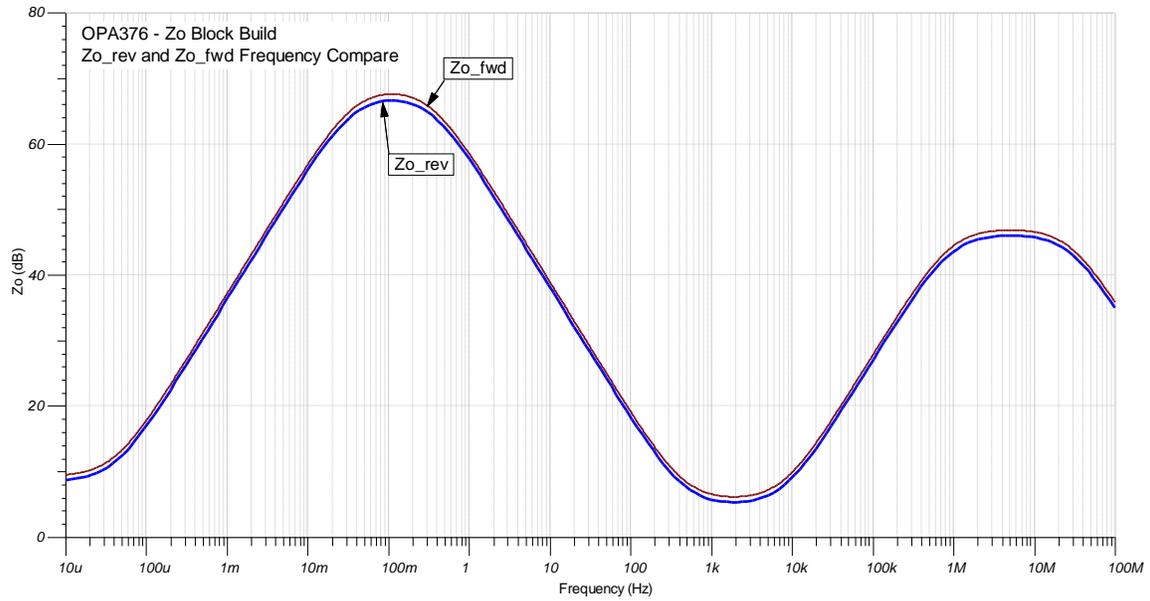


Fig. 11.33: Final OPA376 Zo Block Design – Zo\_fwd and Zo\_rev Frequency Compare

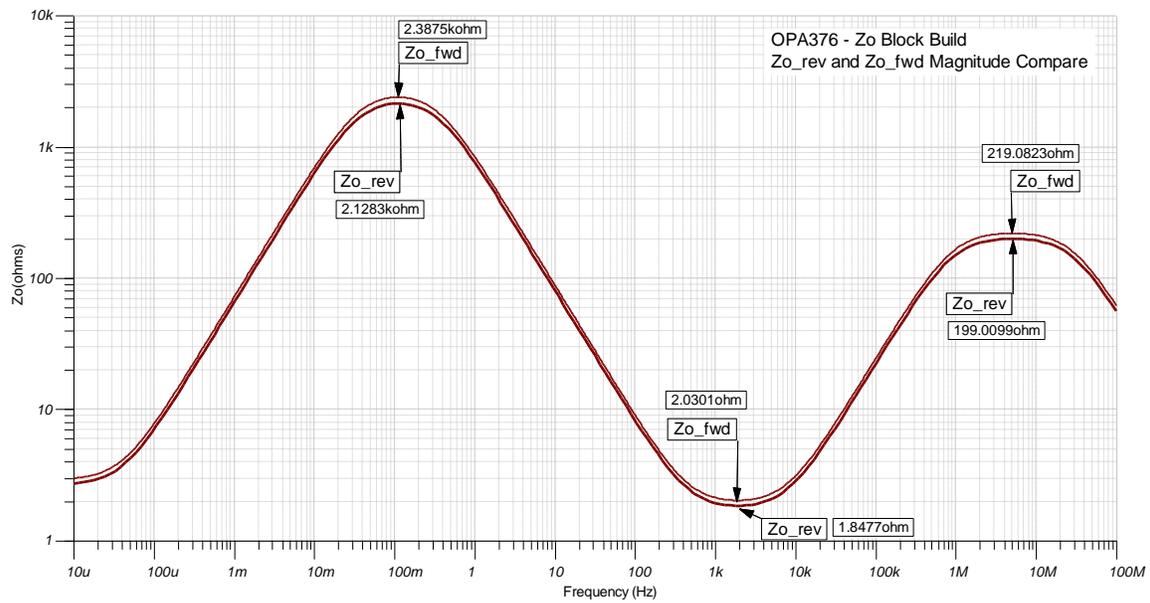


Fig. 11.34: Final OPA376 Zo Block Design – Zo\_fwd and Zo\_rev Magnitude Compare

In Figure 11.35 we add a side note on building Zo External Blocks. Sometimes the Zo\_Lo\_f value is not available from the data sheet as the curve may still be rising at the rate of 20dB/decade as it approaches lower frequencies. Sometimes the op amp Electrical Characteristics table will contain two different test conditions for the Open Loop Gain or Aol specification as shown in Figure 11.35. Based on these measurements one can compute the Zo\_Lo\_f value as shown in Figure 11.35. The detailed derivation of this equation is included in this article's Appendix.

Typical Electrical Characteristics Table Excerpt		
Parameter	Condition	Typical
Open Loop Gain	Vs = 5V, RL = 10k	134dB
	Vs = 5V, RL = 2k	126dB



$$R_o = \frac{RL2 - \left( \frac{Aol1}{Aol2} \cdot \frac{RL2}{RL1} \right) \cdot RL1}{\left( \frac{Aol1}{Aol2} \cdot \frac{RL2}{RL1} \right) - 1}$$

Parameter	Aol	Aol	RLx
	(dB)	(V/V)	(ohms)
Aol1	134	5.01E+06	RL1=10k
Aol2	126	2.00E+06	RL2=2k

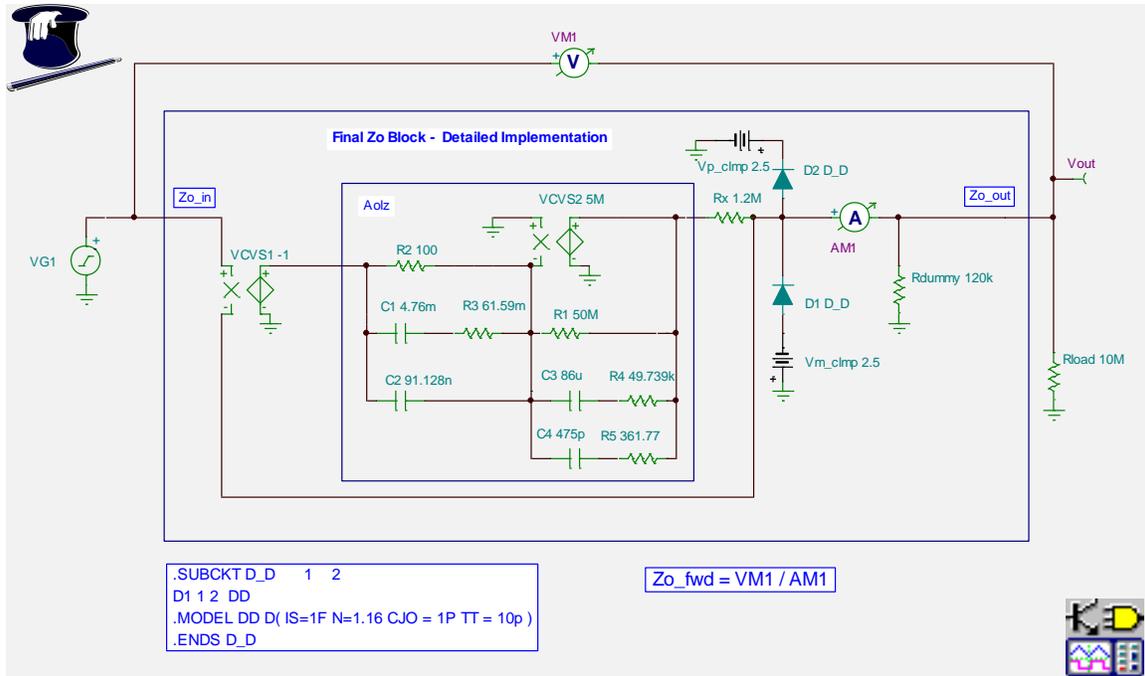
$$R_o := \frac{2000 - (2.50 \cdot 2) \cdot 10000}{(2.50 \cdot 2) - 1}$$

Aol1/Aol2	RL2/RL1
2.5	5

$$R_o = 6 \times 10^3$$

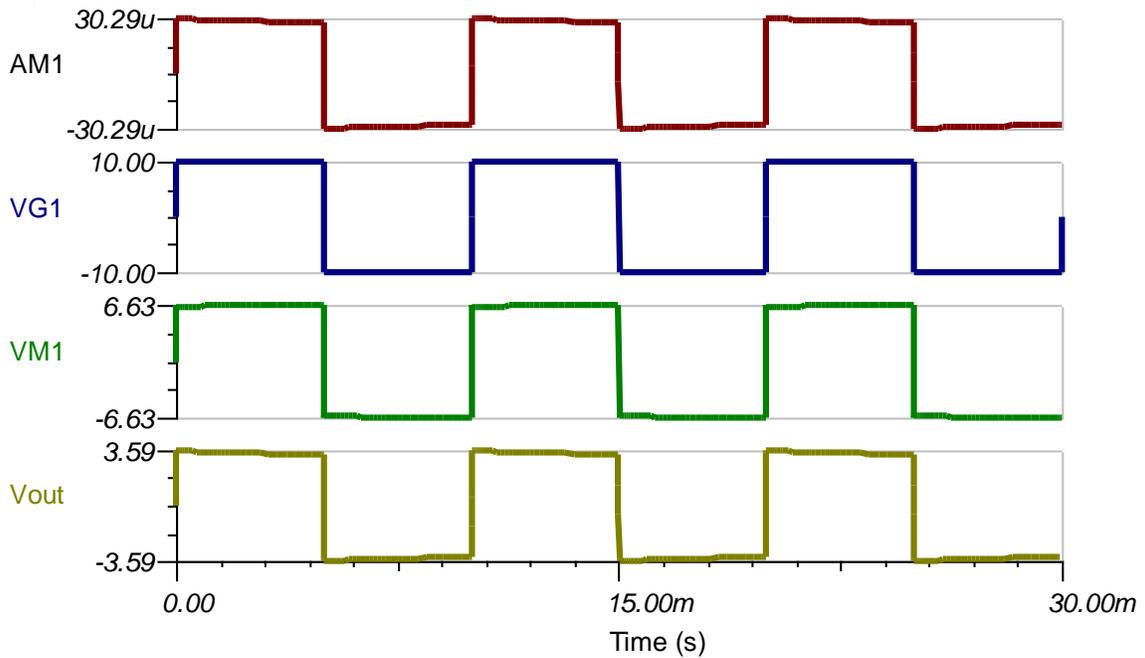
**Fig. 11.35: Zo\_Lo\_f Calculation from Datasheet Aol Tests**

One final trick for our External Zo Block is shown in Figure 11.36. When the External Zo Block is used in an op amp SPICE circuit it is advisable to bound the possible voltages coming out of the External Zo Block to slightly above (typically +/-0.7V) or below the positive and negative supplies used on the respective op amp for which the External Zo Block was created. This helps prevent convergence issues in SPICE especially since VCVS2 in Figure 11.36 can potentially produce very large voltages. D1 and D2 are diode clamps to keep the output of the External Zo Block within a reasonable range close to the positive and negative supplies used for the simulated application circuit. Vp\_clmp and Vm\_clmp DC Sources in Figure 11.36 can be adjusted to move Vout clamp points as close as desired to the supplies used on the op amp for which the External Zo Block was created.



**Fig. 11.36: Final Zo Block with Added Diode Clamps for SPICE Convergence**

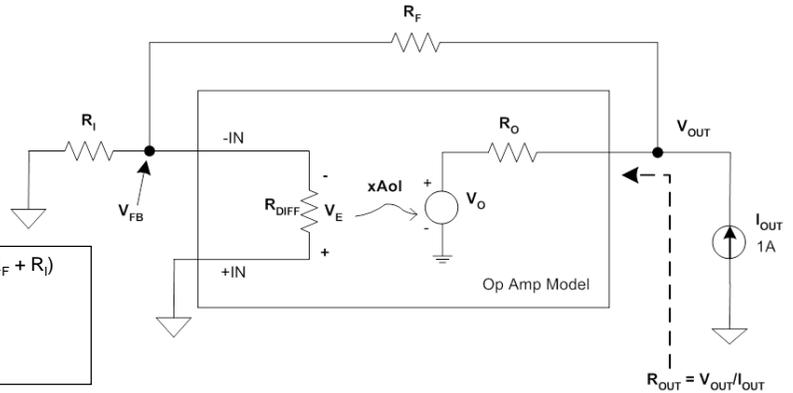
A transient TINA SPICE simulation of our final Zo Block with Added Clamp Diodes is shown in Figure 11.37 and illustrates the point that Vout is clamped within +/-1.1V of the +/-2.5V supplies (Vm\_clmp and Vp\_clmp) used to set the clamp levels. Vm\_clmp and Vp\_clmp may be adjusted as desired to raise or lower these clamping levels.



**Fig. 11.37: Final Zo Block with Added Diode Clamps for SPICE Convergence Test**

## APPENDIX: DETAILED DERIVATIONS

### $R_O$ and $R_{OUT}$ Derivation

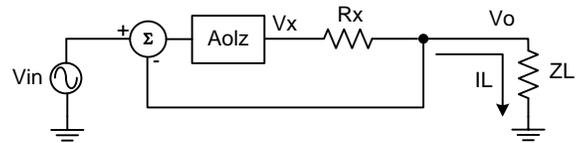


- 1)  $\beta = V_{FB} / V_{OUT} = [V_{OUT} (R_i / (R_f + R_i))] / V_{OUT} = R_i / (R_f + R_i)$
- 2)  $R_{OUT} = V_{OUT} / I_{OUT}$
- 3)  $V_O = -V_E A_{OL}$
- 4)  $V_E = V_{OUT} [R_i / (R_f + R_i)]$

- 5)  $V_{OUT} = V_O + I_{OUT} R_O$
- 6)  $V_{OUT} = -V_E A_{OL} + I_{OUT} R_O$  Substitute 3) into 5) for  $V_O$
- 7)  $V_{OUT} = -V_{OUT} [R_i / (R_f + R_i)] A_{OL} + I_{OUT} R_O$  Substitute 4) into 6) for  $V_E$
- 8)  $V_{OUT} + V_{OUT} [R_i / (R_f + R_i)] A_{OL} = I_{OUT} R_O$  Rearrange 7) to get  $V_{OUT}$  terms on left
- 9)  $V_{OUT} = I_{OUT} R_O / \{1 + [R_i A_{OL} / (R_f + R_i)]\}$  Divide in 8) to get  $V_{OUT}$  on left
- 10)  $R_{OUT} = V_{OUT} / I_{OUT} = [I_{OUT} R_O / \{1 + [R_i A_{OL} / (R_f + R_i)]\}] / I_{OUT}$   
Divide both sides of 9) by  $I_{OUT}$  to get  $R_{OUT}$  [from 2)] on left
- 11)  $R_{OUT} = R_O / (1 + A_{OL} \beta)$  Substitute 1) into 10) ←

From: Frederiksen, Thomas M. Intuitive Operational Amplifiers. McGraw-Hill Book Company. New York. Revised Edition. 1988.

### $Z_o$ forward Derivation



#### $Z_o$ Forward Derivation

Eq1:  $V_X = (V_{in} - V_o) \cdot A_{olz}$

Eq2:  $V_o = \frac{V_X Z_L}{R_X + Z_L}$

Eq3:  $I_L = \frac{V_o}{Z_L}$

Eq4:  $Z_{o\_fwd} = \frac{V_{in} - V_o}{I_L}$

Solve for  $V_X$

Substitute Eq2 into Eq1

$$V_X = \left( V_{in} - \frac{V_X Z_L}{R_X + Z_L} \right) \cdot A_{olz}$$

$$\frac{V_X}{A_{olz}} = V_{in} - \frac{(V_X Z_L)}{R_X + Z_L}$$

$$\frac{V_X}{A_{olz}} + \frac{V_X Z_L}{R_X + Z_L} = V_{in}$$

Eq5:  $V_X = \frac{V_{in}}{\frac{1}{A_{olz}} + \frac{Z_L}{R_X + Z_L}}$

Solve for  $V_o$

Substitute Eq5 into Eq2

Eq6:  $V_o = \frac{\left( V_{in} \cdot \frac{Z_L}{R_X + Z_L} \right)}{\frac{1}{A_{olz}} + \frac{Z_L}{R_X + Z_L}}$

## Zo\_forward Derivation (cont.)

Solve for Zo\_fwd  
Substitute Eq3 into Eq4

$$\text{Eq7: } Z_{o\_fwd} = \frac{V_{in} - V_o}{\left(\frac{V_o}{Z_L}\right)}$$



Solve for Zo\_fwd  
Substitute Eq6 into Eq7

$$Z_{o\_fwd} = \frac{\left[ V_{in} - \frac{\left( V_{in} \cdot \frac{Z_L}{R_x + Z_L} \right)}{\frac{1}{A_{olz}} + \frac{Z_L}{R_x + Z_L}} \right]}{\frac{\left( \frac{V_{in} \cdot \frac{Z_L}{R_x + Z_L} \right)}{\frac{1}{A_{olz}} + \frac{Z_L}{R_x + Z_L}} \cdot \frac{1}{Z_L}}$$

Divide numerator and denominator by

$$Z_{o\_fwd} = \frac{\left[ 1 - \frac{\left( \frac{Z_L}{R_x + Z_L} \right)}{\frac{1}{A_{olz}} + \frac{Z_L}{R_x + Z_L}} \right]}{\frac{\left( \frac{Z_L}{R_x + Z_L} \right)}{\frac{1}{A_{olz}} + \frac{Z_L}{R_x + Z_L}} \cdot \frac{1}{Z_L}}$$

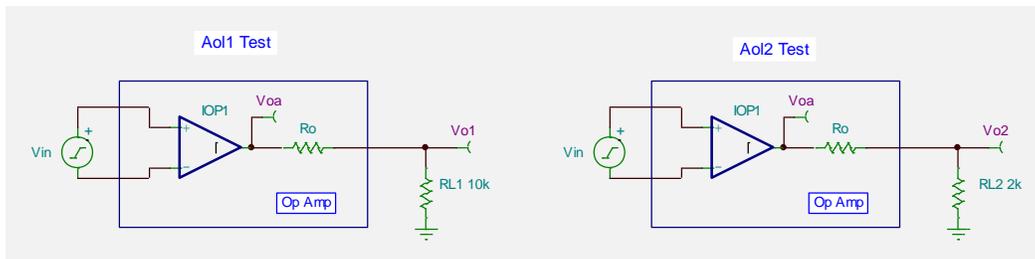
Multiply numerator and denominator by  $\left[ \frac{1}{A_{olz}} + \frac{Z_L}{R_x + Z_L} \right]$

$$Z_{o\_fwd} = \frac{\frac{1}{A_{olz}} + \frac{Z_L}{R_x + Z_L} - \left( \frac{Z_L}{R_x + Z_L} \right)}{\left( \frac{Z_L}{R_x + Z_L} \right) \cdot \frac{1}{Z_L}}$$

$$Z_{o\_fwd} = \frac{\frac{1}{A_{olz}}}{\left( \frac{1}{R_x + Z_L} \right)}$$

$$\text{Eq8: } Z_{o\_fwd} = \frac{R_x + Z_L}{A_{olz}}$$

## Zo\_Lo\_f Calculation from Aol Tests Derivation



$$\text{Eq1.1: } V_{o1} = \frac{V_{oa} \cdot R_{L1}}{R_o + R_{L1}}$$

$$\text{Eq 2.1: } A_{ol1} = \frac{V_{o1}}{V_{in}}$$

$$\text{Eq 3.1: } V_{oa} = V_{o1} \cdot \frac{R_o + R_{L1}}{R_{L1}} \quad \text{From Eq 1.1}$$

$$\text{Eq 4.1: } V_{o1} = A_{ol1} \cdot V_{in} \quad \text{From Eq 2.1}$$

$$\text{Eq 5.1: } V_{oa} = (A_{ol1} \cdot V_{in}) \cdot \frac{R_o + R_{L1}}{R_{L1}} \quad \text{Substitute EQ 4.1 into EQ 3.1}$$

$$\text{Eq 6.1: } \frac{V_{oa}}{V_{in}} = A_{ol1} \cdot \frac{R_o + R_{L1}}{R_{L1}} \quad \text{Divide by } V_{in}$$

$$\text{Eq1.2 } V_{o2} = \frac{V_{oa} \cdot R_{L2}}{R_o + R_{L2}}$$

:

$$\text{Eq 2.2: } A_{ol2} = \frac{V_{o2}}{V_{in}}$$

$$\text{Eq 3.2: } V_{oa} = V_{o2} \cdot \frac{R_o + R_{L2}}{R_{L2}} \quad \text{From Eq 1.2}$$

$$\text{Eq 4.2: } V_{o2} = A_{ol2} \cdot V_{in} \quad \text{From Eq 2.2}$$

$$\text{Eq 5.2: } V_{oa} = (A_{ol2} \cdot V_{in}) \cdot \frac{R_o + R_{L2}}{R_{L2}} \quad \text{Substitute EQ 4.2 into EQ 3.2}$$

$$\text{Eq 6.2: } \frac{V_{oa}}{V_{in}} = A_{ol2} \cdot \frac{R_o + R_{L2}}{R_{L2}} \quad \text{Divide by } V_{in}$$



## Zo\_Lo\_f Calculation from Aol Tests Derivation (cont.)

Set EQ 6.1 = EQ 6.2

$$\text{EQ 7: } A_{o1} \cdot \frac{R_o + RL1}{RL1} = A_{o2} \cdot \frac{R_o + RL2}{RL2}$$

$$\frac{A_{o1}}{A_{o2}} = \frac{R_o + RL2}{RL2} \cdot \frac{RL1}{R_o + RL1}$$

$$\text{EQ 8: } \frac{A_{o1}}{A_{o2}} \cdot \frac{RL2}{RL1} = \frac{R_o + RL2}{R_o + RL1}$$

$$\text{EQ 9: } X = \frac{A_{o1}}{A_{o2}} \cdot \frac{RL2}{RL1}$$

$$\text{Eq 10: } X = \frac{R_o + RL2}{R_o + RL1} \quad \text{Substitute Eq 9 into Eq 8}$$

$$\text{Eq 11: } R_o = \frac{RL2 - X \cdot RL1}{X - 1} \quad \text{Solve Eq 10 for } R_o$$



$$\text{Eq 12: } R_o = \frac{RL2 - \left( \frac{A_{o1}}{A_{o2}} \cdot \frac{RL2}{RL1} \right) \cdot RL1}{\left( \frac{A_{o1}}{A_{o2}} \cdot \frac{RL2}{RL1} \right) - 1} \quad \text{Substitute Eq 9 into Eq 11}$$

### Acknowledgements:

For his countless years of technical dialogue and discussions with the “Wizard of Zo” (aka the author) about Zo - Bill Sands, Analog & RF Models, Purveyor of fine SPICE Macromodels of anything Analog. For SPICE Macromodel Development contact Bill through <http://www.home.earthlink.net/~wksands/>

For his being the first IC Design Engineer to work cooperatively with the author on Zo at the IC simulation level and measure without argument, question, or protest Zo on his op amp - Stephen Sanchez, Manager Design Engineering Apex Precision Power, a division of Cirrus Logic, Inc.

For their easy to use SPICE Simulator – DesignSoft, TINA SPICE link at <http://www.tina.com/>

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After earning a BSEE from the University of Arizona, in 1981, Tim Green has worked as an analog and mixed signal board/system level design engineer for over 30 years, including brushless motor control, aircraft jet engine control, missile systems, power op amps, data acquisition systems, CCD cameras, and analog/mixed signal semiconductors. Tim's recent experience is focused on Power Audio for the automotive market. He is currently a Senior Staff Systems Engineer for Audio Power Products located in Tucson, Arizona at Apex Precision Power, a division of Cirrus Logic, Inc. Tim may be reached at [tim.green@cirrus.com](mailto:tim.green@cirrus.com)