# High-precision composite op-amps

**New technique using cascaded op-amps gives low vector error using simple feedback networks without component matching.**

JOHN D YEWEN

Several recent articles have discussed amplifier configurations with reduced vector error for a given overall bandwidth (see references). The transfer functions of these systems have the first N terms of the numerator and denominator equal for an Nth-order system, for example a third-order system would be of the form

$$\frac{2s^2+2s+1}{s^3+2s^2+2s+1}$$

and in servo terms would have zero position, velocity and acceleration error, giving a finite error only when the input function has a non-zero third derivative.

It can also be seen from the form of the transfer function that for low frequencies, the error term varies as the cube of the frequency and can thus be made very small for operating frequencies relatively near the cut-off frequency where $s \approx 1$.

Previous articles have described non-inverting configurations and rely on matching resitor networks and, for the most part, matching amplifier gains.

The circuits described here are inverting circuits where the gain is set by only two components, which may be resistive or reactive, so that filters and integrators may be made. In addition there is a true virtual earth, so that signals may be summed if required.

The matching of the transfer function numerator and denominator terms is inherent in the circuit topology, and is independent of the precise values of either the resistors or the amplifier cut-off frequencies.

The generalised circuit schematic is shown in Fig.1 For an Nth order system there will be N separate forward paths between the amplifier virtual earth input and the output.

The ω terms are cut-off frequencies of the op-amps in rad/s, and the passive interstage attenuators are used to shape the closed-loop transfer function into a stable and useful form, as shown in the worked third-order example following.

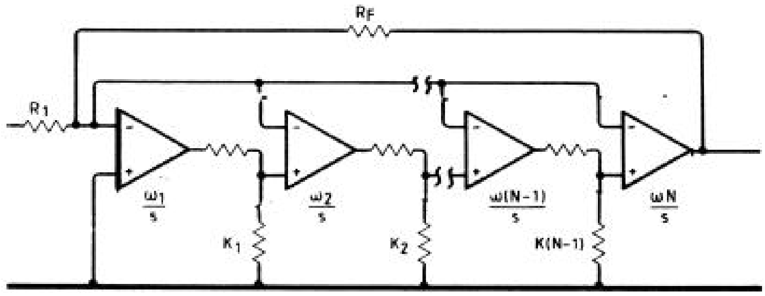


**Fig.1. Nth-order circuit schematic.**

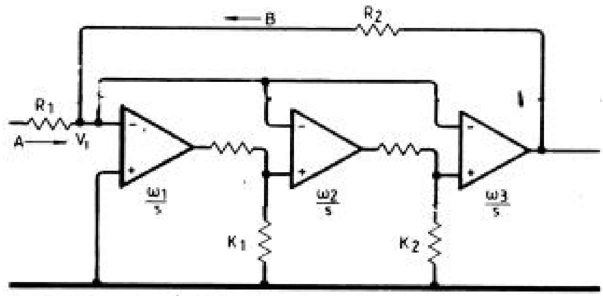


**Fig.2. Third-order system.**

## TRANSFER FUNCTION

Referring to Fig.2 the open-loop transfer function is

$$\frac{V_{out}}{V_{in}}=\frac{\omega_3}{s}+\frac{K_2\omega_2\omega_3}{s^2}+\frac{K_1K_2\omega_1\omega_2\omega_3}{s^3}$$

$$=\frac{s^2\omega_3+sK_2\omega_2\omega_3+K_1K_2\omega_1\omega_2\omega_3}{s^3}$$

So putting in the input and feedback path gains A and B, multiplying top and bottom by $s^3$, and using the standard servo closed-loop formula

$$\frac{G(s)}{1+H(s)G(s)}$$

we get $V_{out}/V_{in}=$

$$\frac{A}{B}\cdot\frac{s^2\omega^3+sK_2\omega_2\omega_3+K_1K_2\omega_1\omega_2\omega_3}{s^3/B+s^2\omega^3+sK_2\omega_2\omega_3+K_1K_2\omega_1\omega_2\omega_3} \quad (1)$$

The last three terms match identically in the numerator and denominator as a direct consequence of the term G(s) in the numerator and denominator of the servo closed-loop formula above, and the matching is totally independent of the actual values K, ω, A and B.


SUMMARY

**The use of cascaded op-amps in a new circuit configuration enables precision inverting amplifier circuits to be constructed using standard feedback networks. The completed circuits have a very low phase and amplitude vector error, and, unlike previously published circuits, do not require elaborate matched feedback networks to maintain high accuracy, are easy to use, and have lower output impedance. In addition, they are also suitable for use in summing, integrating and filtering applications.**

To produce a stable system, the transfer function denominator polynomial must meet the Routh-Hurwitz stability criterion, and in practice must exceed this by a safe margin.

Suitable normalized coefficients could be the binomial series 1,1; 1,2,1; 1,3,3,1; etc. giving coincident negative real poles. A somewhat better approach would be to use the standard Butterworth filter denominator coefficients, which would give better frequency domain performance, at the expense of poorer settling time and transient response.

## WORKED EXAMPLE

Taking the third-order close-loop transfer function (eqn 1). Choose a unity-gain inverter as an example, giving B = ½. Chose third-order Butterworth denominator coefficients 1,2,2,1). Assume that individual amplifier frequency responses are the same – this is not essential but simplifies the example – so $\omega_1 = \omega_2 = \omega_3 = \omega$.

Because we want a normalized form, multiply through by B giving unity for our first denominator term:

Now re-normalize to $\omega_0 = \omega/4$ giving $\omega = 4\omega_0$ which gives the second term equal to 2, as required:

$$s^3 + 2\omega_0 s^2 + 8K_2\omega_0^2 s + 32K_1K_2\omega_0^3,$$

from which $k_2$ = ¼ and $k_1$ = ⅛ to give the other two terms equal to 2 and 1 respectively.

Replacing these values in the original closed-loop transfer function (1) and putting B = ½. A = ½ gives

$$s^3+\tfrac{1}{2}\omega s^2 + \tfrac{1}{2}K_2\omega^2 s + \tfrac{1}{2}K_1K_2\omega^3.$$

$$\frac{s^2\omega+\frac{1}{4}s\omega^2+\frac{1}{32}\omega^3}{2s^3+s^2\omega+\frac{1}{4}s\omega^2+\frac{1}{32}\omega^3}.$$

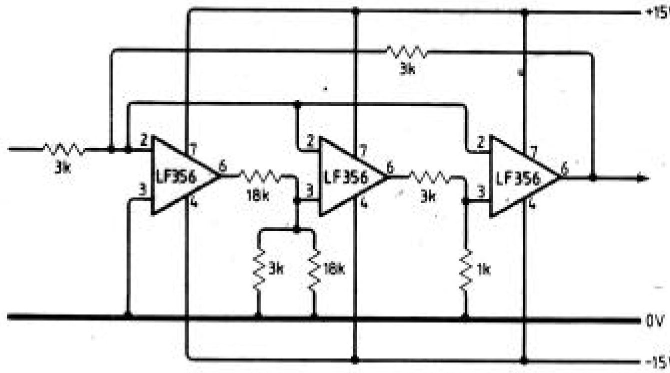


**Fig.3. Completed circuit for worked example.**

At low frequencies the $s^2$ and s terms are negligible compared with the constant term, and the error is represented by the $s^3$ term, where the denominator differs from the numerator. So the low frequency error is approximately.

$$2s^3/(\tfrac{1}{32})\omega^3=(4s/\omega)^3.$$

So using LF356 amplifiers ($\omega \approx$ 4MHz) for example at a frequency of 100kHz in the circuit we get

$$(4\times 100\text{kHz}/4\text{MHz})^3=1/1000$$

i.e. one part per thousand error.

The circuits can be adapted to work at any closed-loop gain; use a variety of amplifiers, not necessarily all identical; have their transfer functions tailored to suit specific applications; and be made of any required order to meet a given specification.

In addition, as large amounts of negative feedback are being applied they have lower closed-loop output impedances than would otherwise be the case.


**References**

Reducing amplifier distortion. A.M. Sandman *Wireless World* October 1974 p.367.

Analgoue Applications Seminar Notes, Precision Monolithics Inc. September 1986.

Active op-amp compensation. A.M. Soliman, *Electronics & Wireless World* November 1986, p.49,50.



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# BOOKS

**Newnes Electronics Pocket Book,** 5th edition, edited by E.A. Parr. Heinemann, 315 pages 95×196mm, hard cover, £8.95. Useful and very portable reference book covering all aspects of the subject from fundamental physics to fault-finding. This edition includes numerous revisions, including new material on op-amp applications and design of digital circuits. Emphasis is on communications and computing, with an entire new chapter on the latter. Not to be confused with the same publisher's Radio and Electronic Engineer's Pocket Book (E&WW, July 1986, p.21), which duplicates very little of the same territory.

**BBC Annual Report and Handbook 1987.** BBC Publications, 291 pages, soft cover, £8. Review of the BBC's activities in 1986 with many colour photographs of the year's radio and television programme highlights. Also included is a great deal of reference material – such as, for example, where to apply for a job. The editors devote 37 pages to the accounts and only about half that to engineering matters, which no doubt shows the way things are these days.

**British Television: the formative years** by R.W. Burns. IEE History of Technology series 7, Peter Peregrinus in association with the Science Museum, London; 488 pages, hard cover, £38.40. Detailed academic history of the medium from Baird's early experiments to the establishment of the Marconi-EMI electronic system and to the suspension of the service at the outbreak of war in 1939, with many photographs and drawings. Absorbing reading for Baird-watchers, since inevitably much of the content deals with the Scottish inventor's endless squabbles with the General Post Office and the BBC; Professor Burns has much more patience with him than others have shown. No doubt it was clear to many that the man was caught in an evolutionary blind alley and that his spinning discs offered no possibility of a service with real entertainment value; but it was hardly dignified of P.P. Eckersley, the BBC's chief engineer to prejudice his Board against the system before they had seen it working. Inevitably, the reader is left filled with admiration for Shoenberg, Blumlein and the rest of the EMI team, who conjured up what is essentially the television standard we have today in the space of a few months.

Baird, incidentally, showed great animosity towards his rivals, which he saw as an American-inspired (EMI had access to RCA patents) campaign to undermine his all-British invention. A parable for our time?

## DOUBLE-NOTCH FILTERING

An unfortunate error in the January part of this article made nonsense of Mr Sokol's argument. The affected part was the first equation on page 47 and the correct version is

$$H(s)=\frac{s^2-\omega^2}{s^2+(\omega/Q)s+\omega^2}$$

Our apologies to readers and to Mr Sokol.