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Harmonic Distortion in Electret Microphones

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ABSTRACT

Using the harmonic-balance method, an approximate solution for the differential equation describing the relationship between the output voltage of the electret microphone and its air-gap thickness is obtained. Closed form expressions are derived for the amplitudes and phases of the fundamental, second- and third-harmonic components of the output voltage.

1 INTRODUCTION

Electret microphones have all the good features of conventional condenser microphones, such as wide flat frequency response, low distortion, low sensitivity to external mechanical and electromagnetic signals, good impulse response, and simplicity in design.[1] Moreover, in contrast with the conventional condenser microphones, electret microphones do not use external dc bias and they have a higher capacitance/unit area.[2] Electrets can now be fabricated using polymer films only a few micrometers thick, ideally suited for use as a microphone diaphragm[3] or sensors.[4] The simplified cross-sectional diagram of Fig. 1[5] shows that a microphone using the electret principle is essentially a very simple device comprising a metallized electret diaphragm located a defined distance in front of a conducting back plate. The surface charges δ_1 and δ_2 of the electret, which are considered to be constant, induce charges δ_{i_1} and δ_{i_2} in the metal layers and generate electric fields E_1, E_2 and E_3 in the three dielectric layers with thicknesses d_1, d_2 and d_3 and dielectric permitivities ε_1, ε_2 and ε_3.

If a sound wave impinges on the metallized foil, the thickness d_3 of the air

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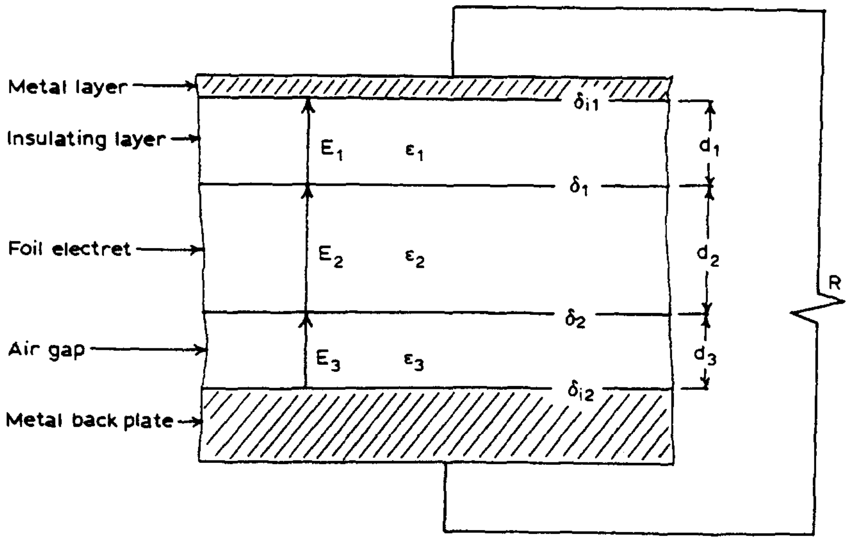

Fig. 1. Small section of electret microphone (not to scale).

gap is changed periodically, thus changing the electric fields and the induced charges, and generating a voltage V across the load resistor R. Applying Gauss's theorem, at the two surfaces of the electret we have

$$\varepsilon_1 E_1 - \varepsilon_2 E_2 = \delta_1 \tag{1}$$

$$\varepsilon_2 E_2 - \varepsilon_3 E_3 = \delta_2 \tag{2}$$

In addition $\oint E ds = 0$ gives

$$V + \sum_{i=1}^{3} E_i d_i = 0 \tag{3}$$

Assuming $\delta_1 = -\delta_2 = -\delta$, $\varepsilon_1 = \varepsilon_2$ setting $d_1 + d_2 = D$ and eliminating E_1 and E_2 from eqns (1–3) then

$$E_3 = \frac{-(\varepsilon_1 V + \delta d_2)}{\varepsilon_3 D + \varepsilon_1 d_3} \tag{4}$$

Now suppose d_3 is changed by a sound wave according to

$$d_3 = d_{3,0} + d_{3,1} \sin wt \tag{5}$$

then the electret output voltage V, determined by the change in δ_{i_2}, is given by

$$V = RA \frac{\mathrm{d}\delta_{i_2}}{\mathrm{d}t} \tag{6}$$

where A is the back-plate area. The induced change δ_{i_2} is determined by E_3, thus

$$\delta_{i_2} = \varepsilon_3 E_3 \tag{7}$$

From eqns (4), (6) and (7) it is easy to show that[5]

$$\frac{\mathrm{d}V}{\mathrm{d}t} + V\left[\frac{\varepsilon_3 D + \varepsilon_1 d_3}{R\varepsilon_1\varepsilon_3 A} - \frac{\varepsilon_1}{\varepsilon_3 D + \varepsilon_1 d_3}\frac{\mathrm{d}}{\mathrm{d}t}(d_3)\right] - \frac{\delta d_2}{\varepsilon_3 D + \varepsilon_1 d_3}\frac{\mathrm{d}}{\mathrm{d}t}(d_3) = 0 \quad (8)$$

By setting

$$\chi = \varepsilon_3 D + \varepsilon_1 d_3 \quad (9)$$

and

$$\frac{\mathrm{d}\chi}{\mathrm{d}t} = \varepsilon_1 \frac{\mathrm{d}}{\mathrm{d}t}(d_3)$$

and eqn (8) reduces to

$$\chi\frac{\mathrm{d}V}{\mathrm{d}t} + V\left[\alpha\chi^2 - \frac{\mathrm{d}\chi}{\mathrm{d}t}\right] - \beta\frac{\mathrm{d}\chi}{\mathrm{d}t} = 0 \quad (10)$$

where $\alpha = 1/R\varepsilon_1\varepsilon_3 A$ and $\beta = \delta d_2/\varepsilon_1$.

Equation (10) is a linear differential equation, for which a solution of the form of eqn (11) can be obtained.[6]

$$V(t) = \mathrm{K}\exp(-\textstyle\int f(t)\,\mathrm{d}t) + \exp(-\int f(t)\,\mathrm{d}t)\int \exp(\int f(t)\,\mathrm{d}t)y(t)\,\mathrm{d}t \quad (11)$$

where K is an arbitrary constant,

$$y(t) = \beta w \varepsilon_1 d_{31}\cos wt$$

and

$$f(t) = \alpha(\varepsilon_3 D + \varepsilon_1 d_{30} + \varepsilon_1 d_{31}\sin wt) - \frac{w\varepsilon_1 d_{31}\cos wt}{\varepsilon_3 D + \varepsilon_1 d_{30} + \varepsilon_1 d_{31}\sin wt}$$

The integration of eqn (11) can be easily performed if the variable part of d_3, i.e. d_{31}, is neglected. This yields a closed-form solution for the electret output voltage given by[5]

$$V(t) = \frac{\delta d_2 d_{31}}{(\varepsilon_3 D + \varepsilon_1 d_{30})(1 + \tan^2\phi)^{1/2}}\sin(wt + \phi) \quad (12)$$

where $\tan\phi = 1/wcR$, $c = \varepsilon_1\varepsilon_3 A/(\varepsilon_3 D + \varepsilon_1 d_3)$. Equation (12) reveals that the electret output voltage is changing linearly with the acoustic input to the electret microphone, i.e. the electret microphone is not exhibiting any harmonic distortion. This is the opposite of what is actually observed in practice, as harmonic distortion in electret microphones has been reported.[3] Although the theory of harmonic distortion in electret microphones has not been studied in the open literature, it is argued that this distortion arises not from the electret but from the transformer used to step-up electret signal voltage.[7] This is very rare these days!

The purpose of this paper is, therefore, to present an approximate solution for eqn (10). This solution permits simple expressions for the harmonic components of the output voltage of the electret microphone.

2 ANALYSIS

As a first approximation, here we assume that the electret output voltage can be expressed as

$$V(t)=\sum_{n=1}^{3} V_n \sin(nwt+\phi_n) \tag{13}$$

Substituting eqns (5), (9) and (13) into eqn (10), we obtain

$$\begin{aligned}(\varepsilon_3 D+\varepsilon_1 d_{30}+\varepsilon_1 d_{31}\sin wt)\sum_{n=1}^{3} nwV_n\cos(nwt+\phi_n)+\sum_{n=1}^{3} V_n\sin(nwt+\phi_n)\\ \times[\alpha(\varepsilon_3 D+\varepsilon_1 d_{30}+\varepsilon_1 d_{31}\sin wt)^2-\varepsilon_1 d_{31}w\cos wt]-\beta\varepsilon_1 d_{31}w\cos wt=0\end{aligned} \tag{14}$$

Using the principle of harmonic balance,[6] six nonlinear equations in six unknowns V_n, ϕ_n, $n=1$–3 can be obtained. In general these equations can be solved using iterative procedures. However, by assuming that V_2 and V_3 are much smaller than V_1; which is usually the case, the phase shift ϕ_n is sufficiently small that

$$\sin\phi_n\simeq\phi_n \qquad \cos\phi_n\simeq 1$$

and neglecting second-order terms, the nonlinear equations can be reduced to linear equations yielding,

$$V_1=\frac{\beta\varepsilon_1 d_{31}w}{w(\varepsilon_3 D+\varepsilon_1 d_{30})+\alpha[(\varepsilon_3 D+\varepsilon_1 d_{30})^2+\frac{1}{4}\varepsilon_1^2 d_{31}^2]\phi_1} \tag{15}$$

$$\phi_1=\frac{\alpha(\varepsilon_3 D+\varepsilon_1 d_{30})^2+\frac{3}{4}\alpha\varepsilon_1^2 d_{31}^2}{w(\varepsilon_3 D+\varepsilon_1 d_{30})} \tag{16}$$

$$\frac{V_2}{V_1}=\frac{\alpha\varepsilon_1 d_{31}}{2w} \tag{17}$$

$$\phi_2=\frac{\alpha[(\varepsilon_3 D+\varepsilon_1 d_{30})\{(\varepsilon_3 D+\varepsilon_1 d_{30})V_2+\varepsilon_1 d_{31}V_1\phi_1\}+\frac{1}{2}V_2\varepsilon_1^2 d_{31}^2]}{2wV_2(\varepsilon_3 D+\varepsilon_1 d_{30})} \tag{18}$$

$$\frac{V_3}{V_1}=\frac{\alpha(\varepsilon_3 D+\varepsilon_1 d_{30})\varepsilon_1 d_{31}(V_2/V_1)-\frac{1}{4}\alpha\varepsilon_1^2 d_{31}^2\phi_1}{3w(\varepsilon_3 D+\varepsilon_1 d_{30})} \tag{19}$$

$$\begin{aligned}\phi_3=\{&\tfrac{1}{2}\varepsilon_1 d_{31}V_2 w+\alpha(\varepsilon_3 D+\varepsilon_1 d_{30})[(\varepsilon_3 D+\varepsilon_1 d_{30})V_3+\varepsilon_1 d_{31}V_2\phi_2]\\ &+\tfrac{1}{2}\alpha\varepsilon_1^2 d_{31}^2(V_3-\tfrac{1}{2}V_1)\}/\{3wV_3(\varepsilon_3 D+\varepsilon_1 d_{30})\}\end{aligned} \tag{20}$$

Further simplification of eqns (15)–(20) can be obtained by noticing that almost any electret microphone will incorporate a high input impedance preamplifier closely coupled to the electret cartridge.[8] Ideal preamplifiers have input impedance equal to infinity and, therefore, $\alpha \to 0$. Moreover, by neglecting terms in d_{31}^2 eqns (15)–(20) reduce to

$$V_1 = \frac{\delta d_2 d_{31}}{(\varepsilon_3 D + \varepsilon_1 d_{30})} \tag{21}$$

$$\phi_1 = 1/wcR \tag{22}$$

$$\frac{V_2}{V_1} = \frac{\frac{1}{2}\varepsilon_1 d_{31}}{\varepsilon_3 D + \varepsilon_1 d_{30}} \frac{1}{wcR} \tag{23}$$

$$\phi_2 \simeq 1/wcR \tag{24}$$

$$\frac{V_3}{V_1} = \frac{1}{6}\left(\frac{\varepsilon_1 d_{31}}{\varepsilon_3 D + \varepsilon_1 d_{30}}\right)^2 \left(\frac{1}{wcR}\right)^2 \tag{25}$$

$$\phi_3 = \frac{1}{2}\frac{1}{wcR} \tag{26}$$

From eqn (21) it is obvious that the amplitude of the fundamental component of the electret output voltage is linearly proportional to the change in the air-gap thickness, which in turn is proportional to the sound impinging on the metallized foil. Also, it is obvious from eqn (22) that the phase angle of the fundamental component of the output voltage is inversely proportional to the load resistance R. This means that for ideal preamplifiers, with $R = \infty$, the phase shift ϕ_1 will be zero. These are exactly the same conclusions as reached by Sessler.[5] From eqns (23) and (25) it is obvious that the second-harmonic component is predominant. This agrees well with the experimentally observed results.[9] Moreover, it is obvious that while the second-harmonic component increases with the normalized air-gap thickness $\varepsilon_1 d_{31}/(\varepsilon_3 D + \varepsilon_1 d_{30})$ at a rate of 6 dB/Octave, the third-harmonic component increases at a rate of 12 dB/Octave. Both the second- and the third-harmonic components decrease as R increases and becomes zero for $R = \infty$, i.e. an ideal preamplifier with input impedance $= \infty$. It can, therefore, be concluded that a high input-impedance preamplifier is necessary not only to convert the high impedance of the electret element to a more practical lower level and to isolate the electret element from any possible electrical loading due to a following system[8] but also to ensure low harmonic distortion in the output voltage of the electret. Furthermore, since the harmonic distortion decreases with the increase of the parameter c; which in turn is a function of the physical dimensions of the electret (d_1, d_2, d_3 and A) and its dielectric permitivities ($\varepsilon_1 = \varepsilon_2$ and ε_3), this may lead to the selection of design parameters for the electret which may result in minimizing the harmonic distortion.

3 CONCLUSIONS

In this paper an approximate solution has been presented for the differential equation describing the relationship between the output voltage of the electret microphone and its air-gap thickness. This solution resulted in closed-form expressions for the amplitudes and phases of the fundamental, second- and third-harmonic components of the output voltage.

In general the second-harmonic is predominant and increases proportionally with the change in the air-gap thickness at a rate of 6 dB/Octave while the third-harmonic increases at a rate of 12 dB/Octave. The second- and third-harmonic distortion decreases as the input resistance of the preamplifier, usually incorporated with the electret microphone, increases. Therefore, an ideal preamplifier, with input impedance equal to infinity, will have no harmonic distortion. Moreover, the expressions obtained for the harmonic distortion components are in terms of the geometrical and physical parameters of the electret microphone. This may help in optimizing the design of electret microphones to minimize the harmonic distortion.

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