

# Why Circuits Oscillate Spuriously, Part 1: BJT Circuits

by Dennis L Feucht

One of the more troubling aspects of analog circuit design is amplifiers that oscillate when they are not supposed to. Engineers commonly respond to spurious oscillations with empirical efforts to defeat the instability, by throwing small-value resistors, ferrite beads, and bypass capacitors at the problem (whether in a hardware prototype or circuit simulation), hoping it will go away. Perhaps these kludged circuits stop oscillating, but then the performance has been degraded. Or perhaps occasional production units oscillate. What is a circuit designer to do?

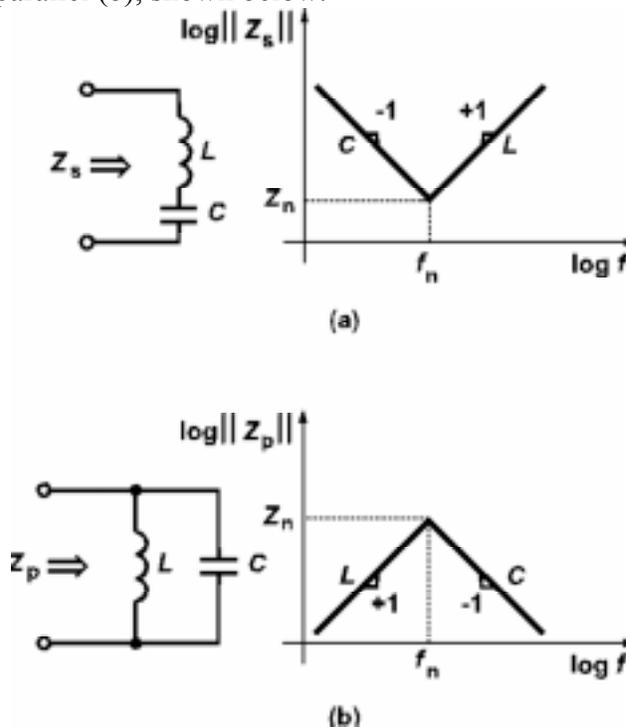
The answer, of course, is to understand why the circuit oscillates. There is always a rational, underlying cause for annoyingly unwanted sinusoids, and finding it can lead not only to a stable circuit, but also a performance-optimized circuit. Not only can the oscillation be modeled and its compensation designed, the margin of stability can also be estimated so that you can know how far from oscillation the circuit design is.

In Part 1 we examine causes and solutions for discrete BJT (bipolar junction transistor) amplifier circuit oscillation. The principles carry over to FETs fairly directly. We will look at two cases: in the first parasitic resonance is the cause; in the second the little-known effect of high-frequency impedance gyration explains why capacitively-loaded emitter followers often oscillate.

## Parasitic Resonance

One of the more annoying (or intriguing, once you become curious instead of frustrated) aspects of circuit design are elements in the design you do not know about. These parasitics arise as integral to components, always imperfect realizations of ideal circuit elements. Sometimes these parasitic elements occur in subtle places, such as board traces, ideally perfect conductors.

Before looking at a BJT amplifier stage let's first review resonant circuits. There are two resonant modes: serial (a) and parallel (b), shown below.



The circuits and their corresponding asymptotic impedance plots are shown.

Define the intersection of the  $L$  and  $C$  plots as the resonant point, and the resonant frequency is:

$$f_n = \frac{1}{2\pi\sqrt{LC}}$$

and the characteristic impedance of the resonance is:

$$Z_n = \sqrt{\frac{L}{C}}$$

$Z_n$  is the reactance of each resonating element at the resonant point. Exact plots require that a vertical asymptote at  $f_n$  be approached on each side by a curve tending to  $\pm\infty$ . For a series resonance,  $Z_s = 0$ , which is at  $-\infty$  on a log-log reactance plot. For a parallel resonance,  $Z_p \rightarrow \infty$ , which is at  $+\infty$  on a reactance plot.

A characteristic feature of resonance is a  $\pm 2$  change of slope on the plot:  $+2$  for a series and  $-2$  for a parallel resonance. In the case of LC circuits, this is a change from  $\pm 1$  to  $\mp 1$ . However, changes from  $\pm 2$  to zero or zero to  $\pm 2$  can also cause resonance and provide the potential for oscillation if not sufficiently damped.

The amount of damping of an RLC resonance is characterized by the damping ratio,  $\zeta$ . For parallel resonances, this is:

$$\text{parallel resonance} \quad \zeta = \frac{Z_n}{2R}$$

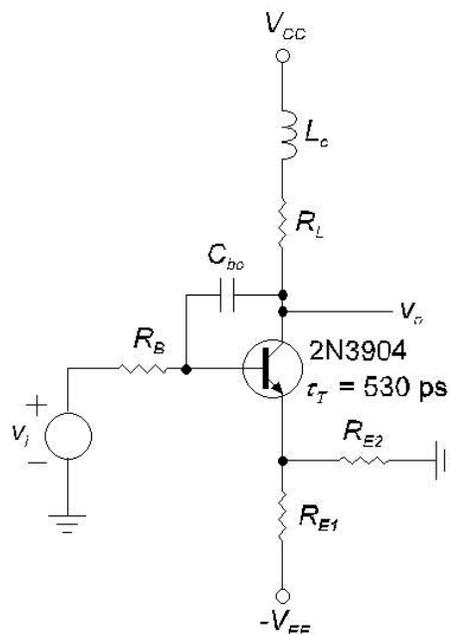
And for series resonances, it is:

$$\text{series resonance} \quad \zeta = \frac{R}{2Z_n}$$

For critical damping,  $\zeta = 1$ . Then  $R_p = Z_n/2$  and  $R_s = 2 \cdot Z_n$ . In both cases, critical damping is achieved by a resistance equal to the combined circuit reactance of the  $L$  and  $C$  at resonance.

An estimate can therefore be made on the reactance plot as to how well a resonance is damped. For a parallel resonance the parallel resistance must be below the  $LC$  resonant  $Z_n$  to be well-damped; for series resonance, it must be above  $Z_n$ .

Spurious resonance in a BJT circuit can exist when an inductance and capacitance inadvertently interact with BJT gain. A common example of how this arises is shown by the following circuit.



The oscillating common-emitter amplifier stage is typically designed for speedy performance, and consequently is driven by a low-impedance source, with small values of  $R_B$  and also  $R_L$ . In this case, the designer did not adequately bypass the collector supply,  $V_{CC}$ . The top of  $R_L$  is connected to a board trace that eventually connects back to a bypass capacitor connected to ground, completing the output loop (and in this case, also the input loop) of the circuit. The long trace introduces the parasitic inductance shown in the circuit diagram as  $L_c$ .

So far the extra collector inductance has the advantage of peaking the circuit, increasing its speed (bandwidth) somewhat. That's not bad; a free inductive-peaking bandwidth extension from a parasitic element. The approximation for trace or wire inductance we used at Tektronix years ago was 10 nH/cm. It is hard to analytically derive this number, for it depends on many factors, but it is useful as a roughly approximate "rule of thumb."

Necessary for oscillation is the other parasitic element, the parasitic capacitance of the collector-base junction, shown as  $C_{bc}$  although largely found within the BJT. Because  $C_{bc}$  provides a feedback path to the base, the series resonance formed by  $L_c$  and  $C_{bc}$  might also be amplified, forming an oscillator. This resonance can be damped with sufficient series resistance. In the more common oscillating (unbypassed-collector) emitter-follower,  $R_L$  is near zero. Damping requires that  $R_B$  be made large enough, according to the above damping equation, to eliminate oscillation. If an increase in  $R_B$  diminishes circuit performance, then  $R_L$  can be added instead. Oscillation-damping explains some of those low-value base resistors in amplifier designs -- usually from 10  $\Omega$  to 100  $\Omega$  in value.

If the collector is bypassed to ground with a capacitor that "shorts"  $L_c$ , all the better. Beware though that the capacitor must be a "high-frequency" kind, one which has low series inductance itself. Its inductance will parallel that of  $L_c$  and reduce  $Z_n$ . This is desirable because with a lower  $Z_n$ , it takes less series resistance to damp the resonance. Any unbypassed  $R_L$  will also contribute to the damping.

The series inductance of small ceramic capacitors of typical bypass value from 10 nF to 200 nF is almost entirely in their leads. Chip capacitors are best for high-frequency bypassing if they can be soldered directly across ground and supply nodes. Running traces from chip capacitors defeats the purpose in having no leads.

In addition, as  $L_c$  is decreased  $f_n$  increases. At some higher frequency the BJT runs out of gain and though the resonance is undamped, its frequency is too high to be amplified enough for oscillation to occur. If the  $f_T$  ( $= \frac{1}{2} \cdot \pi \cdot \tau_T$ ) is increased (that is, a faster BJT is used), then  $C_{bc}$  is typically decreased, raising  $Z_n$  and the range of  $f_n$  for which the faster BJT can sustain oscillation. A slower BJT might be less likely to oscillate.

## High-Frequency Resonance

Before those of you who are not "speed freaks" are turned away by the esoteric notion of "high-frequency" (HF) design considerations, the expression is relative to the bandwidth of the amplifying device, and can be as low as 10 Hz, or less, for op amps. One of the great wonders of the electronics industry is that there is a very basic circuit phenomenon which shows up all over the place yet you will be hard-put to find it in any circuits textbook, no less an engineering-school curriculum. (It is covered in my analog circuit design book at length <http://www.innovatia.com>.) The concept is one of the best-kept secrets of the industry. I was blessed to learn of it early at Tektronix, for it is revealing of much that is otherwise mysterious in circuit behavior. What is strange is that it has propagated negligibly from a few places like Tek. Judging from the kind of

articles being published from HP engineers in the 1970s about emitter-follower oscillation, it was not understood too well either at that bastion of circuits knowledge. Nathan Sokol also published some articles in the EE trade journals back then nipping around the edges of the problem. And some IEEE papers recognized the effect but did not offer a simple, general, theory of it (see References). It really ought to be a part of undergraduate active-circuits courses. It is that basic.

Here's the essence of what is being missed: active devices can usually be characterized as having a gain that is flat out to some bandwidth, such as  $f_\beta$  for a BJT or  $f_{bw}$  for an amplifier. Above this break frequency they exhibit single-pole (-20 dB/decade) roll-off in their gain. Then, at some higher frequency, such as  $f_T$  for a BJT or GBW for an amplifier, the gain becomes flat again at a value of one. Between  $f_{bw}$  and  $f_T$  (to use these symbols generally for all such devices), is the high-frequency (HF) region of active-device operation. In this region device impedances are transformed, or gyrated, to become other kinds of circuit elements. For instance, the resistance in the base of a transistor in the HF region is gyrated to appear as an inductance at the emitter. This inductance can (and often does) resonate with emitter capacitance if the resonant frequency is within the HF region.

This HF resonance effect explains why negative voltage regulator ICs will oscillate if not capacitively loaded sufficiently at their outputs, and why emitter-followers (and source-followers and feedback amplifiers) can oscillate when capacitively loaded.

To develop the concept for BJTs, we start by distinguishing between the low-frequency (quasistatic)  $\beta$  and a frequency-dependent  $\beta$  by denoting low-frequency  $\beta \equiv \beta_o$ .

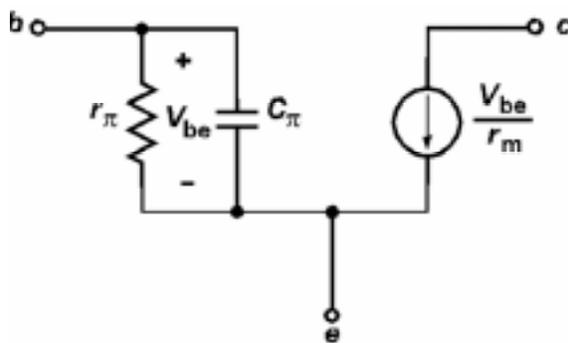
For BJTs, current-gain bandwidth is denoted by  $f_\beta$  and the unity-current-gain frequency by  $f_T$ . They are related by:

$$f_T = \beta_o \cdot f_\beta$$

For a BJT with  $f_T = 300$  MHz and  $\beta_o = 100$ , high-frequency behavior occurs between  $f_\beta = f_T/\beta_o$  and  $f_T$ , or in the HF range, from 3 MHz to 300 MHz. For power BJTs,  $f_\beta$  can be as low as several hundred kilohertz. The open-loop bandwidth of many op-amps is less than 10 Hz when the unity-gain frequency is 1 MHz. This range of rather low frequencies is the op-amp HF region.

### Derivation of BJT High-Frequency Model

We now use a simplified hybrid- $\pi$  BJT model, shown below without  $C_\mu$  or other refinements.



This model is valid for both the low-frequency (LF) and HF regions. The idea of the HF model is that as the BJT input frequency increases above  $1/r_\pi C_\pi$ , a decreasing proportion of base current,  $I_b$ , flows through  $r_\pi$  as the reactance of  $C_\pi$  decreases.

As the base impedance:

$$Z_{\pi} = r_{\pi} \parallel \frac{1}{sC_{\pi}} = \frac{r_{\pi}}{sr_{\pi}C_{\pi} + 1}$$

decreases with frequency,  $V_{be}$  also decreases, resulting in decreased collector current.

Consequently,  $\beta$  also decreases with frequency. The break frequency of  $\beta$  is  $\omega_{\beta} = 1/\tau_{\beta}$ , where:

$$\tau_{\beta} = r_{\pi}C_{\pi}$$

The frequency-dependent form of  $\beta$  is  $I_c(s)/I_b(s)$ . Using the current-divider formula:

$$\beta(s) = \beta_o \left( \frac{1/sC_{\pi}}{1/sC_{\pi} + r_{\pi}} \right) = \frac{\beta_o}{s\tau_{\beta} + 1}$$

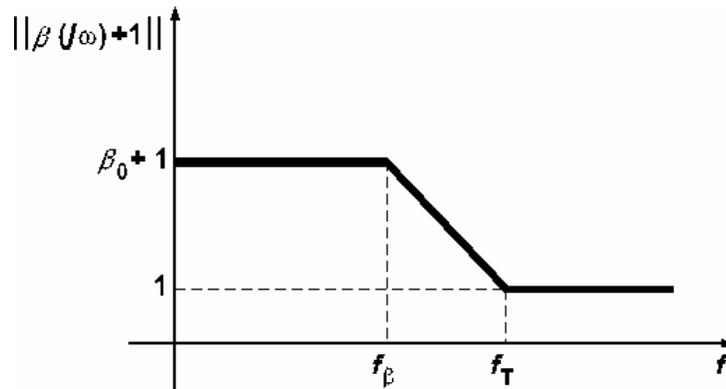
and:

$$\beta(s) + 1 = (\beta_o + 1) \frac{s\alpha_o\tau_T + 1}{s\tau_{\beta} + 1}$$

where:

$$\tau_{\beta} = \beta_o\tau_T$$

The Bode plot of  $\beta(s) + 1$  is shown below.



In the LF region the transistor model does not require reactive elements. In the HF region,  $\beta$  rolls off with frequency, and above  $f_T$  the device has essentially lost its gain (though power gain under the right circuit conditions takes place up to the unity-power-gain frequency,  $f_{MAX}$ ). Other significant factors not accounted for in this model (such as base transit time) cause its error to increase as  $f_T$  is approached. The model predicts less phase shift in  $\beta$  than actually occurs due to other transistor delays yet it is accurate enough to be quite useful.

A simplified model, valid only for the HF region, can be derived by letting  $\beta_o \rightarrow \infty$ . Then:

$$\lim_{\beta_o \rightarrow \infty} \beta(s) = \beta_{hf} = \frac{1}{s\tau_T}$$

These expressions for  $\beta$  can now be used in circuit analysis.

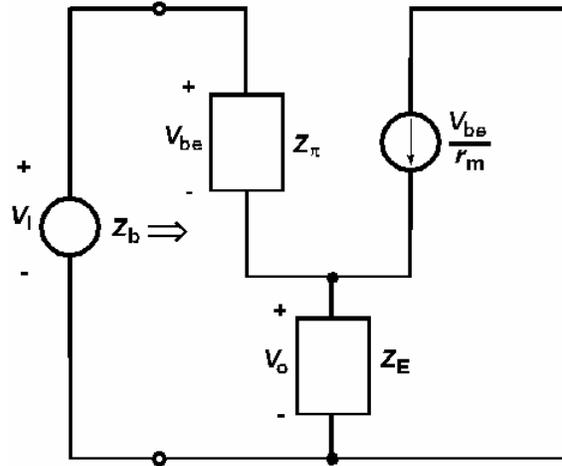
## Impedance Transformations in the High-Frequency Region

The  $\beta$  transform, as applied to non-reactive BJT circuits, can be generalized using  $\beta(s)$  in reactive circuits. The impedance at the base node due to impedance  $Z_E$  at the emitter node is then:

$$Z_b = [\beta(s) + 1] \cdot Z_E$$

and, from the emitter, the impedance in the base circuit,  $Z_B$ , appears at the emitter as:

$$Z_e = \frac{Z_B}{\beta(s) + 1}$$



The corresponding circuit is shown above, from which we can derive  $Z_b$  and verify the above equation for it.

Applying KCL to the emitter node:

$$\frac{V_o}{Z_E} + \frac{V_o - V_i}{Z_\pi} = \frac{V_{be}}{r_m}$$

( $V_{be} = V_i - V_o$ .) Solving for  $Z_b$  gives:

$$Z_b = Z_\pi + Z_E \left( 1 + \frac{\beta_o}{sr_\pi C_\pi + 1} \right)$$

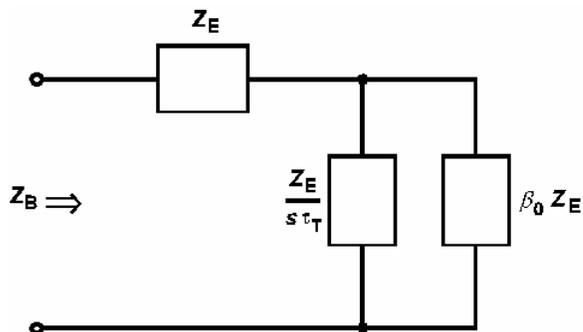
Well above  $f_\beta$ ,  $Z_\pi$  becomes negligible, and:

$$Z_b \cong Z_E \left( 1 + \frac{\beta_o}{sr_\pi C_\pi + 1} \right) = Z_E + \frac{1}{sr_\pi C_\pi / Z_E \beta_o + 1 / Z_E \beta_o}$$

Because  $\tau_\beta = \beta_o \cdot \tau_T$ ,  $Z_b$  can be rewritten as a continued fraction making the topology explicit in equation form:

$$Z_b \cong Z_E + \frac{1}{\frac{1}{Z_E / s\tau_T} + \frac{1}{\beta_o Z_E}}, \quad \tau_T = r_m C_\pi$$

The corresponding circuit topology is shown below.



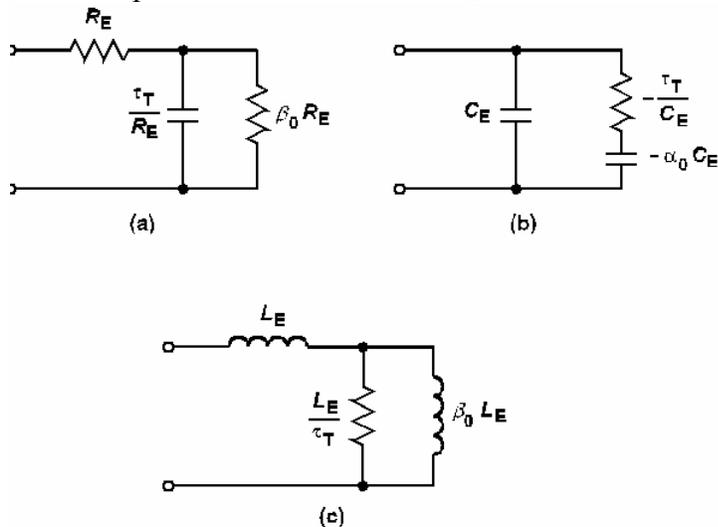
Here  $\beta_o Z_E$  is the LF contribution to  $Z_b$ ,  $Z_E/s\tau_T$  is the HF contribution, and the series  $Z_E$  is common to both. Below  $f_\beta$ ,  $Z_E/s\tau_T$  approaches an open circuit so that  $Z_b$  is consistent with the LF model. In the HF region,  $Z_E/s\tau_T$  dominates  $Z_b$ . Dividing  $Z_E$  by  $s$  gyrates the impedance of  $Z_E$  by  $-90^\circ$  so that:

$$R \rightarrow C$$

$$C \rightarrow -R$$

$$L \rightarrow R$$

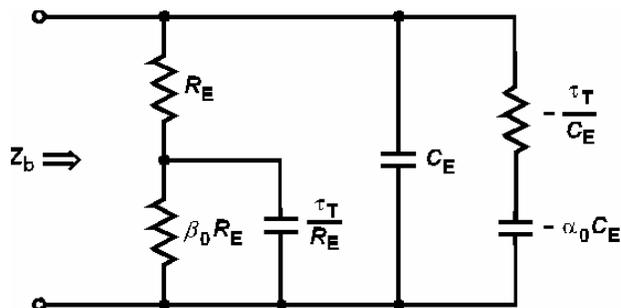
For the three cases of  $Z_E$ , the transformed impedances are shown below:  $R$  in (a),  $C$  in (b), and  $L$  in (c). Because this analysis is linear, combinations of the three elements in  $Z_E$  can be individually transformed and combined to produce the transformed  $Z_E$ .



### Example: Shunt RC-Loaded Emitter-Follower

A common-collector (CC) BJT amplifier has a shunt RC load where  $R_E = 470 \Omega$  and  $C_E = 10 \text{ pF}$ . The BJT has a  $\beta_o = 150$  and  $f_T = 300 \text{ MHz}$  at  $I_E = 10 \text{ mA}$  (typical of a 2N3904). What is  $Z_b$ ?

The combination of gyrated circuits from (a) and (b) above are shown combined below.



For element values, calculate  $\tau_T = \frac{1}{2} \cdot \pi \cdot f_T = 531$  ps and  $\alpha_o = 0.993 \cong 1$ . Then  $\tau_T/R_E = 1.13$  pF and  $-\tau_T/C_E = -53.1$   $\Omega$ . Furthermore,  $\beta_o \cdot R_E = 70.5$  k $\Omega$  and  $-\alpha_o \cdot C_E \cong -10$  pF. An HF equivalent circuit omits  $\beta_o \cdot R_E$ . Whether  $Z_\pi$  is negligible depends on the other elements in the circuit. If base reactance creates a resonance with the emitter impedance near  $f_\beta$ , then  $Z_\pi$  is probably significant. For this circuit,  $r_e \cong 2.6$   $\Omega$  and  $C_\pi = \tau_T/r_e = 204$  pF.

One method of compensating this circuit is to add a series RC circuit from the base to ground that has positive values corresponding to those of the  $-R$ ,  $-C$  branch. These branches cancel, leaving no  $-R$  to form an oscillator.

## Impedance Gyration Viewed From the Emitter

A similar circuit derivation for  $Z_e$  results in

$$Z_e = \frac{Z_\pi + Z_B}{1 + \beta_o / (s\tau_T C_\pi + 1)}$$

Again approximating  $Z_\pi \cong 0$ , we obtain the continued fraction:

$$Z_e \cong \frac{1}{\frac{1}{Z_B} + \frac{1}{s\tau_T Z_B + Z_B / \beta_o}}$$

The topology is the dual of the one above. Below  $f_\beta$ ,  $s\tau_T Z_B$  approaches a short circuit and becomes the LF model. The HF contribution of  $s\tau_T Z_B$  gyrates  $Z_e$  by  $+90^\circ$  so that:

$$R \rightarrow L$$

$$L \rightarrow -R$$

$$C \rightarrow R$$

The expressions for  $Z_b$  and  $Z_e$  are valid from dc to  $f_T$ . A simpler model, applicable only in the HF region, is derived by using  $\beta_{hf}$ . To derive the simplified HF models, assume:

$$\beta_o \rightarrow \infty, \quad Z_\pi = 0$$

Then:

$$Z_b(\text{hf}) = Z_E \left( 1 + \frac{1}{s\tau_T} \right)$$

$$Z_e(\text{hf}) = \frac{Z_B}{1 + 1/s\tau_T}$$

The HF models are based on removal of the break frequency of  $\beta(s) + 1$  at  $f_\beta$  so that  $\beta(s)$  rolls off from infinity at the origin. The expression for  $\beta + 1$  becomes:

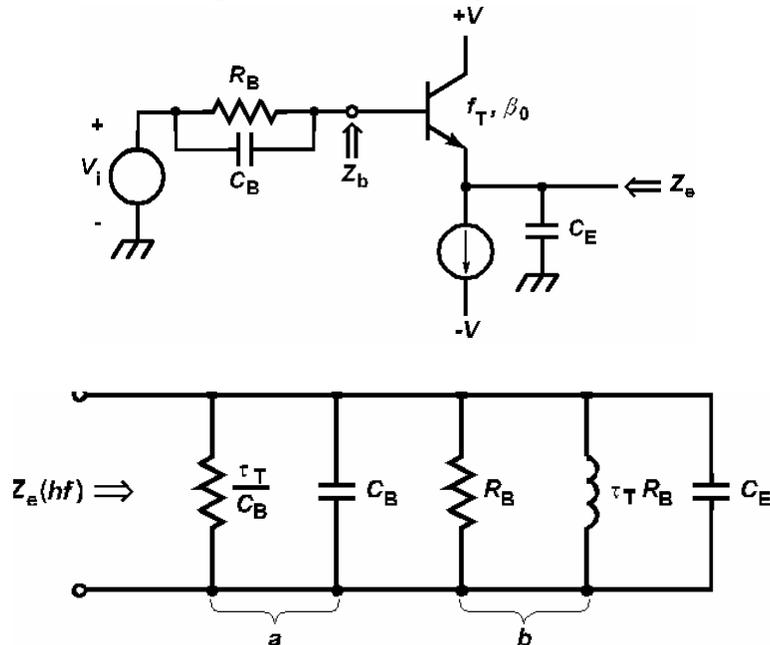
$$\beta_{\text{hf}} + 1 = \frac{1}{s\tau_T} + 1 = \frac{s\tau_T + 1}{s\tau_T}$$

This expression has a pole at the origin which breaks at  $f_T$ . The HF model is consequently not valid in the LF region.

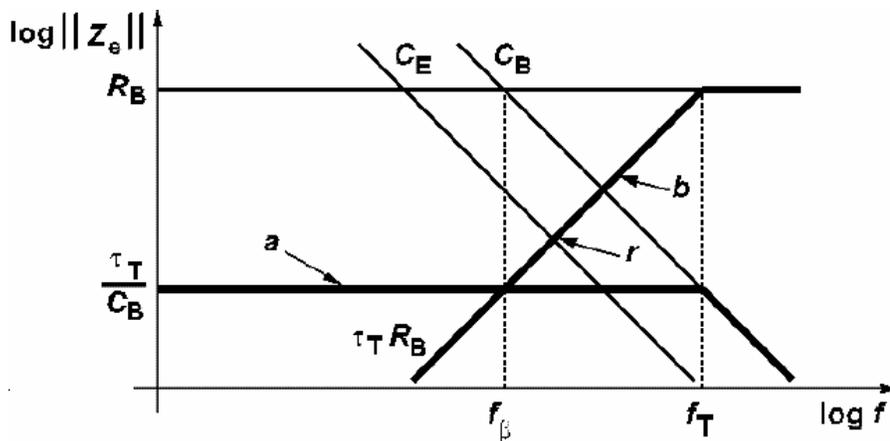
## Emitter-Follower Reactance Plot Stability Analysis

CC stages are commonly used to drive capacitive loads such as transmission lines. The high current gain of the CC configuration allows it to supply high transient currents required to quickly charge the capacitive load. When capacitive loading is combined with base resistance, an HF resonance can occur.

The figure below shows a CC amplifier with its HF equivalent circuit at the emitter node.



The resulting reactance plot for the HF circuit is shown below.



Ordinarily we would combine the parallel  $R$ s and  $C$ s before plotting. Here, sections  $a$  and  $b$  of the HF circuit are plotted separately because the elements within them are interdependent. Section  $a$  of  $Z_e$  is resistive up to  $f_T$  and is capacitive with value  $C_B$  above  $f_T$ . Section  $b$  is inductive up to  $f_T$ , above which it is resistive with value  $R_B$ . On the reactance chart  $R_B$  and  $C_B$  have been chosen so that  $R_B C_B = \tau_\beta$ ; they intersect at  $f_\beta$ .  $C_E$  is much greater than  $C_B$ , and it intersects the plot for section  $b$  at resonant point  $r$ . This is a parallel resonance; a resistance less than the impedance at  $r$ , or  $Z_n$ , is required to damp it. The plot of the impedance from section  $a$  is resistive and less than  $Z_n$  at  $f_n$ ; it damps the resonance. Since this resistance decreases as  $C_B$  increases, then increasing base capacitance tends to stabilize a capacitively-loaded emitter-follower.

From the reactance plot we can see what effect changes in the values of circuit elements have. For section  $a$ , increasing  $C_B$  causes the resistive segment of the plot to move downward and thus

provides more damping at  $r$ . At the same time the diagonal line representing  $C_B$  moves to the left. The break frequency does not move but remains constant at  $f_T$ , so curve  $a$  moves downward as  $C_B$  increases. Similarly, an increase in  $R_B$  increases the inductance below  $f_T$  in curve  $b$  while break frequency  $f_T$  remains fixed. That is,  $R_B\tau_T$  always intersects  $R_B$  at  $f_T$ . Increasing  $R_B$  moves curve  $b$  upward.

From the reactance-plot, we observe that a decrease in  $C_E$  or an increase in  $R_B$  or  $C_B$  tends to stabilize the circuit because  $Z_n$  increases relative to the damping resistance.  $C_E$  has a range in which instability can occur. As  $C_E$  increases its plot moves to the left, and  $r$  moves with it and downward until it intersects curve  $a$  at  $f_\beta$ . Then  $1/sC_E = \tau_T/C_B$  at  $f_\beta$ , and  $r$  is eliminated because impedance gyration does not occur below  $f_\beta$ . This is also true above  $f_T$ . If  $C_E$  decreases until it crosses  $R_B$  at  $f_T$ ,  $r$  vanishes. In addition, transistor gain above  $f_T$  may be insufficient to sustain oscillation anyway. Since the reactance plots are asymptotic approximations, HF effects extend somewhat above and below the HF region.

This analysis assumes that  $C_B \ll C_E$  and  $\tau_T/C_B \ll R_B$ . More generally, as  $C_B$  increases two effects occur: its  $\beta$ -gyrated resistance,  $\tau_T/C_B$ , decreases (increasing damping), and  $C_B$  also adds to  $C_E$ , decreasing  $Z_n$ . Adequate damping can occur only when  $C_E$  dominates. If  $C_E$  becomes negligible relative to  $C_B$ , then the resonance cannot be damped better than  $\zeta = 0.5$ . For maximum circuit speed a minimum  $C_B$  is desirable to minimize the base input time constant.

## Closure

While parasitic-resonance oscillations follow from basic resonance theory applied to active-device gain paths, oscillations caused by HF resonance effects are more complicated, and require development of HF gyration theory. The introduction presented here is a start but there is more to it (in the Dynamic Response volume of Analog Circuit Design at <http://www.innovatia.com>). An equivalent analysis to that given here can be carried out at the base instead of the emitter, with the same results. Various methods of compensating HF resonance effects have been devised, and many of them can be found in the vertical amplifiers of Tektronix' oscilloscopes. Acquisition of some Tek instruction manuals for 1970s-vintage 'scopes will provide industry examples of applied HF theory.

This theory also provides a basis for understanding why feedback-amplifier output impedance increases above the open-loop bandwidth and flattens out again at the GBW. Feedback amplifier outputs are characteristically inductive in their HF region, and for the same basic reason that base resistance appears inductive at the emitter of BJTs in the HF region.

In Part 2, we will move on to reactive loading that causes amplifiers to oscillate because of feedback instability, and we also apply HF theory again in analyzing the amplifier of a commercial pulse generator.

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